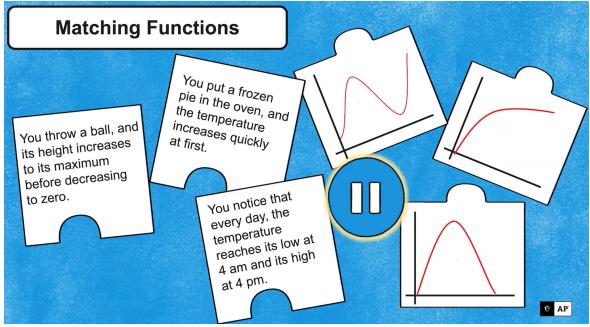
Topic 3.1 Periodic Phenomena (Daily Video 1)

AP Precalculus

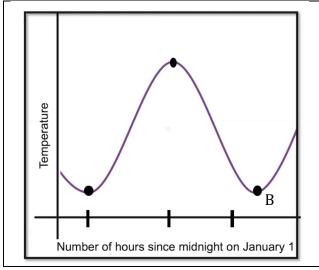
In this video, we will define periodic relationships and explore different contexts that are wellmodeled with periodic functions.

Let's Review!

Match the verbal description of each function with its corresponding graph.



Patterns of Repeating Output Values



You notice that every day, the temperature reaches its low at 4 am and its high at 4 pm.

Every 24 hours, this pattern repeats. Label the values on the x-axis.

The second minimum point (labeled B) occurs _____ hours after midnight on January 1.

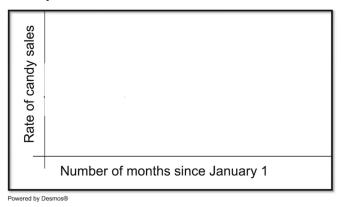
How many hours after midnight on January 1 will another high temperature occur?

A function is periodic if the output values ______ themselves over consecutive ______ intervals of the domain. Why is the previous problem an example of a periodic function?



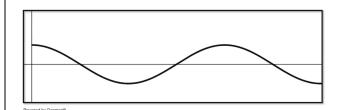
Let's Look at an EXAMPLE!

Candy sales remain steady for the first 8 months of the year, rise quickly until Halloween, and then decrease back to the previous sales rate in January.

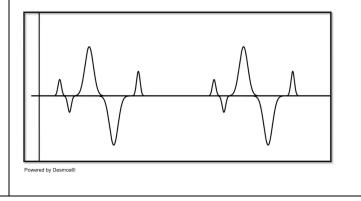


It is the _____ of these functions over equal-sized intervals that makes them periodic.

The displacement of a buoy as it rises and falls with the ocean waves.



An electrocardiogram (EKG) recording of the electrical signal of your heartbeat over time.



- A function is periodic if the _____ of the function demonstrate a repeating pattern over successive, equal-length intervals of the domain.
- One _____ of a periodic function gives enough information to graph the function over any interval of the domain.

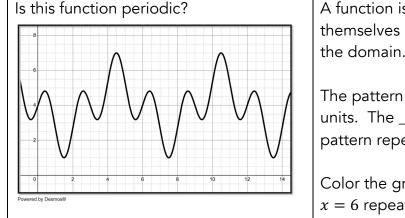


Topic 3.1 Periodic Phenomena (Daily Video 2)

AP Precalculus

In this video, we will describe the characteristics of a periodic function.

Let's look at an EXAMPLE!



A function is periodic if the _____ values repeat themselves over consecutive _____ intervals of the domain.

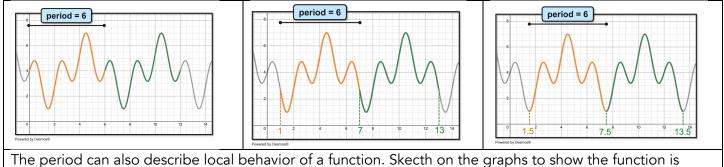
The pattern in the output values repeats every _____ units. The ______ is the length over which the pattern repeats itself.

Color the graph to show the pattern between x = 0 and x = 6 repeats again between x = 6 and x = 12.

Definition: The period of a function is the smallest positive value k such that f(x + k) = f(x) for all x in the domain.

- The period is always a _____ value.
- The period is the _____ interval of the domain over which the pattern of the output values starts to repeat.
- Any one period of the graph of the function will contain _____ the information about the pattern or the phenomena.

We can start at different locations on the graph and the period is the ______. For these graphs, the pattern repeats every six units no matter where we start along the x-axis measuring those six units.



increasing at x = 8, x = 8 - 6 and x = 8 + 6.

- The period is the shortest interval of the domain over which the pattern in the output values starts to repeat.
- We can estimate period by investigating an input interval in which the pattern of the output values starts to repeat.
- Often it is not enough to consider when a particular output value repeats, and we must instead look for when the pattern of output values repeats.



