Precalculus Unit 0: Prerequisites

Lesson	Learning Targets
0.1 The Cartesian Plane	• Connect the distance formula to the Pythagorean theorem by identifying the distance between two coordinate points as the hypotenuse of a right triangle. • Decompose segments on the coordinate plane into horizontal and vertical components to find missing lengths and coordinates. • Understand that the midpoint of a segment is equidistant from both endpoints and can be found by averaging the x and y coordinates.
0.2 Equations of Circles	• Define the graph of a circle to be the set of all points that are exactly one radius length away from a given center. • Identify the center and radius of a circle given an equation in standard form. • Write the equation of a circle given information about its center, radius, or diameter.
0.3 Solving Equations in Multiple Representations	• Use graphs, tables, and algebraic methods to find solutions to an equation or to approximate a solution to an equation. • Interpret a solution to an equation in a real-world context.
0.4 Reasoning with Formulas	• Re-write equations in terms of a different variable. • Explore relationships between the dimensions and volumes of various solids.
0.5 Linear Relationships	• Identify situations with a constant rate of change as describing linear relationships • Interpret a y-intercept and slope in context • Write an equation of a line in slope-intercept and point-slope form
0.6 Reasoning with Slope	• Understand that vertical lines have no "run" and horizontal lines have no "rise" and use this to write their equations • Use properties of parallel and perpendicular lines to reason about their slopes

0.7 Set Notation	• Understand how algebraic, set, and interval notation can be used to describe collections of objects, specifically numbers. • Given a visual or algebraic description of a set, determine the values that are included or excluded.

Precalculus Unit 1: Functions

Lesson	Learning Targets
1.1 Functions and Function Notation	• Understand that functions represent situations where one quantity determines another and each input has exactly one output. • Evaluate functions in function notation to find outputs for provided inputs and solve equations to find an input that yields a given output. • Analyze functions in multiple representations. • Interpret statements that use function notation in a given context.
1.2 Domain and Range	• Connect the domain of a function to its possible inputs and the range of a function to its possible outputs • Determine the domain and range of a function graphically, analytically, and numerically
1.3 Rates of Change and Graph Behavior	• Calculate and interpret an average rate of change over an interval using proper units • Identify the intervals on which a function is increasing, decreasing, or constant • Understand that the transition from increasing to decreasing results in a local maximum and the transition from decreasing to increasing results in a local minimum. • Find and interpret the zeros of a function in context
1.4 Library of Parent Functions	• Describe the key features of six parent functions: identity, absolute value, square root, quadratic, cubic, and reciprocal. • Analyze and compare the key features of parent functions. • Understand that the parent function represents the most basic function in a function family.
1.5 Transformations of Functions	• Apply vertical and horizontal shifts and stretches to parent functions to graph the transformed functions • Given an equation, describe the transformations from the parent function • Use the knowledge of transformations to determine the domain and range of a function.

1.6 Even and Odd Functions	• Write a rule for determining whether a function is even, odd, or neither. • Algebraically prove whether a function is even, odd, or neither. • Visually determine whether a function is even, odd, or neither.
1.7 Building Functions	• Interpret scenarios that require adding, subtracting, multiplying, or dividing functions • Combine standard function types using arithmetic operations. • Find the domain of a combination of functions by selecting the most restrictive domain; when two functions are divided, ensure that the denominator is not zero.
1.8 Compositions of Functions	• Understand that when two functions are composed, the output of one function becomes the input of the other • Interpret the input and output of composite functions in context • Evaluate and write equations for compositions of functions by plugging the inner function in as the independent variable of the outer function • Find the domain of a composition of functions
1.9 Inverse Functions	• Repeatedly solve equations of the form f(x)=c to recognize the need for a function that "undoes" the original function, i.e. to find the value in the domain that generates a certain output. • Read values of an inverse function from a graph or a table, given that the function has an inverse. • Find an inverse function algebraically • Verify by composition that one function is an inverse of another
1.10 Graphs of Inverse Functions	• Explain why a function must be one-to-one in order for the inverse to be a function • Explore relationships between the domains and ranges of functions and their inverses
1.11 Piecewise Functions	• Interpret and evaluate functions that have different rules for certain intervals of the domain. • Graph piecewise-defined functions. • Write equations for piecewise-defined functions given a scenario or a graph.

