Solutions to Practice Question 2: Modeling a Non-periodic Context

2. The temperature of Shania's butternut squash soup when it is first removed from the stove ($t = 0$) is 185˚ Fahrenheit. Ten minutes later ($t = 10$), Shania uses a thermometer to determine that the temperature of the soup has cooled to 130˚ Fahrenheit.

   The temperature of Shania's bowl of soup can be modeled by the function S given by $S(t) = ab^t + 72$, where $S(t)$ is the temperature of the soup, in degrees Fahrenheit (˚F), and t is the number of minutes since the soup was removed from the stove.

(A)	(i)	Since $S(0) = 185$ and $S(10) = 130$, two equations to find a and b are: $$185 = ab^0 + 72$$ $$130 = ab^{10} + 72$$	1 pt
	(ii)	$$185 = ab^0 + 72 \Rightarrow a = 185 - 74 = 113$$ $$130 = 113b^{10} + 72 \Rightarrow b = \left(\frac{130 - 72}{113}\right)^{\frac{1}{10}} = 0.935481$$	1 pt
(B)	(i)	$$\frac{S(10) - S(0)}{10 - 0} = \frac{130 - 185}{10} = -5.5\text{˚F per minute}$$	1 pt
	(ii)	The secant line between $t = 0$ and $t = 10$ passes through the points $(0,185)$ and $(10,130)$ and is given by $y = 185 - 5.5t$ or $y = 130 - 5.5(t - 10)$. For $t = 8$, $y = 185 - 5.5(8) = 141$˚. The temperature of the soup 8 minutes after it was removed from the stove was approximately 141˚.	1 pt
	(iii)	The temperature values estimated using the average rate of change are the y-coordinates of the points on the secant line between $(0,185)$ and $(10,130)$. Since S is an exponential decay function that is decreasing and concave up on its entire domain, the secant line between $t = 0$ and $t = 10$ will be above the graph of S on the entire interval $(0, 10)$. Thus, estimates for the temperature of the soup between $t = 0$ and $t = 10$ found using the average rate of change will be strictly greater than the values predicted by the model.	1 pt
(C)	The value of M represents the ambient temperature of the room the soup is in and $y = M$ represents the horizontal asymptote of S. Analyzing the end behavior of S shows that $M = 72$. Shania finishes the soup by the time the temperature is 10˚ above the room temperature, or 82˚. Solving $S(t) = 82$ gives $t = 36.357$. Thus, an appropriate domain would be $0 \le t \le 36.357$ or $0 \le t \le 36$.		1 pt