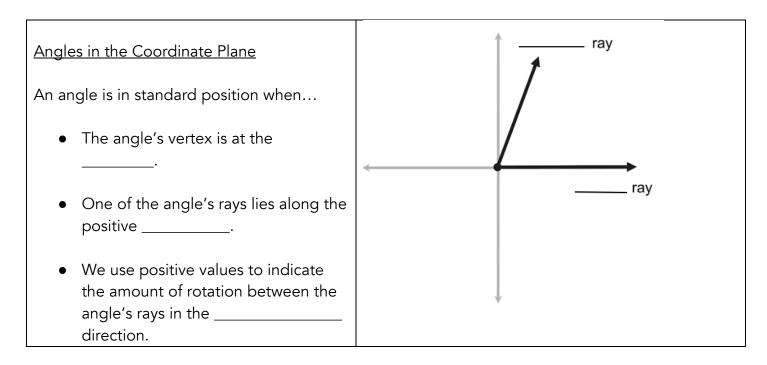
Topic 3.2 Sine, Cosine and Tangent (Daily Video 1)

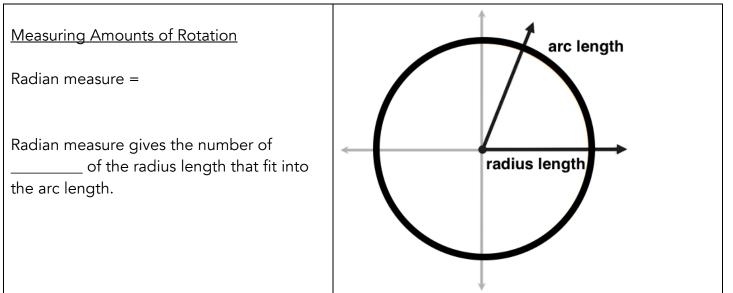
AP Precalculus

In this video, we will examine the radian as a unit of angle measure and explore relationships between the radian measure of an angle and other attributes of the angle.

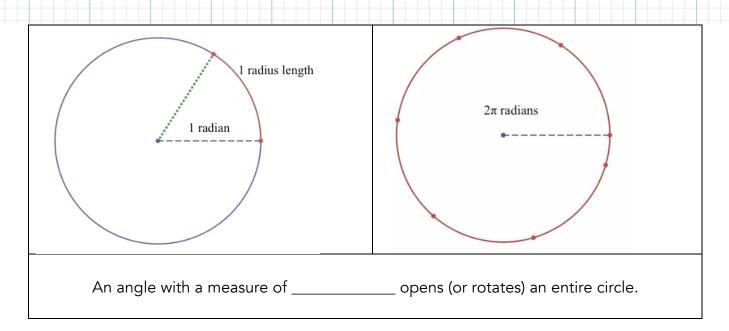
Let's REVIEW!

- An angle is a geometric object that consists of two rays with a ______
- When we measure an angle, we are measuring the amount of ______ between the two rays.

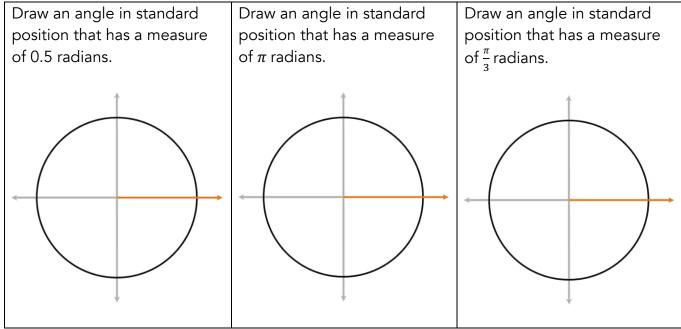








Let's PRACTICE!



- An angle is in ______ position when its vertex is at the origin, one of the angle's rays lies along the positive *x*-axis, and positive values are used to denote the amount of rotation in a counterclockwise direction.
- A radian is a unit of angle measure that relates the arc length cut off by the angle's rays to the _____ length of the circle.
- An angle has a measure of 1 radian when the arc length cut off by the angle's rays is equal to the length of the _____.
- An angle that rotates a full circle has a measure of _____ radians.



Topic 3.2 Sine, Cosine and Tangent (Daily Video 2)

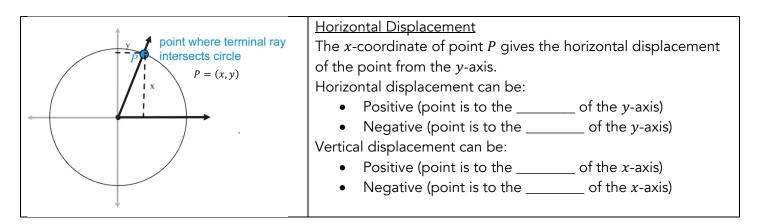
AP Precalculus

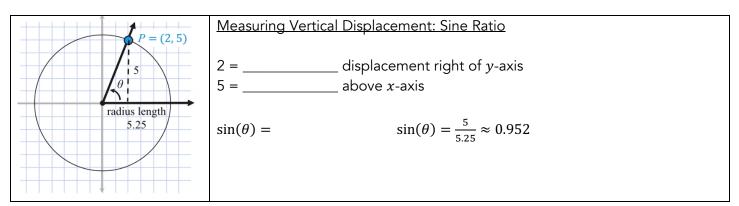
In this video, we will define sine, cosine, and tangent and investigate how to utilize the functions to describe points on the unit circle.

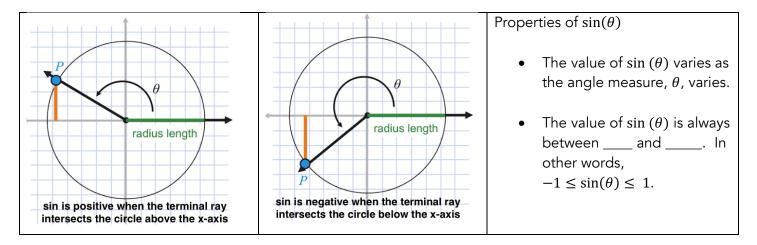
Let's REVIEW!

