

AP Statistics CED 5.1 Daily Video (Skill 1.A)

Introducing Statistics – Why is My Sample Not Like Yours?

What Will We Learn?

Why do different samples produce different statistics when they come from the same population?
How can we estimate the truth?

Dice

Let the random variable x represent the sum of the rolls of 2 fair dice. We would like to take a random sample of 5 rolls and find the sample mean to determine, on average, how far a player would get in 5 turns during a board game.

5 rolls: 7, 4, 9, 3, 12 with \bar{x} = ____ 5 rolls: 4, 3, 10, 3, 6 with \bar{x} = ____ 5 rolls 8, 2, 11, 9, 12, with \bar{x} = ____

Sampling Variability

I think we can agree on a few things:

- Rolling 2 dice produces _____ sums.
- We don't know what results we will get for _____.
- The dice rolls are _____.
- We don't know how close our sample mean for 5 rolls is to the _____ mean of all possible rolls.

A Discrete Random Variable

Let the random variable X represent the sum of the rolls of 2 fair dice. The possible values of X are 2 - 12.

Rolling two 1's is called snake eyes.

$P(\text{snake eyes}) = \frac{1}{36} = \frac{1}{36}$

$P(2 \text{ snake eyes}) = \frac{1}{36} = \frac{1}{36}$

$P(\bar{x} = 2 \text{ when } n=5) \approx \frac{1}{36^5}$

		Die 1 Roll					
		1	2	3	4	5	6
Die 2 Roll	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Sample Means (Using the diagram above)

We can easily roll 2 dice 5 times and calculate the _____.

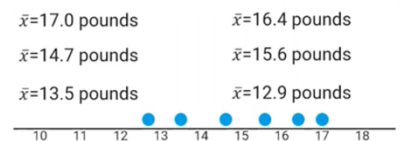
We could determine all possible _____ for 5 rolls.

The sample means _____.

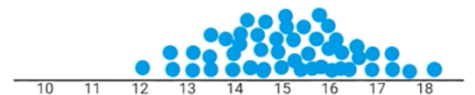
Some sample means are more _____ than others.

Samples from Continuous RVs (random variables)

The weight of adult koalas is about 15 points with a standard deviation of 1.8 pounds. Zoo researchers tracking koala health randomly select a sample of 12 koalas each month and calculate the average weight. Each month they could get a different average weight.



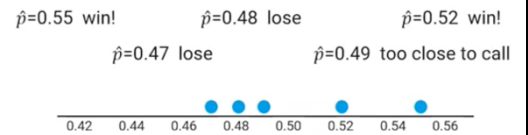
If we could continue to calculate the means of many samples of the same size from the population of all koalas, we would get a distribution of sample means that would help us draw conclusions. And notice that the sample means _____ around the true _____ mean of _____.



Name _____

Sample Proportions

In a close election, a candidate has the support of 51% of the voters. Of course, no one knows this until after the election. Five pollsters each take a random sample of 1,000 voters to determine whether the candidate will win. They find:



The sample proportions will _____. But if we collect _____ sample proportions with a sample size of _____, we start to see a _____ pattern of how the results could look.

How Do Sample Statistics Behave?

Different _____ samples of the _____ from the _____ produce statistics that _____. A statistic from a _____ random sample is not much different that a _____. But, the _____ of a statistic has a _____ pattern of _____ related to the _____.

What Should We Take Away?

Different _____ from the _____ produce _____ statistics. If the _____ of a population is known, then the _____ from many different _____ samples of the _____ from that population is _____.

AP Statistics CED 5.2 Daily Video 1 (Skill 3.A)

The Normal Distribution, Revisited

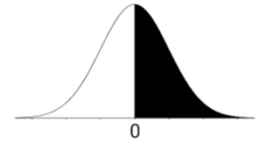
What Will We Learn?

How do we calculate the probability that a particular value lies with a given interval of a normal distribution?

How can we determine the interval associated with a given area in a normal distribution?

The Standard Normal Distribution

In Video 3 of Lesson 1.10, we learned to use z-scores to calculate the proportion of values within a given region of a normal distribution.



Normally Distributed Continuous RVs (random variables)

In video 1 of Lesson 4.7, we learned that continuous random variables can take on an infinite number of values in an interval on a number line. The distribution of all values may be approximately normal.

Proportion of Percent

Looking at the shaded area (above) we could say: The _____ of values above 0 = _____.
OR The _____ of values above 0 is 50%.

Normal Distribution Probabilities

OR, we could say that the _____ of randomly selecting a point in the _____ region of the given normal distribution (above) is _____. Proportion, _____, _____ they all work and they all start with ____.

Giraffes

Giraffes are clearly the tallest living land animal partly due to their lengthy necks. The lengths of adult giraffe necks can be reasonably approximated by a normal distribution with a mean of 5.9 feet and a standard deviation of 0.3 feet. What is the probability that a randomly selected adult giraffe has a neck greater than 6 feet long?

First: define your random variable X.

X = _____

Second: label and shade the graph to the right with the given mean and standard deviation as described in the video.



Third: calculate the z-score using the formula: _____.

Finally: using technology or Table A find: $P(X > 6) =$ _____

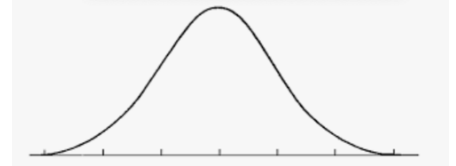
What is important to include in a solution?

- A **defined** random variable.
- A **normal distribution** with **parameters identified**. You can show you used a **normal distribution** by: drawing a sketch as long as you have scaled _____ standard deviations each way; using the _____ coupled with the _____ statement; OR using the _____ statement to clearly state that you are using the normal curve. Identify parameters by simply _____ them; including the _____ in the z-score calculation or including _____ in the technology statement.
- The **value** of interest. You can show this by the _____ in the sketch; by the _____ coupled with the answer or by clearly indicating the _____ and _____ bounds used in the technology statement. (Make sure you identify the values in the technology statement.)
- The **correct** probability

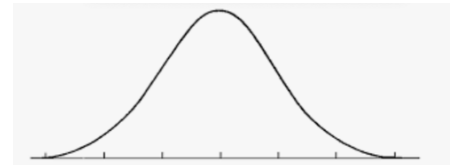
Giraffes (Be sure to stop the video and try these problems before checking!!)

Giraffes are clearly the tallest living land animal partly due to their lengthy necks. The lengths of adult giraffe necks can be reasonably approximated by a normal distribution with a mean of 5.9 feet and a standard deviation of 0.3 feet. What is the probability that a randomly selected adult giraffe has a neck greater than 6 feet long?

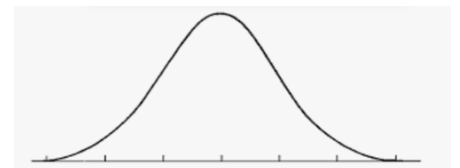
(a) What is the probability that a randomly selected adult giraffe has a neck length less than 5.25 feet?



(b) What is the probability that a randomly selected adult giraffe has a neck length between 5.25 feet and 6.25 feet?



(c) What neck length would place an adult giraffe at the 25th percentile of the distribution?
(Hint this is an **Inverse Normal** problem)



What Should We Take Away?