Precalculus Unit 2: Polynomial and Rational Functions

Lesson	Learning Targets
2.1 Connecting Quadratics	• Describe how standard, vertex, and intercept forms of quadratics demonstrate unique, but related features of parabolas. • Rewrite quadratics into intercept form by factoring; identify zeros from intercept form.
2.2 Completing the Square	• Recognize that quadratics of the form y=a(x-h)^2 are perfect squares, have a vertex on the x-axis, and have identical factors. • Rewrite quadratics into vertex form by completing the square
2.3 Polynomials in the Short Run	• Determine a polynomial's x- and y-intercepts from its equation • Use a root's multiplicity to describe the graph's behavior at an x-intercept • Determine the maximum number of turning points and roots of a polynomial using the Fundamental Theorem of Algebra
2.4 Polynomials in the Long Run	• Identify the degree and leading coefficient of any polynomial function • Describe a polynomial's end behavior by looking at its degree and leading coefficient • Sketch the graph of a polynomial function by attending to its x- and y-intercepts, zeros, and end behavior
2.5 Factor and Remainder Theorem	• Explain why when (x-k) is a factor of a polynomial, x=k is a zero of the polynomial • Interpret the remainder of a polynomial divided by (x-k) • Given one factor of a polynomial function, use division to find the remaining factors
2.6 Complex Zeros	• Identify cases where a polynomial will have a complex zero based on its graph or equation • Understand why complex zeros come in conjugate pairs and use this to describe the zeros of a polynomial • Add, subtract, and multiply complex zeros

2.7 Connecting Zeros Across Multiple Representations	• Given an equation of a polynomial, use a table or graph to find initial zeros, then find remaining zeros • Factor a polynomial completely
2.8 Intro to Rational Functions	• Explore the behavior of rational functions in a real-world context • Solve simple rational equations • Describe a rational function's end behavior by comparing growth rates of numerator and denominator functions • Determine when a rational function will have a slant asymptote and write its equation.
2.9 Graphing Rational Functions	• Distinguish between vertical asymptotes and holes • Use intercepts, asymptotes, and holes to sketch rational functions • Find the domain of a rational function

Precalculus Unit 3: Exponential and Logarithmic Functions

Lesson	Learning Targets
3.1 Exponential Functions	• Recognize scenarios that depict exponential growth by identifying a fixed percent/factor; distinguish exponential growth from linear growth • Write an exponential function modeling a scenario involving growth or decay by a fixed percent/factor • Understand that an exponential function is a function in which a positive constant (b) is raised to a variable (x), where 0<b<1 represents exponential decay and b>1 represents exponential growth; identify the value of b given points on an exponential function
3.2 Graphs of Exponential Functions	• Graph functions of the form y=b^x and identify horizontal asymptotes, domain, range, and key points • Describe transformations of an exponential function from graphs and equations • Reason about equivalent forms of exponential functions
3.3 Compound Interest and an Introduction to "e"	• Model the total value of a loan or investment at the end of a specified term by using repeated multiplication • Describe the effects of compounding quarterly, monthly, weekly, daily, and continually and make use of structure to arrive at the compound interest formula • Understand "e" as the base rate of growth for all continually growing processes
3.4 Logarithmic Functions	• Understand that a logarithm represents the exponent to which the base must be raised in order to attain the input value; use this understanding to evaluate logarithmic expressions. • Use exponential and logarithmic forms to write equivalent statements about powers. • Explain why the logarithmic function has a restricted domain. • Understand the inverse relationship between exponential and logarithmic functions of the same base, including the natural base, e.

3.5 Graphs of Logarithmic Functions	• Sketch logarithmic functions using the key points (1,0) and (b,1) • Connect key features (domain, range, asymptotes, and end behavior) on the graphs of exponential and logarithmic functions • Describe transformations of an logarithmic function and graph using the key points (1, 0) and (b, 1)
3.6 Logarithm Properties	• Discover the sum, difference, and power properties of logarithms by using inductive reasoning • Use properties of logarithms to expand or condense expressions • Identify and write equivalent logarithmic expression
3.7 Solving Exponential and Logarithmic Equations	• Rewrite exponential and logarithmic equations into their alternate form to isolate a variable • Solve exponential and logarithmic equations by applying the one-to-one property • Combine knowledge of inverse operations and logarithm properties to solve logarithmic equation
3.8 Exponential and Logarithmic Modeling	• Create models for half-life or double life problems and use solving techniques to answer questions based on the model. • Use exponential functions to model population growth, disease growth, and compounded interest. • Interpret answers to exponential and logarithmic modeling problems

