Limits, Continuity, Differentiability

Evaluate $\lim_{x \to a} f(x)$

Limits, Continuity, Differentiability

$\lim_{x \to \infty} f(x)$ means

Limits, Continuity, Differentiability

$\lim_{x \to a} f(x)$ exists if

Limits, Continuity, Differentiability

$f(x)$ is continuous at $x = a$ if

Limits, Continuity, Differentiability

$f(x)$ is differentiable at $x = a$ if

Limits, Continuity, Differentiability

L′hôspital′s Rule

Derivatives

Limit definition of $f'(a)$ (Derivative at the point $x = a$)

Derivatives

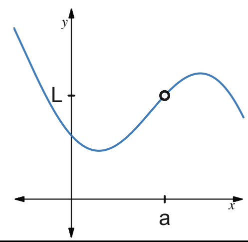
Limit definition of $f'(x)$ (Equation of derivative function)

Derivatives

$$\frac{d}{dx}(c)$$

Derivatives

$$\frac{d}{dx}(x^n)$$

Find end behavior. Use TFEPLC and rational function rules comparing the degree of the numerator and denominator.	Simplify. Let $x = a$. (Direct substitution)
$\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x)$ and $\lim\limits_{x\to a} f(x) = f(a)$	$\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x)$ 
If $\lim\limits_{x\to a} \frac{f(x)}{g(x)}$ is indeterminate $\left(\frac{0}{0} \text{ or} \frac{\infty}{\infty}\right)$ Then $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{f\prime(x)}{g\prime(x)}$	$f(x)$ is continuous at $x = a$ and $\lim\limits_{x\to a^-} f'(x) = \lim\limits_{x\to a^+} f'(x)$
$\lim\limits_{h\to 0} \frac{f(x+h) - f(x)}{h}$	$\lim\limits_{x\to a} \frac{f(x)-f(a)}{x-a}$ or $\lim\limits_{h\to 0} \frac{f(a+h) - f(a)}{h}$
nx^{n-1}	0

Derivatives

$$\frac{d}{dx}(c(f(x))$$

Derivatives

Product Rule

$$\frac{d}{dx}(f(x)\cdot g(x))$$

Derivatives

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$$

Derivatives

Chain Rule – Derivative of a composite function

$$\frac{d}{dx}(f(g(x))$$

Derivatives

$$\frac{d}{dx}(\sin x)$$

Derivatives

$$\frac{d}{dx}(\cos x)$$

Derivatives

$$\frac{d}{dx}(\tan x)$$

Derivatives

$$\frac{d}{dx}(\cot x)$$

Derivatives

$$\frac{d}{dx}(\sec x)$$

Derivatives

$$\frac{d}{dx}(\csc x)$$

$f(x)g'(x) + g(x)f'(x)$	$c \cdot f'(x)$
$g'(x) \cdot f'(g(x))$	$\frac{g(x) \cdot f'(x) - f(x)g'(x)}{\left(g(x)\right)^2}$
$-\sin x$	$\cos x$
$-\csc^2 x$	$\sec^2 x$
$-\csc x \cot x$	$\sec x \tan x$

Derivatives

$$\frac{d}{dx}(\ln x)$$

Derivatives

$$\frac{d}{dx}(e^x)$$

Derivatives

$$\frac{d}{dx}(a^x)$$

Derivatives

$$\frac{d}{dx}(\sin^{-1} x)$$

Derivatives

$$\frac{d}{dx}(\tan^{-1} x)$$

Derivatives

Very rare!

$$\frac{d}{dx}(\sec^{-1} x)$$

Derivatives

Derivatives of inverse cofunctions

$(\arccos x, \operatorname{arccot} x, \operatorname{arccsc} x)$

Derivatives

Derivatives of inverse functions

Let $g(x) = f^{-1}(x)$

What is $g'(x)$?