The probability a value lies within a defined interval of a _____ can be calculated by finding the _____ of the defined interval.

The _____ of an interval associated with a given area in a normal distribution can be determined by using _____ or _____.

AP Statistics CED 5.2 Daily Video 2 (Skill 3.A)

The Normal Distribution, Revisited

What Will We Learn?

When can a linear combination of two random variable be modeled by a normal distribution?
How do we calculate a probability involving such a linear combination?

The Race

The Capitol 10K is a fun run in Austin, Texas, complete with live music along the route, some runners in costumes, but also some very competitive runners. The wide variety of fitness levels creates a large standard deviation of running times for the close to 20,000 participants.

Run Time Parameters

The completion times for each gender are _____ and may be reasonably

approximated by a _____ distribution. The time it takes for each runner to complete the course is measured by a tracking device and recorded for gender.

Females	Males
$X =$ completion time (min) for a randomly selected female.	$Y =$ completion time (min) for a randomly selected male.
$\mu_X = 81$ $\sigma_X = 15.7$	$\mu_Y = 70$ $\sigma_Y = 13.3$

What is the probability that a randomly selected female finishes the race faster than a randomly selected male? (Note: the mean time for males to finish the race is _____; for females it takes them _____ longer.)

First, we need to find the parameters of the distribution of _____ in completion times ($X - Y$). Remember that is ($X - Y$) or (*Females - Males*). Picture randomly selecting a _____ then randomly selecting a male and then _____ their race times. And repeating this process for ALL possible combinations of females and males out of the 20,000 racers.

To find the _____ of the differences, we would just take the _____ of the _____.
 $\mu_{X-Y} = \mu_X - \mu_Y = 81 - 70 = 11$ Thus, on average the males finish the race _____ faster than the females.

To find the standard deviation of the differences you cannot _____ the standard deviations! You must first change them to _____ and then we always _____.

Because X and Y are _____ you have: $\sigma_{X-Y} = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{15.7^2 + 13.3^2} = 20.576$

Shape of the Distribution of $X - Y$.

$$\mu_{X-Y} = 11 \qquad \sigma_{X-Y} = 20.576$$

Still considering the question: What is the probability that randomly selected female finishes the race faster than a randomly selected male?

We know that shape of the distributions of $X - Y$ is _____. The distribution of the _____ of two _____ random variables can be modeled with a _____ if the probability distribution of each random variable is _____.

Note: In the beginning of this problem, we were **given** that the male and female race time were approximately normal. So, we can assume that the distribution of the _____ is also _____.

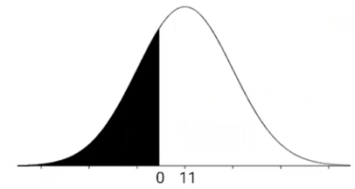
No Difference

What is the probability that a randomly selected female finishes the race faster than a randomly selected male?

$\mu_{X-Y} = 11$ $\sigma_{X-Y} = 20.576$

Using the diagram to the right:

If the randomly selected male and female finish with the _____ time, the difference would be _____.



Understanding the Question

If the female finishes _____, then her time would be _____, making the difference (female-male) _____. So, what we are looking for here is the probability the difference in race time (female-male) is less than _____ or a _____.

Calculating the Probability

What is the probability that a randomly selected female finishes the race faster than a randomly selected male?

$\mu_{X-Y} = 11$ $\sigma_{X-Y} = 20.576$

First, calculate the z-score:

Second, using Table A or technology, find $P(X - Y < 0) =$

Finally, write your answer as a complete sentence.

Shape of the Distribution of X + Y

The distribution of the _____ of two independent random variables can be modeled with a _____ distribution if the probability distribution of each random variable is _____.

$\mu_{X+Y} = \mu_X + \mu_Y$

$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2}$

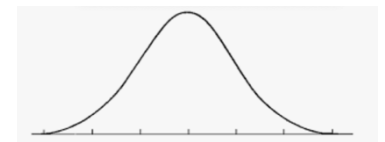
Another 10K

A more competitive 10K has the mean and standard deviations listed to the right for male and female

Females	Males
$X =$ completion time (min) for a randomly selected female.	$Y =$ completion time (min) for a randomly selected male.
$\mu_X = 63$ $\sigma_X = 4$	$\mu_Y = 54$ $\sigma_Y = 3.3$

finishing time in minutes. Times for males and females are independent. It is reasonable to assume that the distribution of times for each gender is approximately normal.

What is the probability that a randomly selected female will finish the race at least 15 minutes slower than a randomly selected male? (Pause the video and complete the problem, then check answer!)



What Should We Take Away?

A linear combination (including a difference or a sum) of two _____ approximately normal random variables can be modeled by a _____.

The procedures learned to find probabilities using a normal distribution also apply when finding probabilities involving a linear combination of _____, approximately _____ random variables.

AP Statistics CED 5.2 Daily Video 3 (Skill 3.C)**The Normal Distribution, Revisited****What Will We Learn?**

How can we decide whether a normal distribution is a good approximation for an unknown distribution?

What characteristics of a normal distribution should we check when assessing normality?

Adulthood

Although a majority of parents disagree, 14% of young adults (18-29) claim their parents are doing too little for them.

What might the results of a random sample of 10 young adults look like on this issue?

Could we use the normal distribution as an approximation for the number of young adults in a random sample of size $n = 10$ who think their parents are doing too little for them?

The Sample Count

14% of young adults claim their parents are doing too little for them. Let X = the number of young adults in a random sample of 10 who think their parents do too little for them.

This is a binomial situation. If it is binomial there has got to be _____ outcomes. The number of trials is _____ at ten. What one young adult says should be _____ of what another young adult says. And, the probability a young adult claims their parent is doing too little is 14% for _____ trial.

Therefore, this is a _____ situation.

The Binomial Formula

For instance, X could be 3. Therefore, $P(X = 3) = \binom{10}{3} 0.14^3 (1 - 0.14)^7 = 0.115$

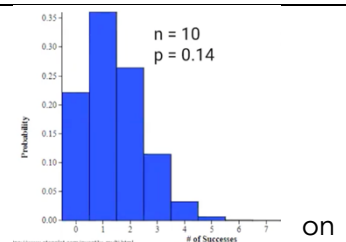
We could then make a graph of _____ values of X and their probabilities to determine whether a normal distribution is an appropriate model.

Is Normal Appropriate?

This distribution is _____ but not _____ or _____.

On the horizontal axis we have _____ possible values of _____. And on the vertical axis we have the _____ for each one.

The distribution is clearly _____, which makes normal distribution inappropriate as an approximation.

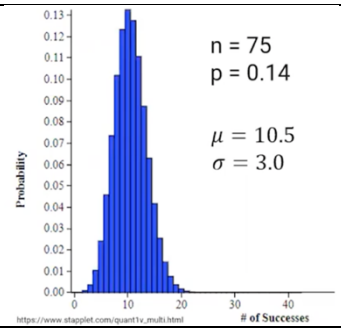


Is Normal Appropriate?

Now, if we do that same thing with a samples size of _____.

This distribution is _____ and looks roughly _____ and _____.

But is it appropriate to use normal distribution to approximate probabilities?