Precalculus Unit 4: Trigonometric Functions

Lesson	Learning Targets
4.1 Right Triangle Trig	• Understand that sine, cosine, and tangent are functions that input angles and output ratios of specific sides in right triangles. • Given one trigonometric ratio, find the other two trigonometric ratios. • Use the trigonometric ratios to find missing sides in a right triangle
4.2 Inverse Trig Ratios	• Understand that the angles in a right triangle are determined by the ratio of the sides • Use the trigonometric ratios to find missing angles in a right triangle
4.3 Radians and Degrees	• Understand that a radian is an angle measure with an arc length of one radius • Use circumference to explain why 2pi radians corresponds to one full rotation • Use proportional reasoning to convert between angles measured in radians and degrees
4.4 Unit Circle	• Use the legs of special right triangles with hypotenuse of 1 to find ordered pairs on the unit circle at key angles. • Evaluate sine, cosine, and tangent values for angles in the first quadrant.
4.5 Unit Circles	• Evaluate sine, cosine, and tangent at angles on the unit circle. • Find angles on the unit circle that satisfy trigonometric equations with sine, cosine, and tangent. • Use reference angles to explain the symmetry of the unit circle in the four quadrants.
4.6 Other Trig Functions	• Define secant, cosecant, and cotangent functions as reciprocals of cosine, sine, and tangent, respectively. • Evaluate secant, cosecant, and cotangent functions at angles on the unit circle. • Find angles on the unit circle that satisfy a trigonometric equation with all six trig functions.

4.7 Graphing Sine and Cosine	• Understand that sine and cosine functions can be graphed by plotting angles on the x-axis, and ratios on the y-axis • Explain why the range of sine and cosine is [-1,1] • Use amplitude and period to describe key characteristics of the parent functions sin(x) and cos(x)
4.8 Transformations of Sine and Cosine Graphs	• Determine how the amplitude and period are affected by transformations • Graph transformed sine and cosine functions given an equation
4.9 Graphing Secant and Cosecant	• Understand how asymptote equations are found for secant and cosecant by finding when the function in the denominator is equal to 0. • Graph secant and cosecant and identify the period and asymptote equations. • Write equations of secant and cosecant when provided with key features of the graph
4.10 Graphing Tangent and Cotangent	• Understand how asymptote equations are found for tangent, and cotangent by finding when the function in the denominator is equal to 0. • Graph tangent and cotangent and identify the period and asymptote equations. • Write equations of tangent and cotangent when provided with key features of the graph
4.11 Inverse Trig Functions	• Explain why and how the domains of sine, cosine, and tangent must be restricted to create an inverse function. • Use the restricted domains of the sine, cosine, and tangent, and reason to reason about the domains and ranges of the inverse functions. • Evaluate inverse trig expressions and equations.
4.12 Trigonometric Modeling	• Use trigonometric equations to model real-world periodic behavior. • Interpret period and amplitude in context.
4.13 Trigonometric Identities	• Explain the relationship between the six trigonometric ratios. • Simplify trigonometric expressions using trigonometric and Pythagorean identities.

Precalculus Unit 5: Applications of Trigonometry

Lesson	Learning Targets
5.1 Law of Sines	• Discover the relationship between sides and their opposite angles in any triangle • Identify the conditions needed to use the Law of Sine • Solve for missing sides and angles using the Law of Sines
5.2 The Ambiguous Case (SSA)	• Understand why when given two sides and a non-included angle, there could be 0, 1, or 2 triangles formed. • Determine the number of triangles that can be formed when given two sides and the non-included angle. • Solve triangles using the Law of Sines.
5.3 Law of Cosines	• Understand the Law of Cosines as a more general form of the Pythagorean Theorem for oblique triangles • Solve for missing sides and angles using the Law of Cosines
5.4 Area and Applications of Laws	• Apply Law of Sines and Law of Cosines to applied problems • Identify general area formulas for oblique triangles based on the given parts • Reason about the validity of a mathematical model
5.5 Vectors	• Interpret vectors as quantities that have both magnitude and direction • Write vectors in component form • Add, subtract, and scale vectors algebraically and graphically
5.6 Polar Coordinates	• Understand the polar system as an alternate way of describing locations by using a radius and an angle • Use coterminal angles and reflected radii to name polar points in multiple ways • Convert between polar and Cartesian coordinates
5.7 Equations in Polar and Cartesian Form	• Use the conversion formulas to rewrite equations of graphs in their alternate forms. • Recognize that some graphs are more easily described in polar coordinates whereas others are more easily described in Cartesian coordinates

