### AP Calculus Syllabus (Math Medic)

### **Course Overview**

AP Calculus focuses on students' understanding of calculus concepts and provides experience with methods and applications. The emphasis on the big ideas of calculus (e.g., modeling change, approximation and limits, and analysis of functions) allows learners to make connections between content and see calculus as a unified whole, rather than a list of fragmented, separate topics.

The courses feature a multi-representational approach to calculus, with concepts, results, and problems expressed graphically, numerically, analytically, and verbally. Exploring connections among these representations builds understanding of how calculus applies limits to develop important ideas, definitions, formulas, and theorems. A sustained emphasis on clear communication of methods, reasoning, justifications, and conclusions is woven throughout the course.

This course will be taught using an Experience First, Formalize Later (EFFL) learning model, where students work collaboratively to think, to discuss, and to construct their own understanding of new content before the teacher helps students to arrive at formal definitions and formulas.

### **Course Expectations**

Students are expected to fully participate in small groups when working through the activity for each new lesson. Each member of the group should contribute to the discussion in the group, as well as to listen to and critique ideas from others.

During the whole-class debrief of the collaborative activity, students will be asked to share ideas generated in their groups. The teacher will guide the discussion towards a more formal understanding of what was learned in the activity. Students will be expected to record any new learning that results from the class discussion.

At the end of each lesson, students work individually or in small groups to complete the Check Your Understanding questions. These formative assessments reinforce the key ideas of the activity and extend students' thinking to other contexts, representations, or applications.

Students are expected to complete all homework problems to the best of their ability. If they need additional support, they can refer to the additional resources listed below.

### Additional Resources

The Math Medic Assessment Platform for AP Calculus provides pre-made homework, quizzes, and tests perfectly aligned to the Math Medic lessons, and covering every topic of the CED (Units 1-8). This resource also allows teachers to edit assessments and build their own assessments from a bank of high-quality questions.

The College Board also provides a plethora of resources to help students learn, practice, and review the content in AP Precalculus.

- AP Daily videos are short 5 9-minute videos found in AP Classroom that cover all of the content in the AP Precalculus course.
- Students looking for more practice can request access to additional questions in AP Classroom.

At the end of the year, students will use the Math Medic AP Calculus Exam Review Course, which includes videos, practice problems, and AP Exam Tips.

### **Student Practice**

The Math Medic Assessment Platform for AP Calculus provides high-quality homework assignments, quizzes, and tests that can be assigned to students digitally or in print. These questions are perfectly aligned to the Math Medic AP Calculus lessons and cover every topic in the CED (Units 1-8). The questions vary in difficulty, feature multiple representations, and give students ample opportunities to practice the Mathematical Practices outlined for AP Calculus.

Throughout each unit, Topic Questions from AP Classroom will also be provided to help students check their understanding. The Topic Questions are especially useful for confirming understanding of difficult or foundational topics before moving on to new content or skills that build upon prior topics. Topic Questions can be assigned before, during, or after a lesson, and as in-class work or homework. Students will get rationales for each Topic Question that will help them understand why an answer is correct or incorrect, and their results will reveal misunderstandings to help them target the content and skills needed for additional practice.

At the end of each unit or at key points within a unit, Personal Progress Checks will be provided in class or as homework assignments in AP Classroom. Students will get a personal report with feedback on every topic, skill, and question that they can use to chart their progress, and their results will come with rationales that explain every question's answer.

### Textbook Requirement **CR1**

The Math Medic AP Calculus lessons have been vetted by the College Board and meet the Course Audit curricular requirements. They appear on the textbook list published by the College Board on the AP Calculus AB Course Audit page on AP Central:

• Stecher, Sarah. Math Medic AP Calculus. Math Medic.

### Course Outline and Pacing

- September/October- CED Units 1, 2, and 3
- November/December- CED Units 4 and 5
- January/February– CED Units 6 and 7
- March CED Unit 8
- April/May AP Exam Review

### Course Outline and Description **GR2**

All lessons are from the AP Calculus curriculum on Math Medic.