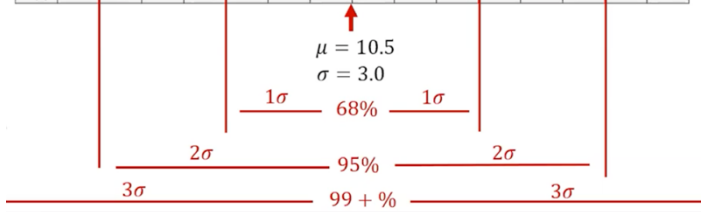


Partial Probability Distribution

X = the number of young adults in a random sample of 75 who think their parents do too little for them.

x	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
P(x)	.010	.02	.05	.07	.10	.12	.13	.13	.11	.09	.06	.04	.03	.01	.01

Note how this partial probability distribution fit the 68-95-99.7 rule for a normal distribution.

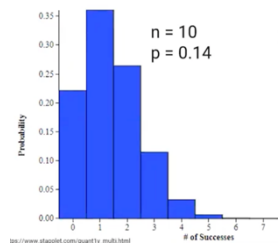


When the sample size is _____ enough, the _____ distribution is a good approximation to a binomial distribution.

Assessing Normality

This distribution is _____ but not _____ or _____.

When the sample size is _____, the normal distribution is _____ a good approximation to a binomial distribution.



What Should We Take Away?

Characteristics to check for an approximately normal distribution:

1. Unimodal, roughly symmetric, bell-shaped
2. Empirical rule generally applies (68, 95, 99.7)

AP Statistics CED 5.3 Daily Video 1 (Skill 3.C)**The Central Limit Theorem****What Will We Learn?**

What is a sampling distribution?

How can we use simulation to approximate the sampling distribution of a statistic?

What does the central limit theorem say?

Sampling Distribution Definition

A sampling distribution of a statistic is the distribution of values for the statistics for _____ of the _____ from a given population.

You are interested in the average resting heart rate of teenagers. You use a sample size of _____. You take a _____ sample of _____ teens. Determine the sample _____ resting heart rate. And then do it again, and again and again. Until you have _____ possible samples of 28 and their sample _____ resting heart rate. The collection of _____ those sample means is a sampling distribution. That is nearly impossible to do for most populations

Rice University Applet

http://onlinestatbook.com/stat_sim/sampling_dist/

Follow along as the video demonstrates this sampling distribution applet.

If the population distribution is approximately _____, it appears the sampling distribution is approximately _____ even with a _____ sample.

If the population distribution is skewed and the sample size is small, the sampling distribution is also _____. However, if the sample size is large enough, the sampling distribution is _____, _____ and appears _____.

It appears that no matter what the population looks like, or one single sample, the _____ of sample means will be approximately normal **IF** the sample size is _____.

What Should We Take Away?

The sampling distribution of a statistic is the distribution of values for the statistic of _____ possible samples of the _____ from a given population.

The sampling distribution of a statistic can be _____ by generating repeated random samples from a population.

The central limit theorem _____ state that when the sample size is _____, a sampling distribution of the _____ of a random variable will be approximately _____ distributed.

AP Statistics CED 5.3 Daily Video 2 (Skill 3.C)

The Central Limit Theorem

What Will We Learn?

What is a randomization distribution?

What can a randomization distribution tell us about the results of an experiment?

The Experiment

Newborns often sleep more during the day than the night. In a double-blind experiment to increase nighttime sleep for newborns, 16 nursing mothers were randomly assigned to take either melatonin or a placebo.

Experimental Result

During the night, newborns wake up to be fed and then, hopefully, return to sleep. The total number of hours each newborn slept for the right was recorded.

Melatonin				Placebo			
11.1	10.4	8.8	10.4	7.2	10.8	9.3	10.6
10.7	13.0	11.4	12.8	9.4	8.0	10.1	9.6
$\bar{x}_M = 11.07$ hours				$\bar{x}_P = 9.38$ hours			
$\begin{aligned} \bar{x}_M - \bar{x}_P &= 11.07 - 9.38 \\ &= 1.69 \text{ hours} \end{aligned}$							

Randomization Distribution

To investigate whether the difference in sleep time could reasonably be due to chance, we will _____ a randomization distribution. A randomization distribution is a collection of statistics generated by _____ reassigning the response values to the treatment group.

Random Reallocation

If you take all of the values out of their treatment group and _____ assign eight of the values in the melatonin group the remaining values will be in the placebo group.

11.1	10.4	8.8	10.4	7.2	10.8	9.3	10.6
10.7	13.0	11.4	12.8	9.4	8.0	10.1	9.6

Calculating Statistics

Once they are separated, with total disregard for where they really came from, calculate the _____ of each treatment group and find the _____ in those means.

Melatonin				Placebo			
13.0	8.8	11.4	12.8	11.1	10.7	10.4	10.4
10.8	8.0	10.1	9.6	7.2	9.4	9.3	10.6
$\bar{x}_M = 10.56$ hours				$\bar{x}_P = 9.89$ hours			
$\begin{aligned} \bar{x}_M - \bar{x}_P &= 10.56 - 9.89 \\ &= 0.67 \text{ hours} \end{aligned}$							

Find the difference in mean sleep time. Here (because our value is positive) it looks like the melatonin group got _____ sleep on average then the placebo group.

Another Random Reallocation

Randomly assign another 8 values for the melatonin group. The remaining vales will be in the placebo group.

11.1	10.4	8.8	10.4	7.2	10.8	9.3	10.6
10.7	13.0	11.4	12.8	9.4	8.0	10.1	9.6

Calculate Statistics

Calculate the mean of each treatment group and find the difference in means.

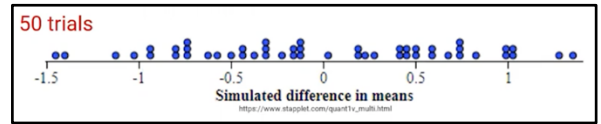
Melatonin				Placebo			
10.4	10.7	13.0	11.4	11.1	10.4	8.8	12.8
7.2	8.0	10.6	9.6	9.4	10.8	10.1	9.3
$\bar{x}_M = 10.11$				$\bar{x}_P = 10.33$			
$\begin{aligned} \bar{x}_M - \bar{x}_P &= 10.11 - 10.33 \\ &= -0.22 \end{aligned}$							

Here (because the value is negative) it looks like the placebo group got _____ sleep on average than the melatonin group.

Random Assignment Possibilities

If we continue to _____ the response values to the treatment groups, find the _____ of sample means, and _____ the differences, we will create a _____.

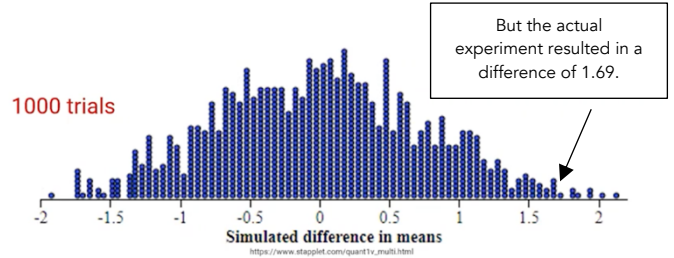
This dotplot represents a _____ difference in means. (Note that each point (dot) represents the times from eight babies sleeping in the _____ group and eight babies sleeping the _____ group, finding the _____ of each and _____ them.) This is what the distribution looks like for 50 trials.



