Practice Question 2: Modeling a Non-periodic Context

2. The temperature of Shania's butternut squash soup when it is first removed from the stove ($t = 0$) is 185° Fahrenheit. Ten minutes later ($t = 10$), Shania uses a thermometer to determine that the temperature of the soup has cooled to 130° Fahrenheit.

   The temperature of Shania's bowl of soup can be modeled by the function S given by $S(t) = ab^t + 72$, where $S(t)$ is the temperature of the soup, in degrees Fahrenheit (°F), and t is the number of minutes since the soup was removed from the stove.

   (A)   (i)   Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $S(t)$.

   (ii)   Find the values of a and b as decimal approximations.

   (B)   (i)   Use the given data to find the average rate of change of the temperature of the soup, in degrees Fahrenheit per minute, from $t = 0$ to $t = 10$. Express your answer as a decimal approximation. Show the computations that lead to your answer.

   (ii)   Use the average rate of change found in (i) to estimate the temperature of the soup when $t = 8$. Show the work that leads to your answer.

   (iii)   The average rate of change calculated in part (i) can be used to estimate the temperature of the soup at time t for values of t between $t = 0$ and $t = 10$. Will the values estimated using the average rate of change be strictly greater than, strictly less than, or sometimes greater and sometimes less than the values predicted by the model, S, for $0 \le t \le 10$? Explain why this is the case.

   (C)   According to the model, the temperature of the soup will always exceed a certain temperature, M, which corresponds to the temperature of the room in which the soup is cooling. In other words, $S(t) > M$ for all t. However, Shania finishes her soup by the time the soup's temperature has cooled to 10° Fahrenheit above the room temperature, M. Explain how this information could be used to determine an appropriate domain for S based on the context.