5.8 Polar Graphs Part 1	• Generalize polar equations for circles and roses based on inquiry • Describe key features of roses from their equation (symmetry, number of petals, location of petals) • Understand graphing polar equations through point-by-point graphing
5.9 Polar Graphs Part 2	• Identify special types of limacons by comparing values of the parameters, a and b • Describe the key features of limacons from their equation • Reason about the range and intercepts of limacons
5.10 Parametric Equations	• Define a parameter as a third variable that is used to generate values of x and y. • Graph non-trigonometric parametric equations from tables • Convert between parametric and Cartesian equations by eliminating or adding a parameter
5.11 Parametric Equations (With Trigs)	• Graph parametric equations involving trigonometry using tables • Use the Pythagorean identity to convert between parametric and Cartesian equations of circles and ellipses • Understand the advantages of parameterizing a curve

Precalculus Unit 6: Systems of Equations

Lesson	Learning Targets
6.1 What is a Solution?	• Define a solution as an ordered pair that satisfies an equation and is thus on the graph of that equation • Use algebraic and graphical methods to find solutions to systems of equations • Determine when a system of equations will be inconsistent
6.2 Solving Systems with Substitution	• Identify equivalent expressions in order to make substitutions • Interpret scenarios that represent breaking even • Select an appropriate strategy for solving a system
6.3 Solving Systems with Elimination	• Explain the method of elimination using scaling and comparison • Determine the conditions that result in dependent, independent, and inconsistent systems • Connect contextual, graphical, and analytical representations of dependent, independent, and inconsistent systems
6.4 Solving Systems in 3 Variables	• Explain the importance of row-echelon form in solving a three variable linear system • Solve independent linear systems in three variables using Gaussian elimination • Determine when a system in three variables results in an inconsistent or dependent system • Write the general solution to a dependent system of equations
6.5 Partial Fractions	• Apply rules for adding and subtracting fractions to rational function • Decompose rational functions with distinct, linear factors into partial fractions

Precalculus Unit 7: Sequences and Series

Lesson	Learning Targets
Unit Intro: Introducing Sequences	• Describe visual patterns with equations • Explore sequences with a constant second difference
7.1 Using Sequences and Series to Describe Patterns	• Use sequence and series notation to describe patterns • Distinguish between explicit and recursive formulas • Find partial sums by hand and with a calculator
7.2 Arithmetic Sequences and Series	• Write explicit rules to describe sequences with a common difference • Generate a sum formula for arithmetic sequences using the idea of averages • Find missing terms of an arithmetic sequence • Solve for the term number in which a sequence reaches a particular sum.
7.3 Geometric Sequences and Finite Series	• Write explicit rules to describe sequences with a common ratio • Generalize a pattern to find the sum of a finite geometric sequence • Solve for the term number in which a sequence reaches a particular sum
7.4 Infinite Geometric Sequences and Series	• Explore the behavior of a geometric sequence as n approaches infinity. • Understand when and how adding infinitely many terms can lead to a finite sum.
7.5 Proof by Induction	• Understand what constitutes as mathematical proof • Explain the importance of the base case, induction hypothesis, and induction step in writing a proof by induction • Write a four-step proof by induction

Precalculus Unit 8: Limits

Lesson	Learning Targets
8.1 What is a Limit?	• Understand limits as predicted or intended outputs based on surrounding behavior • Evaluate limits using graphs and tables • Use limit notation to describe function behavior to the left and right of a particular x-value
8.2 Evaluating Limits Graphically	• Evaluate limits using graphs • Connect expressions of limits across multiple representations
8.3 Evaluating Limits with Direct Substitution	• Use direct substitution to evaluate limits • Evaluate limits of piecewise functions • Determine when direct substitution will work as a strategy for evaluating limits
8.4 Evaluating Limits Analytically	• Connect factors and zeros of rational functions to holes and vertical asymptotes • Use limits to describe function behavior at holes and asymptotes • Use known limits to reason about unknown limits • Choose an appropriate strategy for evaluating a limit
8.5 Continuity	• Classify discontinuities as jump, removable, or infinite from a graph or equation • Justify whether a function is continuous at a particular x-value using the definition of continuity • Determine when and how discontinuous functions can be made continuous
8.6 Intermediate Value Theorem	• Verify the conditions of the Intermediate Value Theorem • Make conclusions about the outputs of a function using the Intermediate Value Theorem • Apply the Intermediate Value Theorem to prove the existence of roots.
8.7 Limits at Infinity	• Describe vertical and horizontal asymptotes using limit notation • Evaluate limits as x approaches infinity by comparing growth rates of numerator and denominator functions