*Note: There are _____ different possibilities for reallocating the treatments!!!

Random Assignment Possibilities

Using this simulated data we could find the probability that we would get _____ 1.69 or greater hours of sleep by chance alone.



It looks like there are $\frac{12}{1000} =$ _____

Conclusion

$\bar{x}_M =$ _____

$\bar{x}_P =$ _____

$P(\bar{x}_M - \bar{x}_P \geq \text{_____} | \text{_____}) = \text{_____} = \text{_____}$

The probability of the _____ for the melatonin group being _____ greater than the _____ for the placebo group in a _____ of values to treatment groups is _____. So it is _____ that the _____ happened by _____. This experiment provides _____ evidence that the melatonin given to nursing mothers _____ increases the _____ number of hours that newborns sleep.

What Should We Take Away?

A _____ distribution for a randomized experiment is a collection of statistics generated by _____ to _____ response values to treatment groups.

A randomized distribution can be used to assess the _____ of an observed outcome happening by _____.

AP Statistics CED 5.4 Daily Video 1 (Skill 4.B)

Biased and Unbiased Point Estimates

What Will We Learn?

What is a point estimate?

How do we determine if an estimator is unbiased?

Weimaraner

A breeder has 5 Weimaraner dogs. A sample of 3 dogs will be randomly selected for a study. The ages of the dogs (in years) are: 0 2 5 8 10

Is the sample mean an unbiased estimator of the mean age of the population?

Unbiased Estimator. Population ages: 0 2 5 8 10 $\mu = 5$

When estimating a parameter, an estimator is _____ if, _____, the value of the estimator is _____ to the population parameter.

The population is small enough that we can actually investigate all possible samples of size 3.

One Sample Mean Population ages: 0 2 5 8 10 $\mu = 5$

One possible sample of three dog ages is: 0, 2, 5

$$\bar{x} = 2.3$$

A sample statistic is a _____ of the corresponding population parameter

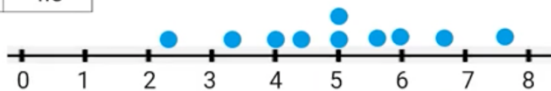
Then continue to find the sample mean for all possible samples of size 3.

All Possible Sample Means

Sample	\bar{x}	Sample	\bar{x}	Sample	\bar{x}
0, 2, 5	2.3	0, 5, 10	5.0	2, 5, 10	5.7
0, 2, 8	3.3	0, 8, 10	6.0	2, 8, 10	6.7
0, 2, 10	4.0	2, 5, 8	5.0	5, 8, 10	7.7
0, 5, 8	4.3				

$$\mu_{\bar{x}} = 5$$

When estimating a _____ parameter, an _____ exhibits _____ that can be modeled using probability.



A Second Look Population ages: 0 2 5 8 10 $\mu = 5$

$$\mu_{\bar{x}} = 5$$

Let's look at the definition of an unbiased estimator again.

Unbiased Estimator – Mean

An estimator is unbiased if _____, the value of the _____ is _____ to the _____. Here, with the mean, it is!

One Sample Range Population ages: 0 2 5 8 10 population range = 10

One possible sample of three dog ages: 0, 2, 5

$$\text{range} = 5$$

point **estimate**

Is the sample range an unbiased estimator of the population range?

point **estimator**

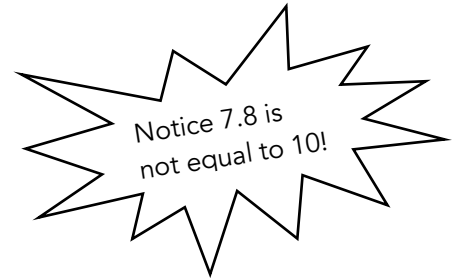
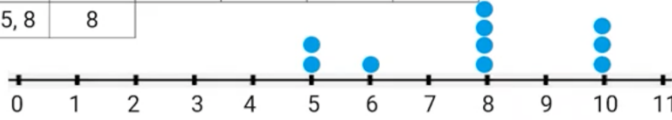
Continue to find the sample range for all possible samples of size three.

All Possible Sample Ranges:

Population ages: 0 2 5 8 10 population range = 10

Sample	Range	Sample	Range	Sample	Range
0, 2, 5	5	0, 5, 10	10	2, 5, 10	8
0, 2, 8	8	0, 8, 10	10	2, 8, 10	8
0, 2, 10	10	2, 5, 8	6	5, 8, 10	5
0, 5, 8	8				

mean of sample ranges = 7.8



Notice that the different values of the range for different samples of size 3 vary, but they do NOT vary on either side of the population range. They vary only on the _____ side!

Biased Estimator – Range

An estimator is unbiased if, _____, the value of the _____ is equal to the _____. Here _____ the value of the _____ does not equal the population parameter. So, range is a _____ estimator of the _____ range.

What Should We Take Away?

A sample statistic is a _____ of the corresponding population parameter.

When estimating a _____, an estimator exhibits _____ that can be modeled using probability.

An estimator is _____ if, _____, the value of the estimator is _____ to the population parameter.

AP Statistics CED 5.5 Daily Video 1 (Skill 3.B)

Sampling Distributions for Sample Proportions

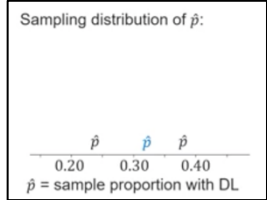
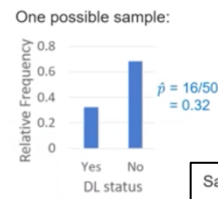
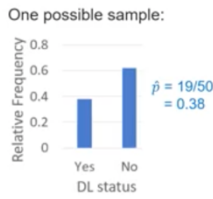
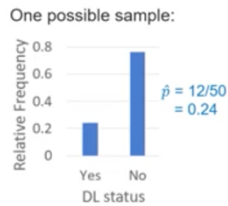
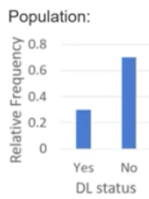
What Will We Learn?

How do we determine the parameters of a sampling distribution of a sample proportion?

How do we determine if the shape of a sampling distribution of a sample proportion is approximately normal?

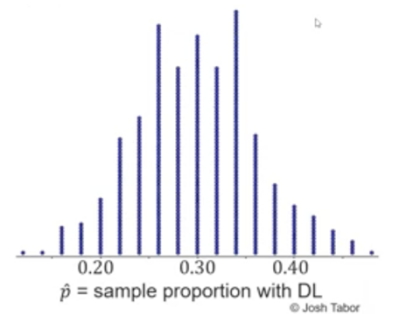
Sampling From a High School

In a high school with 2000 students, 30% of the students have a driver's license. Suppose we select a random sample of $n = 50$ students from the high school, ask each student in the sample if he or she has a driver's license, and calculate the sample proportion \hat{p} with a driver's license.



We won't always get the same value when we take a sample, that is called _____. There are lots of possible samples of size 50 that we could take.

Here is a simulated sampling distribution of _____ = the sample proportion of students with a driver's license, based on 500 samples of size $n = 50$. When you think about describing this distribution you should always think about _____ and _____ and _____.