Derivatives

$$\frac{d}{dx}(|x|)$$

Derivatives

Implicit differentiation

e^x	$\frac{1}{x}$
$\frac{1}{\sqrt{1-x^2}}$	$a^x \cdot \ln a$
$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\frac{1}{1+x^2}$
$g'(x) = \frac{1}{f'(g(x))}$ Slopes at inverse points are reciprocals.	Opposite of the derivative of the corresponding function
Differentiate both sides with respect to x. Treat y as a function of x and apply chain rule.	Find the slope using the graph. There is no shortcut rule for this function!

Other Average rate of change of f on $[a, b]$	Other How to find critical points
Other How to find the absolute maxima and minima of f	Other How to find inflection points
Other How to find vertical asymptotes	Other How to find horizontal asymptotes
Other How to solve related rates problems	Other
Integration $\int dx$	Integration $\int a\,dx$

A critical point is any interior point where $f'(x) = 0$ or $f'(x)$ is undefined.	Find the slope of the secant line. $\frac{f(b) - f(a)}{b - a}$
Inflection points will exist where f'' changes signs.	Candidate's test: Compare function outputs at endpoints and points where f' changes sign. The *y*-value is the max/min!
Use end behavior. Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$	Set the denominator equal to 0 and solve for *x*.
	Write an equation that models the situation. Implicitly differentiate all variables that are changing with respect to time.
$ax + C$	$x + C$

Integration $$\int x^n \, dx$$	Integration $$\int x^{-1} \, dx \text{ OR } \int \frac{1}{x} \, dx \text{ OR } \int \frac{dx}{x}$$
Integration $$\int \cos x \, dx$$	Integration $$\int \sin x \, dx$$
Integration $$\int \sec^2 x \, dx$$	Integration $$\int \csc^2 x \, dx$$
Integration $$\int \sec x \tan x \, dx$$	Integration $$\int \csc x \cot x \, dx$$
Integration $$\int e^x \, dx$$	Integration $$\int a^x \, dx$$

$\ln\lvert x\rvert + C$	$\frac{x^{n+1}}{n+1} + C$
$-\cos x + C$	$\sin x + C$
$-\cot x + C$	$\tan x + C$
$-\csc x + C$	$\sec x + C$
$\frac{1}{\ln a} \cdot a^x + C$	$e^x + C$

Integration

$$\int \frac{1}{\sqrt{1-x^2}}\,dx$$

Integration

$$\int \frac{1}{1+x^2}\,dx$$

Integration

Very rare!

$$\int \frac{1}{|x|\sqrt{x^2-1}}\,dx$$

Integration

$$\int_a^b \sqrt{r^2-x^2}\,dx$$

Interpretations and Justifications

Intermediate Value Theorem (IVT)

Interpretations and Justifications

Mean Value Theorem

Interpretations and Justifications

A particle is speeding up if

Interpretations and Justifications

A particle is slowing down if

Interpretations and Justifications

Interpreting a derivative at $x = a$ in context

Interpretations and Justifications

Interpreting an integral over $[a, b]$ in context

$\tan^{-1} x + C$	$\sin^{-1} x + C$
Use geometry! This is a semicircle of radius r!	$\sec^{-1} x + C$
If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) then $\frac{f(b) - f(a)}{b - a} = f'(c)$ For some value c in (a, b). Slope of the secant line = slope of the tangent line. Average rate of change = instantaneous rate of change.	If $f(x)$ is continuous on $[a, b]$ then $f(x)$ takes on every value between $f(a)$ and $f(b)$.
$v(t)$ and $a(t)$ have opposite signs	$v(t)$ and $a(t)$ have the same sign
From $x =$ ___to x = ___the net change in _________ is ___ (units). 1. Interval 2. Context 3. Net change or accumulation 4. Units	At $x =$ ___, the _________is changing at a rate of _______ (units). 1. Instant 2. Context 3. Rate of change 4. Units