Radian measure =

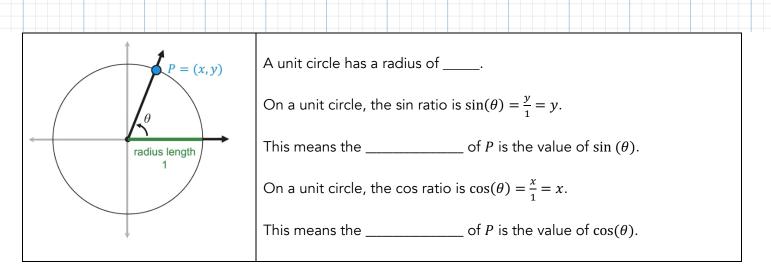
When measuring in radians, θ gives the number of copies of the radius length that fit into the arc length. We are measuring the arc length with a ruler the length of the radius.

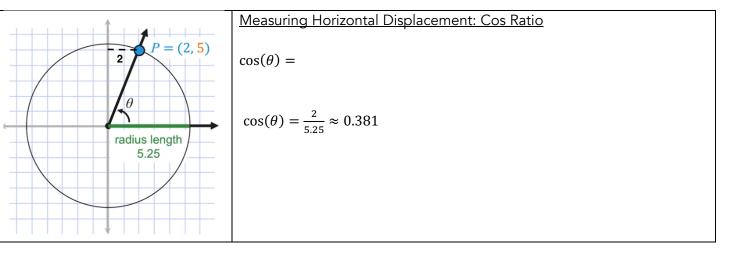


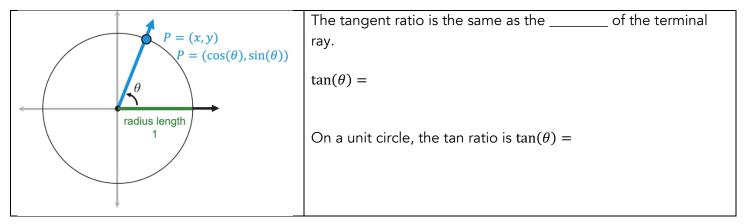




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- The sin ratio is the vertical displacement of point *P* from the *x*-axis to the radius of the circle. In a unit circle, $sin(\theta)$ is the value of the *y*-coordinate of point *P*.
- The cos ratio is the horizontal displacement of point *P* from the *y*-axis to the radius of the circle. In a unit circle, $cos(\theta)$ is the value of the *x*-coordinate of point *P*.
- Tan(θ) is the ratio of the vertical displacement of point P from the x-axis to the horizontal displacement of point P from the y-axis. In the unit circle, tan(θ) is the ratio of sin(θ) to cos(θ).

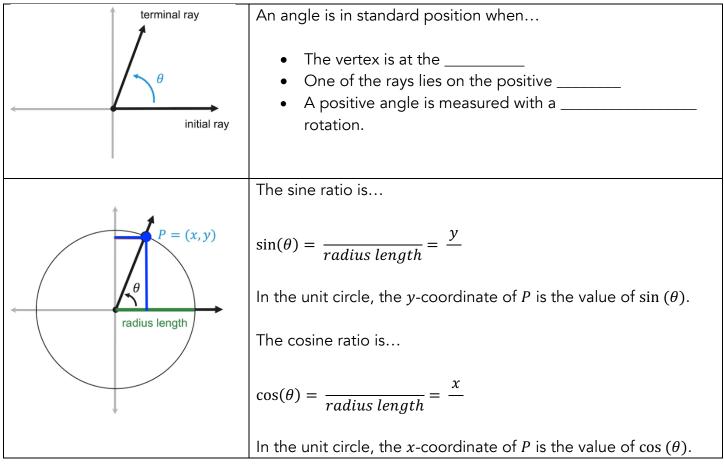


Topic 3.3 Sine and Cosine Function Values (Daily Video 1)

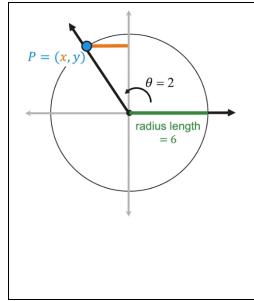
AP Precalculus

In this video, we will learn how to determine the coordinates of a point where the terminal ray of an angle intersects a circle.

Let's REVIEW!



Let's look at an EXAMPLE!



Determining Coordinates: Suppose an angle with a measure of 2 radians is inscribed in a circle with a radius of 6 units. What are the coordinates where the terminal ray intersects the circle?