The distribution of the sample proportion is:

- _____
- _____
- _____

Mean and Standard Deviation

Let \hat{p} be the sample proportion of _____ in a random sample of size n selected from a population with proportion of successes p . Then,

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

*assuming the sample size is less than _____ of the population size OR if we sample with replacement from the population. This condition is met in almost all circumstances. Good news! These are on the AP Stat formula sheet!

In our example, \hat{p} is the _____ proportion of students with a driver's license in samples of size $n = \underline{\quad}$ from a population where _____. So that tells us that:

the mean of the \hat{p} distribution in that context is: $\mu_{\hat{p}} = p = 0.3$
and the standard deviation is:

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.3(1-0.3)}{50}} = 0.065^*$$

In this case we are taking samples of size _____ from a large high school of 2000 student clearly < _____ of the population.

Shape

Let _____ be the sample proportion of success in a random sample of size n selected from a population with a proportion of successes _____. Then, the sample distribution of _____ will be approximately normal when: $np \geq 10$ and $n(1 - p) \geq 10$

For our example about students with driver's licenses, _____ and _____ :

$$np = \text{_____} \text{ and}$$

$$n(1 - p) = \text{_____}$$

What Should We Take Away?

How do we determine the parameters of a sampling distribution of a sample proportion?

Mean: _____

Standard Deviation : _____,

***assuming the sample size is _____ than _____ of the population size.**

How do we determine if the shape of a sampling distribution of a sample proportion is approximately normal?

The sampling distribution of _____ will be approximately normal when:

_____ and _____

AP Statistics CED 5.5 Daily Video 2 (Skill 4.B)

Sampling Distributions for Sample Proportions

What Will We Learn?

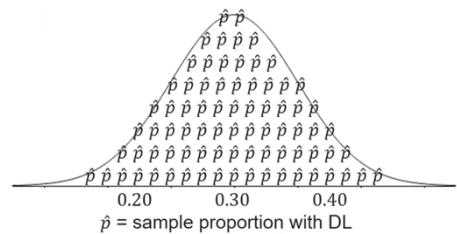
How do we interpret the parameters of a sampling distribution of a sample proportion?
 How do we calculate and interpret probabilities involving a sampling distribution of a sample proportion?

Sampling From a High School

In a high school with 2000 students, 30% of the students have a driver's license. Suppose we select a random sample of $n = 50$ students from the high school, ask each student in the sample if he or she has a driver's license, and calculate the sample proportion \hat{p} with a driver's license.

Here is what we learned from the 5.5 video 1. The sampling distribution of \hat{p} is:

- \hat{p} _____
- $\mu_{\hat{p}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
- $\sigma_{\hat{p}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$



Interpreting the Mean

Interpret the value of $\mu_{\hat{p}} = .30$

For all _____ samples of size _____ from this population, the sample proportions of students who have a driver's license will have a _____ of _____.

Interpreting the Standard Deviation

Interpret the value $\sigma_{\hat{p}} = 0.065$

For all _____ samples of size _____ from this population, the sample proportions of students who have a driver's license _____ by about _____ from the _____ proportion of _____.

*Knowing how much a _____ statistics _____ varies from the _____ is one of the most important reasons to study _____.

Calculating Probabilities

Would it be unusual to get a sample proportion of $\hat{p} = 0.12$ or less in a random sample of size $n = 50$ from this population? (Using the normal curve below to label and shade.)

$Z = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

Use table A or technology to calculate:

$P(\hat{p} < 0.12) = \underline{\hspace{2cm}}$



Interpreting Probabilities

Would it be unusual to get a sample proportion of $\hat{p} = 0.12$ or less in a random sample of size $n = 50$ from this population?

Getting a _____ proportion of _____ or _____ happens in only about _____ of all possible samples of _____ from _____ population. This is _____.

*Knowing what values of a statistic count as _____ is one of the most important reasons to study _____.

What Should We Take Away?

How do we interpret the parameters of a sampling distribution of a sample proportion?

- In _____
- Reference _____ possible samples _____ from the population
- For standard deviation, include _____ or _____.

How do we calculate and interpret probabilities involving a sampling distribution of a sample proportion?

- Use _____ procedures from Topic 5.2 with the parameters of the sampling distribution of _____ (Topic 5.5, Video 1)
- Interpretations should be _____ and reference _____ samples of _____ from the population.

AP Statistics CED 5.6 Daily Video 1 (Skill 3.B)

Sampling Distributions for Differences in Sample Proportions

What Will We Learn?

How do we determine the parameters of a sampling distribution of a difference in sample proportions?

How do we determine if the shape of a sampling distribution of a difference in sample proportions is approximately normal?

Sampling From a High School

In high school A, 30% of the students have a driver's license. In high school B, 22% of the students have a driver's license. Suppose we select a random sample of $n = 50$ students from each high school, ask each student in each sample if he or she has a driver's license, calculate the sample proportion with a driver's license in each sample, and compute the difference in sample proportions $\hat{p}_A - \hat{p}_B$.

One possible sample from A: $\hat{p}_A = 18/50 = 0.36$

One possible sample from B: $\hat{p}_B = 10/50 = 0.20$

$$\hat{p}_A - \hat{p}_B = \underline{\hspace{2cm}}$$

One possible sample from A: $\hat{p}_A = 13/50 = 0.26$

One possible sample from B: $\hat{p}_B = 14/50 = 0.28$

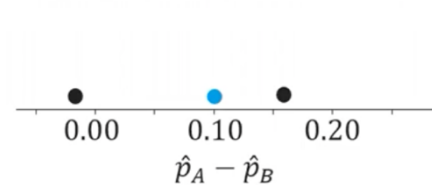
$$\hat{p}_A - \hat{p}_B = \underline{\hspace{2cm}}$$

One possible sample from A: $\hat{p}_A = 14/50 = 0.28$

One possible sample from B: $\hat{p}_B = 9/50 = 0.18$

$$\hat{p}_A - \hat{p}_B = \underline{\hspace{2cm}}$$

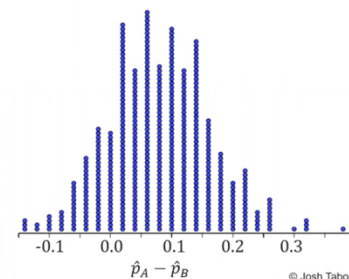
Sampling distribution of $\hat{p}_A - \hat{p}_B$:



Here is a simulated sampling distribution of $\hat{p}_A - \hat{p}_B$ based on 500 random samples of size $n = 50$ from high school A and 500 random samples of size $n = 50$ from high school B.

The distribution of $\hat{p}_A - \hat{p}_B$ is:

- _____
- _____
- _____



Mean and Standard Deviation

Let _____ be the sample proportion of successes in a _____ sample of size, _____ selected from population 1 with proportions of successes _____ and let _____ be the sample proportion of successes in a random sample of size _____ selected from population 2 with proportion of successes _____. Then,

$$\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2 \quad \sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

Assuming the samples sizes are less than 10% of the population sizes and that the two samples are independent.

Mean and Standard Deviation

In high school A, $\hat{p}_A =$ _____ and $n_A =$ _____.

In high school B, $\hat{p}_B =$ _____ and $n_B =$ _____.

