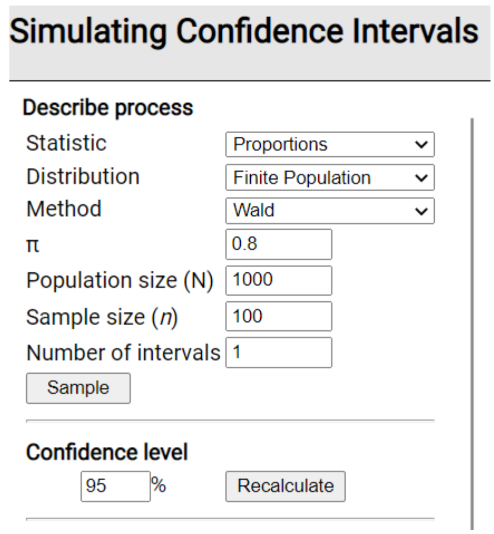
$n \leq (0.10)N$

What Happens When the 10% Condition Isn't Satisfied?

$\sqrt{1-\frac{n}{N}}$

The principal of a school wants to estimate the proportion of students at her school that have internet access at home. To do this, she will select a random sample of students, calculate the proportion of students in the sample with internet access at home, and use this proportion to create a 95% confidence interval. Suppose that 800 of the 1000 students in the school have internet access at home.

1. Launch the *Simulating Confidence Intervals for a Population Parameter* applet at www.rossmanchance.com/applets.



2. Change the Distribution to "Finite Population" and change π (the population proportion) to 0.80, keeping everything else the same. This tells the applet to select a random sample of *n* = 100 students from a population of *N* = 1000 students, where 800/1000 = 0.80 have internet access at home and then construct a 95% confidence interval for *p* using the method taught in AP Statistics (which is sometimes called the Wald method, named for Abraham Wald).

3. Click the Sample button once and notice what happens.

   (a) What do the blue bars in the upper-right represent?

   the number of failures & successes in one sample of 100

   (b) What does the green/red rectangle in the lower-right represent?

   the proportion for that one sample

   (c) Does the confidence interval from this sample capture π = 0.80? How do you know? Click the Sample button a few more times to confirm your description.

   yes because the vertical bar at 0.80 is captured by my interval (no if the value 0.80 is not captured)

4. Change the Number of intervals to 50 and click the Sample button many times. What percent of the intervals captured the population proportion of π = 0.80? How does this compare with the stated confidence level of 95%?

   Running total containing π = 94.8% which is very close to 95%.

In Steps 2–4, both the Random and Large Counts conditions are met. The applet selects a random sample of students and both $np = 100(0.80) = 80$ and $n(1 - p) = 100(1 - 0.80) = 20$ are at least 10. However, we are on the boundary for the 10% condition, as $n = 100$ is exactly 10% of $N = 1000$. To understand why we check the 10% condition, let's violate it and see what happens.

5. In the applet, change the sample size to $n = 300$ (30% of the population!) and click the Sample button. What do you notice about the length of the intervals compared to when $n = 100$? Why does this make sense?

the intervals are narrower because increasing n lowers the margin of error

6. Click the Sample button many times. What percent of the intervals captured the population proportion of $\pi = 0.80$? How does this compare with the stated confidence level of 95%?

Running total containing π = 97.8% which is larger than 95%.

7. Now change the sample size to $n = 1000$ (100% of the population!). Before doing anything else, what percent of the intervals do you think will capture $\pi = 0.80$? Explain your answer.

100% because we are sampling the whole population and every sample proportion will be $\hat{p} = 0.80$.

8. Click the Sample button many times. Was your answer in Step 7 correct?

Yes!

9. Based on your answers in Steps 5–8, what is the result of violating the 10% condition? Is this a big deal?

Increasing the sample size past 10% of the population increases the capture rate past the advertised 95%. This is a good problem to have!

Key takeaway

STOP **OPTIONAL EXTENSION:** In courses beyond AP Statistics, a "correction" factor can be used to account for sampling without replacement, eliminating the need for the 10% condition.

This is called the finite population correction factor = $\sqrt{1-\frac{n}{N}}$.

factor decreases from 1 as sample size is a larger percent of the population

When 10% condition is satisfied: $ME = z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

When 10% condition is not satisfied: $ME = z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\sqrt{1-\frac{n}{N}}$

10. In the Method menu, change from Wald to Finite correction. What happened to the length of the intervals? Why does it make sense that the margin of error = 0 when n = N = 1000?

The intervals shrunk to a single value. When the sample is the whole population, there is no sampling variability.

11. Change the sample size back to n = 300. Click the Sample button many times. What percent of the intervals captured the population proportion of π = 0.80? How does this compare with your answer in Step 6? With the stated confidence level of 95%?

Running total containing π = 95.1%, which is lower than 97.8% from Step 6 & very close to advertised 95%.

12. Change the Method back to Wald (our method in AP Stats) and describe what happens to the length of the intervals.

The intervals are wider with the Wald method.

13. Although more complicated to calculate, what are some benefits of using the finite correction?

① The margin of error is smaller
② The capture rate is close to the advertised confidence level.
③ We don't have to worry about the 10% condition.