Using $\cos(\theta) = \frac{x}{r}$

6 * cos(__) = ____

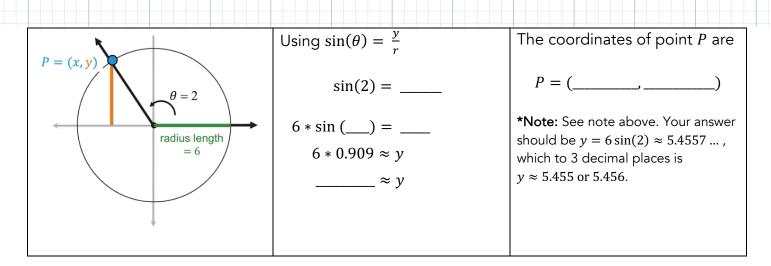
 $6 * -0.416 \approx x$

$$\sim x$$

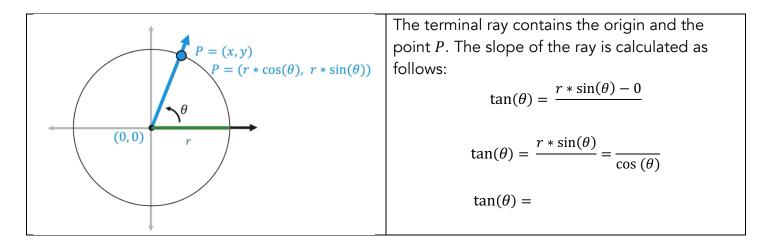
Make sure your calculator is in radian mode

*Note: On the AP Exam you should not use a rounded off answer ($\cos(2) = -0.416$) in a later calculation. Your answer should be $x = 6\cos(2) \approx -2.4968...$, which to 3 decimal places is $x \approx -2.497$ or -2.496.





Coordinates of Points in a Plane: If point P = (x, y) is on the terminal side of an angle in standard position with radius, r, and angle measure of θ , then the coordinates of the point can be written in terms of r and θ as $P = (r * \cos(\theta), r * \sin(\theta))$.



- The coordinates (______, _____) describe the location where the terminal ray of an angle with measure θ intersects a circle with radius r.
- The slope of the terminal ray, $\tan(\theta)$, is always given by the ratio .

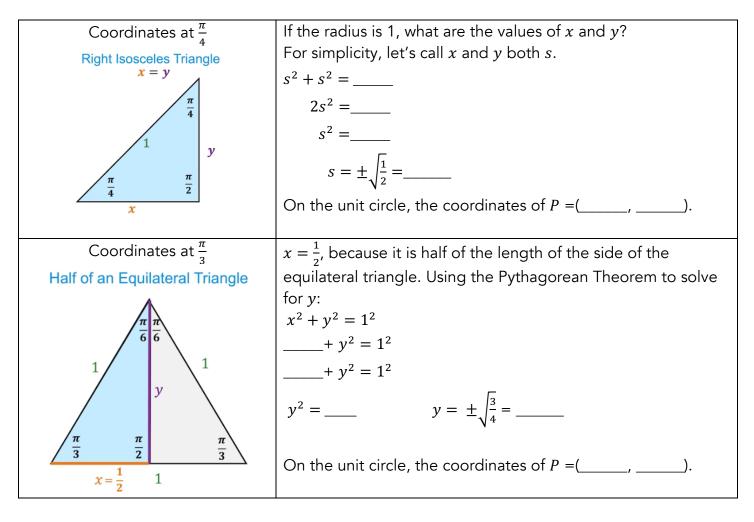


Topic 3.3 Sine and Cosine Function Values (Daily Video 2) AP Precalculus

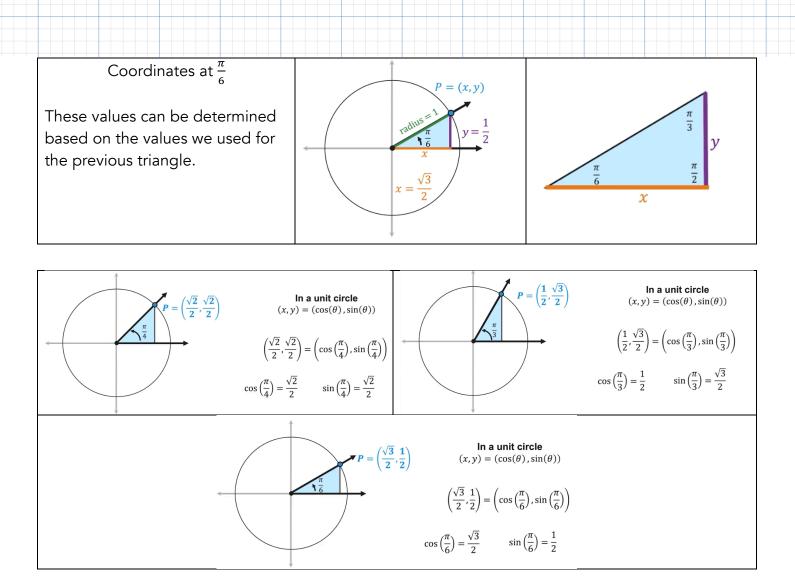
In this video, we will learn how to use the geometries of isosceles and equilateral triangles to determine the exact values of the sine and cosine functions for certain angle measures.

Let's REVIEW!

If point P = (x, y) is on the terminal side of an angle in standard position with radius, r, and angle measure of θ , then the coordinates of the point can be written in terms of r and θ as $P = (_ , _ _).$ Label each angle of the special triangles in terms of radians. $I = (_ , _ _).$ Equilateral Triangle $I = (_ , _ _).$ $I = (_ , _].$ $I = (_ , _$







What should we take away?