Therefore,

$$\mu_{\hat{p}_A - \hat{p}_B} = \underline{\hspace{4cm}}$$

$$\sigma_{\hat{p}_A - \hat{p}_B} = \sqrt{\frac{p_A(1-p_A)}{n_A} + \frac{p_B(1-p_B)}{n_B}}$$

$$= \underline{\hspace{4cm}} = \underline{\hspace{4cm}}$$

Assuming the sample sizes are less than _____ and the two samples are _____.

Mean and Standard Deviation

Good news! Both of the formulas above can be found on the AP Statistics formula sheet!!! Take a moment to locate these formulas now!

Shape

Let \hat{p}_1 be the sample proportion of successes in a random sample of size n_1 selected from population 1 with proportion of successes p_1 , and let \hat{p}_2 be the sample proportion of successes in a random sample of size n_2 selected from population 2 with proportion of successes p_2 . Then, the sampling distribution of $\hat{p}_1 - \hat{p}_2$ will be approximately normal when:

$$\underline{\hspace{2cm}} \geq \underline{\hspace{2cm}}; \quad \underline{\hspace{2cm}} \geq \underline{\hspace{2cm}}; \quad \underline{\hspace{2cm}} \geq \underline{\hspace{2cm}}; \quad \underline{\hspace{2cm}} \geq \underline{\hspace{2cm}}$$

For our example about students with driver's licenses from high school A and high school B, $n_A =$ _____; $p_A =$ _____; $n_B =$ _____; and $p_B =$ _____

Check conditions: _____ ≥ 10 ; _____ ≥ 10
 _____ ≥ 10 ; _____ ≥ 10

Therefore the (sampling) distribution of $\hat{p}_A - \hat{p}_B$ is _____ normal in this context.

What Should We Take Away?

How do we determine the parameters of a sampling distribution of a difference of sample proportions? **MEAN:**

$$\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2$$

SD:

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

Assuming the sample sizes are less than 10% of the population sizes and samples are independent.

How do we determine if the shape of a sampling distribution of a difference in sample proportions is approximately normal?

The sampling distribution of $\hat{p}_1 - \hat{p}_2$ will be approximately normal when:

$$n_1 p_1 \geq 10, n_1(1 - p_1) \geq 10, n_2 p_2 \geq 10, \text{ and } n_2(1 - p_2) \geq 10$$

AP Statistics CED 5.6 Daily Video 2 (Skill 4.B)

Sampling Distributions for Differences in Sample Proportions

What Will We Learn?

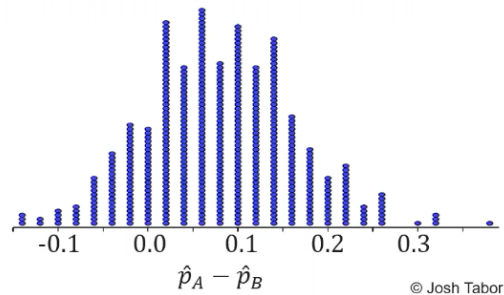
How do we interpret the parameters of a sampling distribution of a difference in sample proportions?
How do we calculate and interpret probabilities involving a sampling distribution of a difference in sample proportions?

Sampling From a High School

In high school A, 30% of the students have a driver's license. In high school B, 22% of the students have a driver's license. Suppose we select a random sample of $n = 50$ students from each high school, ask each student in each sample if he or she has a driver's license, calculate the sample proportion with a driver's license in each sample, and compute the difference in sample proportions $\hat{p}_A - \hat{p}_B$.

The distribution of $\hat{p}_A - \hat{p}_B$ is

- Approximately normal
- $\mu_{\hat{p}_A - \hat{p}_B} = 0.30 - 0.22 = 0.08$
- $\sigma_{\hat{p}_A - \hat{p}_B} = \sqrt{\frac{0.30(1-0.30)}{50} + \frac{0.22(1-0.22)}{50}} = 0.087$



Interpreting the Mean: Interpret the value $\mu_{\hat{p}_A - \hat{p}_B} = 0.08$.

"For all _____ samples of _____ students from high school A and _____ students from high school B, the _____ (A - B) in sample proportions of students _____ will have a _____ of 0.08." (See graph above)

*Note: Always be clear what order you are subtracting in.

Interpreting the Standard Deviation: Interpret the value $\sigma_{\hat{p}_A - \hat{p}_B} = 0.087$

"For all _____ samples of size _____ students from high school A and _____ students from high school B, the _____ (A - B) in sample proportions of students who _____ typically vary by about _____ from the _____ difference of 0.08." (See graph above)

*Note: Knowing how much a sample statistic _____ varies from the truth is one of the _____ important reasons to study sampling distributions.

Calculating Probabilities

In high school A, 30% of the students have a driver's license. In high school B, 22% of the students have a driver's license. Suppose we select a random sample of $n = 50$ students from each high school, would it be unusual to get a greater proportion of students with a driver's license in the sample from school B?

We want to find: _____

Use the blank normal curve at the right to sketch a picture then calculate the z-score:



$z =$ _____ Using technology: $P(\hat{p}_A - \hat{p}_B < 0) =$ _____

Interpreting Probabilities:

Would it be unusual to get a greater proportion of students with a driver's license in the sample from school B?

"Getting a difference (A – B) in sample proportions of _____ happens in about _____ of all possible samples of size 50 from these populations. This is not unusual.

*Note: Knowing what values of a statistic count as unusual is one of the most important reasons to study sampling distributions.

What Should We Take Away?

How do we interpret the parameters of a sampling distribution of a difference in sample proportions?

- In _____
- Reference _____ of these _____ from the populations.
- For standard deviation, include " _____ " or " _____ "

How do we calculate and interpret probabilities involving a sampling distribution of a difference in sample proportions?

- Use normal distribution procedures from Topic 5.2 with the _____ of the sampling distributions of $\hat{p}_A - \hat{p}_B$ (Video 1)
- Interpretations should in _____ and reference _____ samples of _____ from the populations.

AP Statistics CED 5.7 Daily Video 1 (Skill 3.B)

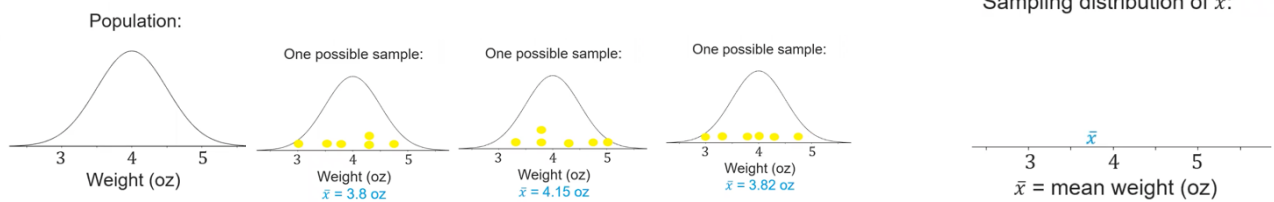
Sampling Distributions for Sample Means

What Will We Learn?

How do we determine the parameters of a sampling distribution of a sample mean?
 How do we determine if the shape of a sampling distribution of a sample mean is approximately normal?

Sampling Lemons:

A large tree produces lemons with weights that are approximately normally distributed with a mean of 4 ounces and a standard deviation of 0.5 ounces. Suppose we select a random sample of $n = 6$ lemons from the tree, weigh each lemon, and calculate the sample mean weight \bar{x} for each sample.