Precalculus Unit 9: Derivatives

Lesson	Learning Targets
Unit 9 Intro: Introduction to Derivatives	• Explore rates of change in context • Estimate and compare rates of change from a graph • Interpret and calculate average rate of change
9.1 Average versus Instantaneous Rates of Change	• Understand average rate of change as the slope over an interval and instantaneous rate of change as the slope at a point • Estimate instantaneous rate of change by calculating the average rate of change over a short interval
9.2 Calculating Instantaneous Rate of Change	• Understand that limits turn an estimate of the instantaneous rate of change into the exact value of the slope • Set-up and evaluate a limit expression that gives the slope at a single point • Write the equation of a tangent line to a curve at a given point
9.3 The Derivative Function	• Define a derivative as the slopes graph of an original function • Understand that the derivative is itself a function that outputs the slope of the curve at any x-value • Find an equation for the derivative function using the limit definition of the derivative • Use derivative notation to refer to derivative functions and derivatives evaluated at a point
9.4 Derivative Shortcuts	• Recognize patterns to find shortcuts for derivatives of constant, linear, and power functions • Apply derivative shortcuts to polynomials and other power functions
9.5 Differentiability	• Understand the conditions needed to draw a tangent line at a point and define this property as differentiability • Explain the relationship between continuity and differentiability • Justify whether a function is differentiable using the limit definition

9.6 Connecting f and f′	• Use the first derivative to justify whether a function is increasing, decreasing, or not changing. • Use rates of change to describe when a function has a relative maximum or minimum and relate this to optimization • Justify where a function has a relative extrema using the first derivative test • Connect graph features on the graphs of f and f′
9.7 Derivatives of Sine and Cosine	• Connect the behavior of sine and cosine functions to their derivatives • Evaluate derivatives that include sine and cosine • Write equations of lines tangent to the sine and cosine graphs
9.8 Product Rule	• Use visuals to make sense of the product rule • Evaluate derivatives using the product rule
9.9 Quotient Rule	• Evaluate derivatives using the quotient rule

Precalculus Unit 10: Conic Sections

Lesson	Learning Targets
10.1 Intro to Conic Sections	• Understand that a conic section is the shape formed when a plane and a cone intersect. • Identify the 4 shapes formed as parabola, circle, ellipse, and hyperbola.
10.2 Defining Parabolas	• Understand that a parabola is the set of points equidistant from a fixed point, called the focus, and a line, called the directrix. • Given the equation of a parabola, identify the focus, vertex, directrix, and two additional points. • Graph a parabola from its key features.
10.3 Working with Parabolas	• Given the equation of a parabola, identify the focus, vertex, directrix, and two additional points. • Graph a horizontal parabola and identify the key features. • Complete the square to re-write the equation of a parabola in vertex form.
10.4 Defining Ellipses	• Understand that an ellipse is like a circle, but with different horizontal and vertical dimensions. • Graph an ellipse and identify key features such as vertices, major axis length, and minor axis length. • Write the equation of an ellipse in standard conic form by completing the square.
10.5 Working with Ellipses	• Understand that for any point on an ellipse, the sum of the distances from the point to each focus is the length of the major axis. • Derive the equation of c for an ellipse by investigating the relationship between a, b, and c. • Write the equation of an ellipse when given key features such as the foci, vertices, and major or minor axis length.
10.6 Defining Hyperbolas	• Explore the graph and key features of a hyperbola. • Write the equation of a hyperbola when given the key features. • Use the center, vertices, and asymptotes of a hyperbola to construct the graph.

10.7 Working with Hyperbolas	• Understand that for any point on a hyperbola, the difference between the distances from any point to each focus is the length of the transverse axis. • Write the equation of a hyperbola when given key features such as the foci, vertices, transverse or conjugate axis length, or asymptote equations.