The exact values of $\sin(\theta)$ and $\cos(\theta)$ for angle measures of $\theta = \frac{\pi}{4}, \frac{\pi}{3}$, and $\frac{\pi}{6}$:

$$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \qquad \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$
$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \qquad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$
$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \qquad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

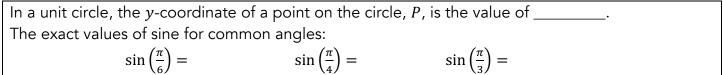


Topic 3.4 Sine and Cosine Function Graphs (Daily Video 1)

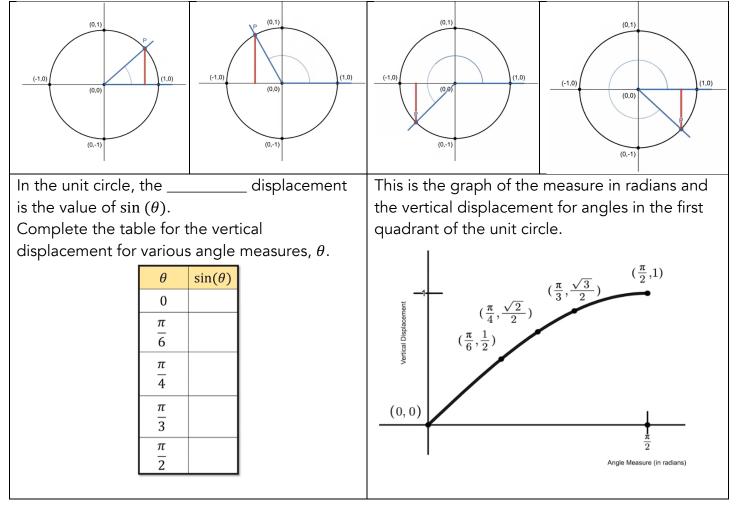
AP Precalculus

In this video, we will construct a graph of the sine function and explore the properties of this function.

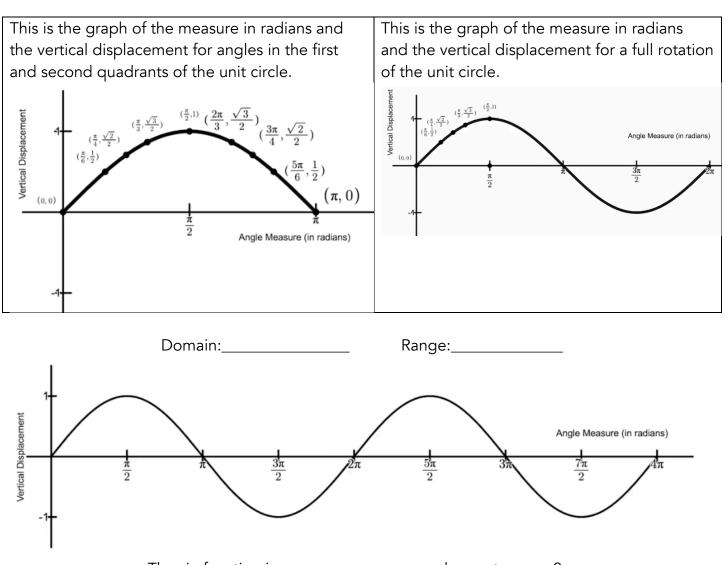
Let's REVIEW!



How the value of sin (θ) changes as the angle measure, θ , changes?







The sin function is _____ and repeats every 2π .

- Tracking the vertical displacement from the *x*-axis of a point *P* on the unit circle as the angle measure varies gives the graph of the ______ function.
- The range of the sine function is [-1, 1], because the point is never more than 1 unit from the
- The ______ of the sine function is all real numbers, representing all possible angle measures.
- The sine function is ______.



Topic 3.4 Sine and Cosine Function Graphs (Daily Video 2)

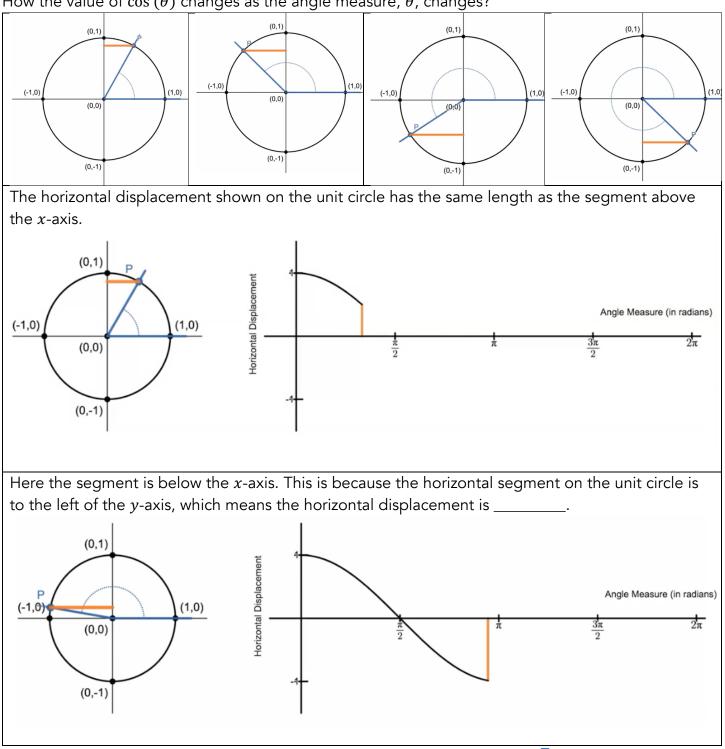
AP Precalculus

In this video, we will construct a graph of the cosine function and explore the properties of this function.