Interpretations and Justifications

f is increasing when

Interpretations and Justifications

f is decreasing when

Interpretations and Justifications

f is concave up when

Interpretations and Justifications

f is concave down when

Interpretations and Justifications

f has a relative max when

Interpretations and Justifications

f has a relative min when

Interpretations and Justifications

When a question asks you to "interpret the meaning"

Interpretations and Justifications

When a question asks you to "justify"

Interpretations and Justifications

Interpretations and Justifications

$f' < 0$	$f' > 0$
$f'' < 0$	$f'' > 0$
$f'(x) = 0$ or undefined and f' changes from – to + OR $f'(x) = 0$ and $f''(x) > 0$	$f'(x) = 0$ or undefined and f' changes from + to – OR $f'(x) = 0$ and $f''(x) < 0$
The response must include a definition or theorem. Be sure to verify the conditions of the theorem!	The response must include: 1. Meaning in context 2. Units 3. Time

Area / Volume

Area by Riemann sums (rectangles)

Area / Volume

Left Riemann sums are an overestimate if / underestimate if

Right Riemann sums are an overestimate if / underestimate if

Area / Volume

Area by trapezoids

Area / Volume

Area using trapezoids will be an overestimate if… underestimate if…

Area / Volume

Total area =

Area / Volume

How to find the area between curves

Area / Volume

Volume: Disks

Area / Volume

Volume: Washers

Area / Volume

Volume: Cross-sections

Area / Volume

Riemann Sum	Over estimate when f is	Under estimate when f is
Left	decreasing	increasing
Right	increasing	decreasing

$$\sum_{k=1}^{n} f(x_k) \cdot \Delta x$$

height width

The y-value/height could be at the left, right, or midpoint of each interval.

f is concave up

f is concave down

$$A = \frac{1}{2}\Delta x(y_0 + y_1) + \frac{1}{2}\Delta x(y_1 + y_2) + \dots$$

$$\int_a^b (\text{higher} - \text{lower})\, dx = \int_a^b [f(x) - g(x)]\, dx$$

OR

$$\int_c^d (\text{right} - \text{left})\, dy = \int_c^d [f(y) - g(y)]\, dy$$

$$\int_a^b |f(x)|\, dx$$

$$\int_a^b \pi(R^2 - r^2)dx$$

OR

$$\int_c^d \pi(R^2 - r^2)dy$$

R = larger measurement from axis of revolution

$$\int_a^b \pi(\text{radius})^2 dx$$

OR

$$\int_c^d \pi(\text{radius})^2 dy$$

$$\int_a^b (\text{area of one cross section})\, dx$$

OR

$$\int_C^d (\text{area of one cross section})\, dy$$

Other $\int_a^b f'(x)\,dx$	Other Final amount / Final position $f(b) =$
Other Total distance traveled	Other Position Velocity Acceleration
Other Speed	Other If $F(x) = \int_a^{v(x)} f(t)dt$ then $F'(x) =$
Other Average value of a function f on $[a, b]$	Other Solving a differential equation
Other Solve $\frac{dy}{dx} = ky$	Other Displacement

$f(a)+\int_a^b f'(x)\,dx$ Starting amount + accumulation	$f(x)\Big]_a^b = f(b)-f(a)$ This gives the net change in the amount or position.
$f(t), x(t)$, or $y(t)$ $f'(t), x'(t)$, or $y'(t)$ $f''(t), x''(t)$, or $y''(t)$	$\int_a^b \lvert v(t)\rvert\,dt$
$f(v(x))\cdot v'(x)$ Plug in the upper limit of integration. Multiply by the derivative of the upper limit.	$\lvert v(t)\rvert$
Separate Integrate Solve for C Isolate y Select the proper solution	$\frac{1}{b-a}\int_a^b f(x)\,dx$
$\int_{t_1}^{t_2} v(t)\,dt$	Exponential growth! $y=y_0e^{kt}$