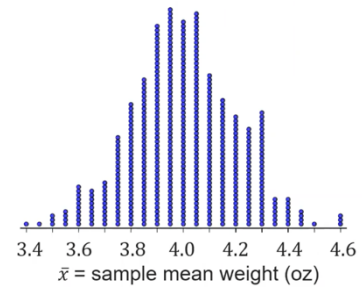


Add the \bar{x} from the second and third sample to the sampling distribution to the right.

Remember, this is just three possible samples of size six from the population. There are MANY, MANY more.

Sampling Lemons

Here is a simulated sampling distribution of \bar{x} = sample mean lemon weight (oz), based on 500 samples of size $n = 6$ from an approximately normal population with $\mu = 4$ and $\sigma = 0.5$.



When thinking about how you describe a distribution, you should always think about _____, _____ and _____.

The distribution of sample means is:

- _____
- _____
- _____

Mean and Standard Deviation

Let \bar{x} be the mean of a sample of size n selected from a population with mean μ and standard deviation σ . Then,

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

*assuming the sample size is less than 10% of the population size.

Good News! These are on the formula sheet! Take a minute to find them now!

Mean and Standard Deviation

In our example, we selected samples of size $n = 6$ from an approximately normal population with

$\mu = 4$ and $\sigma = 0.5$. Therefore:

$$\mu_{\bar{x}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\sigma_{\bar{x}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}; \text{ assuming } 6 \text{ is less than } \underline{\hspace{2cm}} \text{ of all lemons on the tree.}$$

Shape

Let \bar{x} be the mean of a sample of size n selected from a population with mean μ and standard deviation σ . Then, the sampling distribution of \bar{x} will be approximately normal when:

- The _____ distribution is _____ normal (as in the lemon example).

OR

- The _____ distribution is _____ approximately normal but the _____ size is _____ *e.g., greater than or equal to _____). This is called the _____ (see Topic 5.3).

What Should We Take Away?

How do we determine the parameters of a sampling distribution of a sample mean?

Mean: $\mu_{\bar{x}} = \mu$

Standard Deviation: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$, assuming the sample size is less than 10% of the population size.

How do we determine if the shape of a sampling distribution of a sample mean is approximately normal?

The _____ distribution of \bar{x} will be _____ normal when the population distribution is _____ normal or the sample size is _____ enough ($n \geq 30$).

AP Statistics CED 5.7 Daily Video 2 (Skill 4.B)

Sampling Distribution for Sample Means

What Will We Learn?

How do we interpret the parameters of sampling distribution of a sample mean?

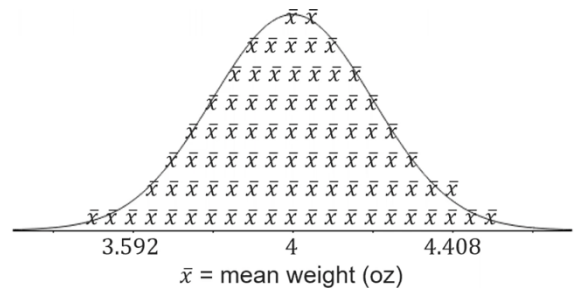
How do we calculate and interpret probabilities involving a sampling distribution of a sample mean?

Sampling Lemons:

A large tree produces lemons with weights that are approximately normally distributed with a mean of 4 ounces and a standard deviation of 0.5 ounces. Suppose we select a random sample of $n = 6$ lemons from the tree, weigh each lemon, and calculate the sample mean weight \bar{x} for each sample.

The sampling of \bar{x} is:

- _____ (because _____ distribution is - approximately normal.
- $\mu_{\bar{x}} = \mu = \underline{\hspace{2cm}}$
- $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$



the

Interpreting the Mean

Interpret the value $\mu_{\bar{x}} = 4$:

For _____ of size $n = \underline{\hspace{2cm}}$ from _____ population, the _____ mean weights of lemons will have a mean of _____.

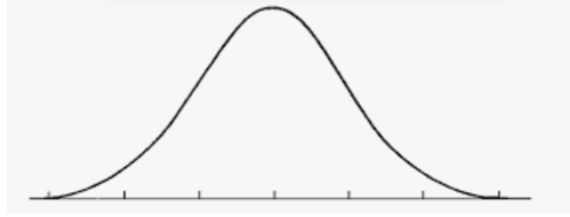
Interpreting the Standard Deviations

Interpreting the value $\sigma_{\bar{x}} = 0.204$

For _____ of size $n = \underline{\hspace{2cm}}$ from _____ population, the _____ mean weights of lemons will _____ vary by about _____ from the _____ mean of _____. *Note, that knowing how much a _____ typically varies from the _____ is one of the most important reasons to study _____ distributions and is the basis of the study of confidence intervals.

Calculating Probabilities

A large tree produces lemons with weights that are approximately normally distributed with a mean of 4 ounces and a standard deviation of 0.5 ounces. Suppose we select a random sample of $n = 6$ lemons from the tree. Would it be **unusual** to get a sample mean of $\bar{x} = 4.5$ or greater in a random sample of size $n = 6$ from this population?



Step 1: Draw a picture (Sketch from video)

Step 2: Calculate z-score = _____

Step 3: Calculate $P(\bar{x} \geq 4.5) =$ _____

Step 4: Interpret the probability – Getting _____ mean of 4.5 ounces or more happens in only about _____ of _____ samples of size _____ from this tree. This is _____ unusual.

*Know what values of a statistic count as _____ is one of the most important reasons to study _____ distributions.

What Should We Take Away?

How do we interpret the parameters of sampling distribution of a sample mean?

- In _____, with _____
- Reference _____ of this size from the population.
- For standard deviation, include _____ or _____.

How do we calculate and interpret probabilities involving a sampling distribution of a sample mean?

- Use _____ procedures from Topic 5.2 with the parameter of the _____ distribution of \bar{x} (Video 1)
- Interpretations should be _____, _____ and reference _____ of this size from the _____.

AP Statistics CED 5.8 Daily Video 1 (Skill 3.B)

Sampling Distributions for Difference in Sample Means

What Will We Learn?

How do we determine the parameters of a sampling distribution of a difference in sample means?
 How we determine if the shape of a sampling distribution of a difference in sample means is approximately normal?

Sampling Citrus

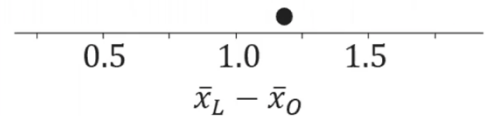
A large tree produces lemons with weights that are approximately normally distributed with $\mu_L = 4$ ounces and $\sigma_L = .5$ ounces. A different tree produces oranges with weights that are approximately normally distributed with $\mu_O = 3$ ounces and $\sigma_O = .4$ ounces. Suppose we select a random sample of $n = 6$ fruit from each tree, calculate the mean weight for each sample, and compute the difference in sample means $\bar{x}_L - \bar{x}_O$. (Label and plot possible samples points on graph as you watch video)

One possible sample of lemons: $\bar{x}_L = 4.1$ ounces
 One possible sample of oranges: $\bar{x}_O = 2.9$ ounces
 $\bar{x}_L - \bar{x}_O = 1.2$ ounces

One possible sample of lemons: $\bar{x}_L =$ _____
 One possible sample of oranges: $\bar{x}_O =$ _____
 $\bar{x}_L - \bar{x}_O =$ _____

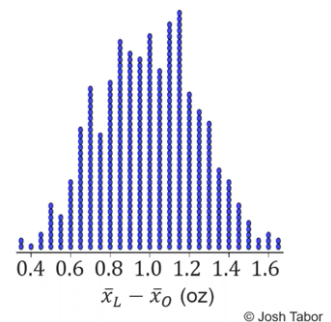
One possible sample of lemons: $\bar{x}_L =$ _____
 One possible sample of oranges: $\bar{x}_O =$ _____
 $\bar{x}_L - \bar{x}_O =$ _____

Sampling distribution of $\bar{x}_L - \bar{x}_O$:



Of course, these are only three of the MANY possible differences.