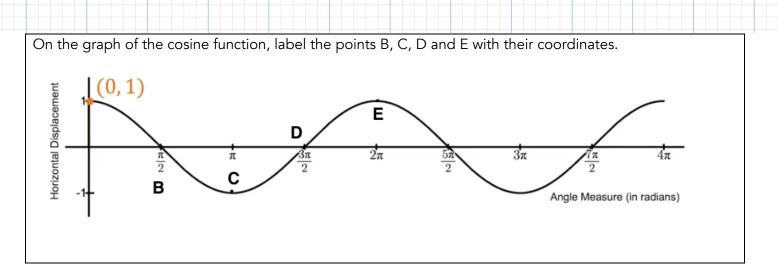
Let's REVIEW!

In a unit circle, the x-coordinate of a point on the circle, P, is the value of _____

How the value of $\cos(\theta)$ changes as the angle measure, θ , changes?



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- Tracking the horizontal displacement from the *y*-axis of a point *P* on the unit circle as the angle measure varies gives the graph of the ______ function.
- The range of the cosine function is [-1, 1], because the point is never more than 1 unit from the ______.
- The _____ of the cosine function is all real numbers, representing all possible angle measures.
- The cosine function is _____.



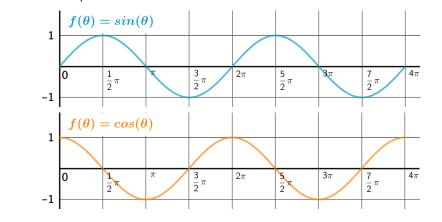
Topic 3.5 Sinusoidal Functions (Daily Video 1)

AP Precalculus

In this video, we will establish the properties of the sinusoidal functions $f(\theta) = \sin(\theta)$ and $f(\theta) = \cos(\theta)$, and we will relate these properties back to the unit circle.

Let's REVIEW!

You can use the website <u>www.geogebra.org/m/tyaseusf</u> to explore the relationships between the graphs of the sine function and cosine function with the vertical and horizontal displacement on the unit circle. Here are two periods of the sin and cos functions.



- When the cosine graph is shifted _____ units to the right, it is the same as the sine graph. This means $f(\theta) = \sin(\theta) = \cos(_____)$.
- When the sine graph is shifted _____ units to the left, it is the same as the cosine graph. This means $f(\theta) = \cos(\theta) = \sin($ _____).
- In other words, there is a phase shift of _____ units between the sine and cosine graphs.

Let's Look at an EXAMPLE!

- The **period** of the graphs of $f(\theta) = \sin(\theta)$ and $f(\theta) = \cos(\theta)$ is ______ radians because that is one revolution around the unit circle.
- The **frequency** of the graphs of $f(\theta) = \sin(\theta)$ and $f(\theta) = \cos(\theta)$ is the reciprocal of the period. So, it is _____.
- The **amplitude** of the graphs of $f(\theta) = \sin(\theta)$ and $f(\theta) = \cos(\theta)$ is ____, because it is the distance from the midline to the maximum or minimum on the graph.
- The midline of the graphs of $f(\theta) = \sin(\theta)$ and $f(\theta) = \cos(\theta)$ is _____ and it splits the graph horizontally down the middle.

- CALC MEDIC

- The symmetry of the graph of $f(\theta) = \sin(\theta)$ is rotationally symmetric about the orgin and that means it has _____ symmetry. In other words, $\sin(-\theta) = -\sin(\theta)$.
- The symmetry of the graph of $f(\theta) = \cos(\theta)$ is reflectively symmetric about the *y*-axis it has _____ symmetry. In other words, $\cos(-\theta) = \cos(\theta)$.
- As input values increase, the graphs of sine and cosine oscillate between concave _____and concave _____.
- As input values increase, the graphs of sine and cosine oscillate between _____ and

- The graphs of $y = \sin(\theta)$ and $y = \cos(\theta)$ have
 - period 2π and frequency of $\frac{1}{2\pi}$
 - amplitude |a| = 1 and midline y = 0.
- The graph of $y = \sin(\theta)$ has odd symmetry and the graph of $y = \cos(\theta)$ has even symmetry.
- As input values increase, the graphs of sinusoidal functions oscillate between increasing and decreasing and concave up and concave down.
- The properties of the sine and cosine graphs are tied into positions of a point on the unit circle.



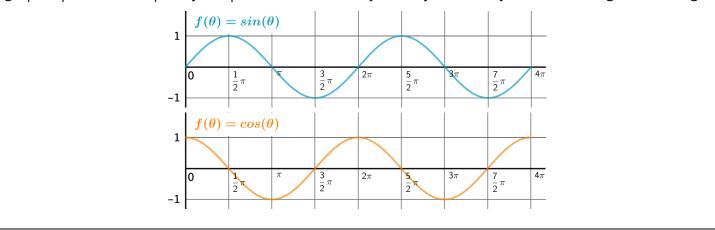
Topic 3.5 Sinusoidal Functions (Daily Video 2)

AP Precalculus

In this video, we will determine the properties of sinusoidal functions based on their graphs.

Let's REVIEW!

