

Key

A. Is g differentiable at $x = 2$? Justify your answer.

$$\lim_{x \rightarrow 2^-} g(x) = 3(2)^2 - 5(2) + 2 = 4$$

$$\lim_{x \rightarrow 2^-} g'(x) = \lim_{x \rightarrow 2^-} (6x - 5) = 7$$

No, because $\lim_{x \rightarrow 2^-} g'(x) \neq \lim_{x \rightarrow 2^+} g'(x)$.

$$\lim_{x \rightarrow 2^+} g(x) = \sqrt{5(2) + 6} = 4$$

$$\lim_{x \rightarrow 2^+} g'(x) = \lim_{x \rightarrow 2^+} \frac{5}{2\sqrt{5x+6}} = \frac{5}{8}$$

B. Which is greater: the average rate of change of k on $[1, 4]$ or the instantaneous rate of change of k at $x = 1$? Explain.

The instantaneous rate of change at $x=1$ because the slope of the tangent line at $x=1$ is greater than the slope of the secant line between $x=1$ and $x=4$.

C. Let $m(x) = h(f(x))$. Find $m'(3)$.

$$m'(x) = f'(x) \cdot h'(f(x)) \quad m'(3) = f'(3) \cdot h'(f(3)) = -1 \cdot h'(1) = -1 \cdot \frac{1}{2} = -\frac{1}{2}$$

D. Let $n(x) = h(x) \cdot f(x)$. Find $n'(3)$.

$$n'(x) = h(x) \cdot f'(x) + f(x) \cdot h'(x)$$

$$n'(3) = h(3) \cdot f'(3) + f(3) \cdot h'(3) = 5 \cdot -1 + 1(-4) = -5 - 4 = -9$$

E. Use a right Riemann sum with the three subintervals indicated by the table to estimate $\int_2^8 h(x) dx$.

$$\int_2^8 h(x) dx \approx 1(5) + 2(-5) + 3(14) = 37$$

F. Write the equation of the line tangent to the graph of h at $x = 8$.

$$y - 14 = -2(x - 8)$$

G. At which x -value(s) does the graph of j have horizontal tangent lines?

$$\text{When is } j'(x) = 0? \quad j'(x) = 12e^{3x} - \frac{5}{2} \quad \frac{5}{2} = 12e^{3x}$$

$$0 = 12e^{3x} - \frac{5}{2}$$

$$\frac{5}{24} = e^{3x}$$

$$\frac{\ln(5/24)}{3} = x$$

H. Find $\lim_{h \rightarrow 0} \frac{j(2+h) - j(2)}{h}$.

$$\text{Find } j'(2). \quad j'(2) = 12e^6 - \frac{5}{2}$$

I. Let $p(x) = \frac{k(x)}{h(x)}$. Find $p'(3)$.

$$p'(x) = \frac{h(x)k'(x) - k(x)h'(x)}{(h(x))^2} \quad p'(3) = \frac{5(0) - 4(-4)}{5^2} = \frac{16}{25}$$

J. Find $\lim_{x \rightarrow 5} \frac{k(x) - k(5)}{x - 5}$.

$$\lim_{x \rightarrow 5} \frac{k(x) - k(5)}{x - 5} = k'(5) = -4$$

K. Find $g'(6)$ and write a sentence interpreting its meaning.

For $x > 2$, $g'(x) = \frac{5}{2\sqrt{5x+6}}$ $g'(6) = \frac{5}{2\sqrt{36}} = \frac{5}{12}$

At $x=6$, the instantaneous rate of change of g is $\frac{5}{12}$

L. Are we guaranteed a value c for $4 < c < 7$ such that $f'(c) = \frac{1}{3}$? Explain. (or, slope of tangent line to g at $x=b$ is $\frac{5}{12}$)

No, f is not differentiable on the interval

so the MVT does not apply.

M. Does h have a relative maximum, minimum, or neither at $x = -1$? Justify your answer.

rel max because $h'(-1) = 0$ and $h''(-1) < 0$
(2nd deriv test)

N. Find $\lim_{x \rightarrow 2^-} f'(x)$.

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O. Give two x -values where the graph of h is above the x -axis and h is decreasing at an increasing rate.

$x = -3$ and $x = 3$ ($x = 8$ also works)

P. Let q be a function such that $q'(x) = g(x)$. At which x -values does the graph of q have a point of inflection? Explain.

$q''(x) = g'(x)$ $q'' = g'$ changes signs when $g'(x) = 0$
 $g'(x) = 6x - 5$ for $x \leq 2$

Q. Let $w(x) = \int_{-2}^x j(t) dt$. Find $w'(0)$.

$g'(x) > 0$ for $x > 2$

$6x - 5 = 0 \Rightarrow x = \frac{5}{6}$

$w'(0) = j(0) = 4e^0 - \frac{5}{2}(0) = 4$

R. Let $r(x) = \cos(\pi x) \cdot f(x)$. Find $r'(4.5)$.

$r'(x) = \cos(\pi x) \cdot f'(x) + f(x) \cdot -\pi \sin(\pi x)$

$r'(4.5) = \cos\left(\frac{9\pi}{2}\right) \cdot f'(4.5) + f(4.5) \cdot (-\pi \sin\left(\frac{9\pi}{2}\right)) = 0 + -0.5(-\pi \cdot 1) = \boxed{\frac{\pi}{2}}$

S. For $x \leq 2$, find $\lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$.

For $x \leq 2$, $g'(x) = 6x - 5$

T. Would the line tangent to the graph of h at $x = 5$ give an under- or over-approximation for $h(5.1)$? Explain.

underapproximation; the graph of h is concave up at $x=5$ so the tangent line lies below the curve at $x=5$.

U. Find $\int_1^3 f'(2x) dx$.

$$\frac{1}{2} f(2x) \Big|_1^3 = \frac{1}{2} (f(6) - f(2)) = \frac{1}{2} (1 - 2) = -\frac{1}{2}$$

V. Let $w(x) = \int_{-2}^x j(t) dt$. Find $w(0)$.

$$\begin{aligned} w(0) &= \int_{-2}^0 \left(4e^{3x} - \frac{5}{2}x \right) dx \\ &= \left[\frac{4}{3}e^{3x} - \frac{5x^2}{4} \right]_{-2}^0 = \frac{4}{3} - \left(\frac{4}{3}e^{-6} - 5 \right) \end{aligned}$$