Here is a simulated sampling distribution of $\bar{x}_L - \bar{x}_O$ based on 500 random samples of size $n_L = 6$ from the lemon tree and 500 random samples of size $n_O = 6$ from the orange tree. How would you describe this distribution?



The distribution of $\bar{x}_L - \bar{x}_O$ is:

- _____
- Centered at _____
- Varies from _____ to about _____.

Mean and Standard Deviation

Let \bar{x}_1 be the sample mean in a random sample of size n_1 selected from population 1 with μ_1 and standard deviation σ_1 , and let \bar{x}_2 be the sample mean in a random sample of size n_2 selected from population 2 with μ_2 and standard deviation σ_2 . Then,

$$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2$$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

*assuming the sample sizes are less than 10% of the population sizes and that the two samples are independent.

Mean and Standard Deviation

For the lemons, $\mu_L = 4$ ounces and $\sigma_L = .5$ ounces.

For the oranges, $\mu_O = 3$ ounces and $\sigma_O = .4$ ounces.

Therefore,

$$\mu_{\bar{x}_L - \bar{x}_O} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\sigma_{\bar{x}_L - \bar{x}_O} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

* Assuming the samples sizes are _____ of the population sizes and the two samples are _____.

Mean and Standard Deviation

Good News! These are all on the formula sheet! Take a moment to find them NOW!

Shape

The sampling distribution of $\bar{x}_1 - \bar{x}_2$ will be approximately normal when:

- _____ population distributions are approximately normal (as in the citrus example).

OR

- The populations are _____ approximately normal but both of the sample sizes are _____ (e.g., _____)

What Should We Take Away?

How do we determine the parameters of a sampling distribution of a difference in sample means?

Mean: $\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2$

SD: $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$, assuming the sample sizes are less than 10%

of the population sizes and the samples are independent.

How we determine if the shape of a sampling distribution of a difference in sample means is approximately normal?

The sampling distribution of $\bar{x}_1 - \bar{x}_2$ will be approximately normal when the population Distributions are both approximately normal OR when both sample sizes are large enough (e.g., greater than or equal to 30).

AP Statistics CED 5.8 Daily Video 2 (Skill 4.B)

Sampling Distributions for Differences in Sample Means

What Will We Learn?

How do we interpret the parameters of a sampling distribution of a difference in sample means?

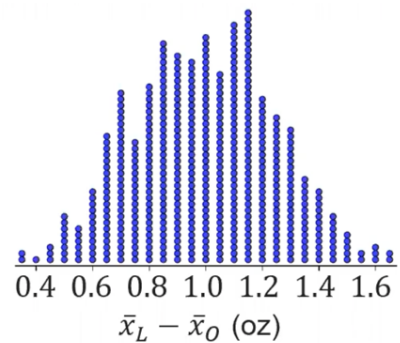
How do we calculate and interpret probabilities involving a sampling distribution of a difference in sample means?

Sampling Citrus

A large tree produces lemons with weights that are approximately normally distributed with $\mu_L = 4$ ounces and $\sigma_L = .5$ ounces. A different tree produces oranges with weights that are approximately normally distributed with $\mu_O = 3$ ounces and $\sigma_O = .4$ ounces. Suppose we select a random sample of $n = 6$ fruit from each tree, calculate the mean weight for each sample, and compute the difference in sample means $\bar{x}_L - \bar{x}_O$.

The distribution of $\bar{x}_L - \bar{x}_O$ is:

- _____ normal
- $\mu_{\bar{x}_L - \bar{x}_O} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$
- $\sigma_{\bar{x}_L - \bar{x}_O} = \underline{\hspace{2cm}}$



© Josh Tabor

Interpreting the Mean

Interpret the value of $\mu_{\bar{x}_L - \bar{x}_O} = 1$

For _____ random samples of _____ from the lemon tree and _____ from the orange tree, the differences ($L - O$) in _____ mean weight will have a _____ of _____.

Interpreting the Standard Deviation

Interpret the value $\sigma_{\bar{x}_L - \bar{x}_O} = 0.26$.

For _____ random samples of _____ from the lemon tree and _____ from the orange tree, the differences ($L - O$) in the _____ mean weight will _____ vary by _____ 0.26 ounces from the _____ difference in means of 1 ounce.

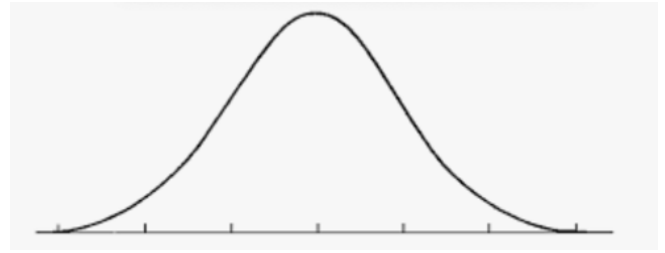
*Knowing how much a _____ statistic _____ varies from the _____ is one of the most important reasons to study _____ distributions!!

Calculating Probabilities

Suppose we selected a random sample of 6 oranges and 6 lemons. Would it be unusual to get a sample mean weight for the lemons that was more than 1.2 ounces greater than the sample mean weight for the oranges?

We want to find $P(\bar{x}_L - \bar{x}_O > 1.2)$.

Step 1: Draw a picture.



Step 2: Calculate z-score: _____

Step 3: Find probability using Table A or Technology: $P(\bar{x}_L - \bar{x}_O > 1.2) =$ _____

Interpreting Probabilities

Would it be unusual to get a sample mean weight for the lemons that was more than 1.2 ounces greater than the sample mean weight for the oranges?

Getting a _____ (L – O) is _____ mean weights of more than 1.2 ounces happens in _____ of _____ possible samples of _____ and _____ from these trees. This is _____.

*Knowing what values of a _____ count as _____ is one of the most important reasons to study _____ distributions.

What Should We Take Away?

How do we interpret the parameters of a sampling distribution of a difference in sample means?

- In _____, with _____
- Reference _____ possible samples of _____ sizes from the populations.
- For standard deviation, include _____ or _____.

How do we calculate and interpret probabilities involving a sampling distribution of a difference in sample means?

- Use _____ distribution procedures from Topic 5.2 with the parameters of the _____ distribution of $\bar{x}_1 - \bar{x}_2$ (Video 1).
- Interpretations should be in _____, with _____ and reference _____ possible samples of these _____ from the populations.