Look at your notes to refresh your memory about the following features of the sine and cosine graphs: period & frequency, amplitude & midline, symmetry, concavity and increasing/decreasing.



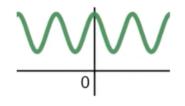
Let's PRACTICE!

1) The period of $y = \sin(\theta)$ and $y = \cos(\theta)$ is
This is because a point rotates through a radian angle around the before repeating itself.
2) The range of $h(t) = \sin(t)$ and $j(t) = \cos(t)$ is
This is because the coordinates of all points on a unit circle are between (and include) and
 If you know the frequency of a sinusoid, then you can also determine the period. True or false? Justify your answer.
, because the period and frequency are of each other.
4) Is it possible for a sinusoid to have neither even nor odd symmetry? Justify your answer.
, because the graph could be transformed. (Sketch an example.)



5) Because of its even symmetry, this curve was created with the cosine function, not the sine function. True or False? Justify your answer.

_____, because the graphs can be made identical with a_____ translation (phase shift).

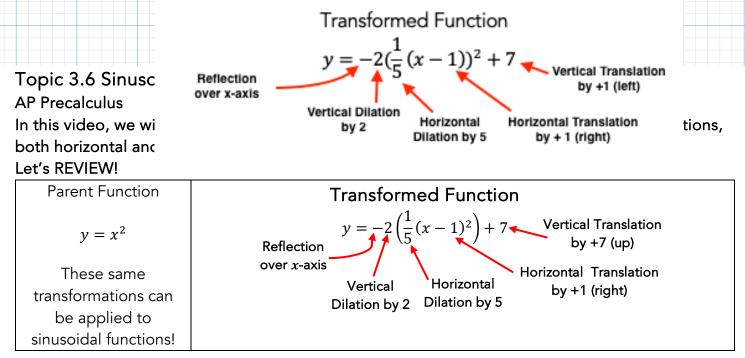


- 6) We know that $\cos(\theta) = \sin(\theta + \frac{\pi}{2})$ for all angles θ . Which of the following is also true for all θ ?
 - (a) $\cos(\theta) = \sin(\theta \frac{\pi}{2})$ (b) $\cos(\theta) = \sin(\theta + \frac{3\pi}{2})$
 - (c) $\cos(\theta) = \sin(\theta + \frac{5\pi}{2})$ (d) $\cos(\theta) = \sin(\theta + \frac{7\pi}{2})$

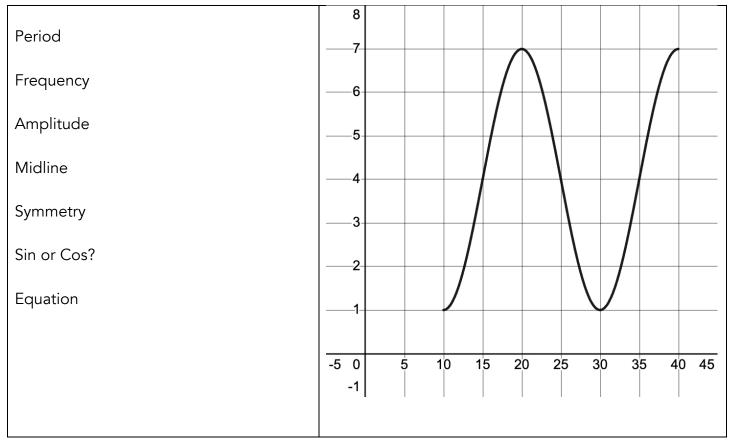
Note that angles of $\theta + \frac{\pi}{2}$ and _____ differ by 2π . And this means the angles are

- The unit circle is useful for helping to understand the parent graphs of $y = \sin(\theta)$ and $y = \cos(\theta)$.
- We can extend our knowledge to answer questions about all sinusoids, including those that have been transformed.





Let's Look at an EXAMPLE!



- Much of the new vocabulary we used to describe sinusoids relates to transformation vocabulary from earlier in this course.
- By relating the properties of a transformed sinusoid to the properties of a parent/basic sinusoid $y = \sin(\theta)$ or $y = \cos(\theta)$, we can write a function equation for a given sinusoid graph.



Topic 3.6 Sinusoidal Function Transformations (Daily Video 2)

AP Precalculus

In this video, we will write the sinusoidal function equation for a provided graph by determining and applying the necessary transformations to a parent sinusoidal function.

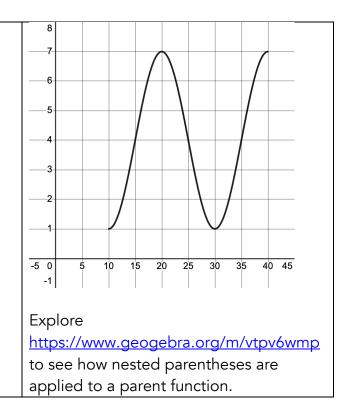
Let's REVIEW!

For the given sinusoid graph, write a function equation.

$$P = 20$$
 $F = \frac{1}{20}$ $|a| = 3$

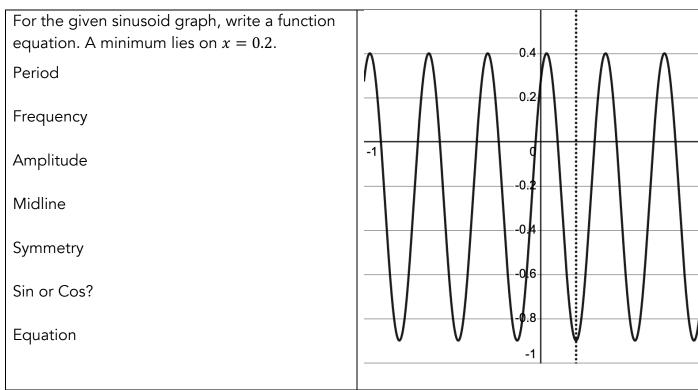
Vertical translation = 4 even

What are 3 different equations to represent this function?

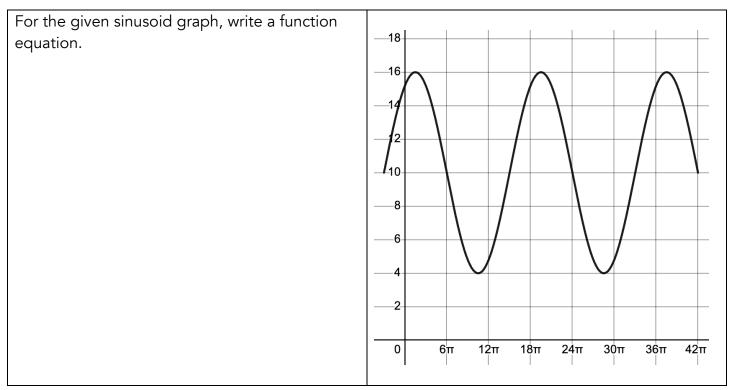


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Let's PRACTICE!



Let's PRACTICE!



What should we take away?

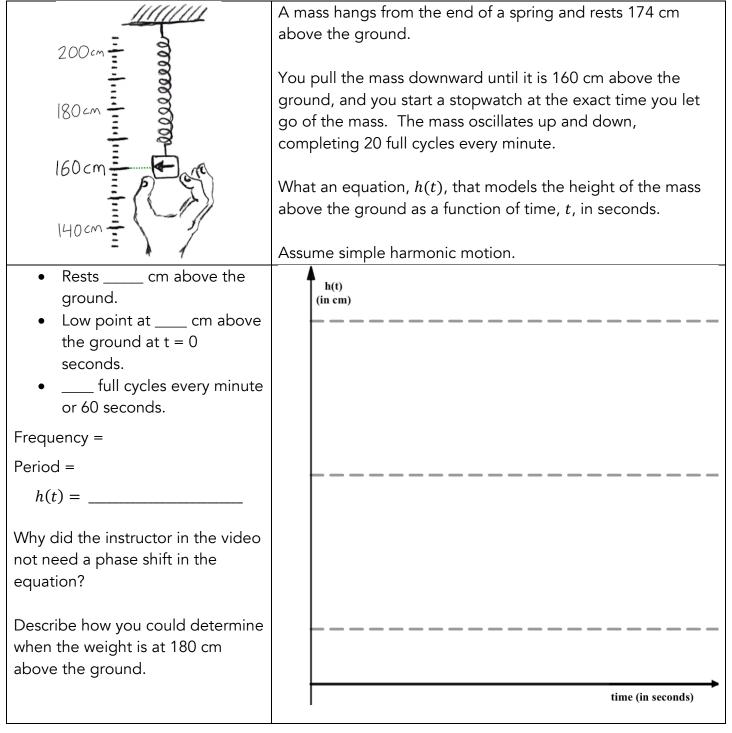
• Given properties about a sinusoid (numerically or graphically), we can write a sinusoid equation that models it.



Topic 3.7 Sinusoidal Function Context and Data Modeling (Daily Video 1) AP Precalculus

In this video, we will use numerical data describing a periodic phenomenon to construct a sinusoidal function model.

Let's Look at an EXAMPLE!



What should we take away?

Given numerical and/or graphical data about a periodic phenomenon, we can construct a sinusoidal function model for it.

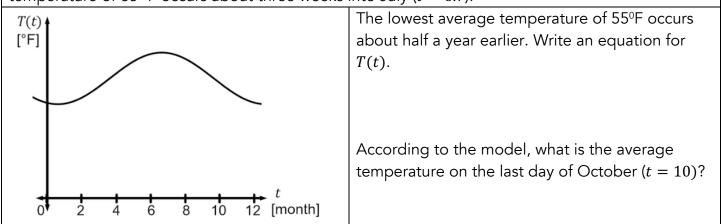


Topic 3.7 Sinusoidal Function Context and Data Modeling (Daily Video 2) AP Precalculus

In this video, we will continue practicing the construction of sinusoidal function models from provided numerical data.

Let's PRACTICE!

Example 1: A sinusoidal function, T(t), models average New Orleans temperatures (in ⁰F) as a function of time, t, in months after January 1. According to the model, the highest average temperature of 85⁰ F occurs about three weeks into July (t = 6.7).



Example 2: A Ferris wheel with a 34-foot diameter rotates at a constant angular speed, taking 50 seconds to complete one full rotation. The lowest position a rider will ever reach is 12 feet off the ground. At time t = 20 seconds, Bill the kitten is at the position indicated in the figure, traveling in the indicated direction.

A	Write an equation for $B(t)$, Bill's height off the ground, as a function of time.
	What is Bill's height when $t = 0$?