#### Math Medic Unit 1: Intro to Calculus

#### Big Ideas: CHA, LIM, FUN

CED Topic 1.1 Introducing Calculus: Can Change Occur at an Instant? (Skill 2.B)

• Lesson 1.1: A Wonder-fuel Intro to Calculus

CED Topic 1.2 Defining Limits and Using Limit Notation (Skill 2.B)

- Lesson 1.2: Can You Predict the Height of Steve Nash's Free Throw? CED Topic 1.3 Estimating Limit Values from Graphs (Skill 2.B)
  - Lesson 1.2: Can You Predict the Height of Steve Nash's Free Throw?
  - Lesson 1.2 (Part 2): Quiz, Quiz Trade

CED Topic 1.4 Estimating Limit Values from Tables (Skill 2.B)

- Lesson 1.2: Can You Predict the Height of Steve Nash's Free Throw?
- CED Topic 1.5 Determining Limits Using Algebraic Properties of Limits (Skill 1.E)
  - Lesson 1.5: Contestants, Can You Solve This Limit?
- CED Topic 1.6 Determining Limits Using Algebraic Manipulation (Skill 1.C)
  - Lesson 1.5: Contestants, Can You Solve This Limit?

CED Topic 1.7 Selecting Procedures for Determining Limits (Skill 1.C)

- Lesson 1.5: Contestants, Can You Solve This Limit?
- Review 1.1-1.5: Solve and Spell
- CED Topic 1.8 Determining Limits Using the Squeeze Theorem (Skill 3.C)
  - Lesson 1.6: How Many Reese's Puffs Are in the Jar?
- CED Topic 1.9 Connecting Multiple Representations of Limits (Skill 2.C)
  - Unit 1 Review: Limit Card Sort
- CED Topic 1.10 Exploring Types of Discontinuities (Skill 3.B)
  - Lesson 1.7: Frozen Yogurt Feuds
- CED Topic 1.11 Defining Continuity at a Point (Skill 3.C)
  - Lesson 1.7: Frozen Yogurt Feuds
- CED Topic 1.12 Confirming Continuity over an Interval (Skill 1.E)
  - Lesson 1.7: Frozen Yogurt Feuds

CED Topic 1.13 Removing Discontinuities (Skill 1.E)

• Lesson 1.8: Can We Fix What's Broken?

CED Topic 1.14 Connecting Infinite Limits and Vertical Asymptotes (Skill 3.D)

- Lesson 1.4: How Much Do We Remember from School?
- Lesson 1.2 (Part 2): Quiz, Quiz Trade

CED Topic 1.15 Connecting Limits at Infinity and Horizontal Asymptotes (Skill 2.D)

• Lesson 1.4: How Much Do We Remember from School?

CED Topic 1.16 Working with the Intermediate Value Theorem (Skill 3.E)

• Lesson 1.9: Are You a 5-Star Uber Driver?

### Math Medic Unit 2: Differentiation

Big Ideas: CHA, LIM, FUN

CED Topic 2.1 Defining Average and Instantaneous Rates of Change at a Point (Skill 2.B)

• Lesson 2.1: Can a Human Break the Sound Barrier?

CED Topic 2.2 Defining the Derivative of a Function and Using Derivative Notation (Skill 1.D, 4.C)

• Lesson 2.2: Can We Find a Slope There?

CED Topic 2.3 Estimating Derivatives of a Function at a Point (Skill 1.E)

- Lesson 2.1: Can a Human Break the Sound Barrier?
- Lesson 2.3: Is There a Shortcut?

CED Topic 2.4 Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist (Skill 3.E)

• Lesson 2.3: Is There a Shortcut?

CED Topic 2.5 Applying the Power Rule (Skill 1.E)

• Lesson 2.4: Toothpick Tangents

CED Topic 2.6 Derivative Rules: Constant, Sum, Difference, and Constant Multiple (Skill 1.E)

• Lesson 2.4 Toothpick Tangents

CED Topic 2.7 Derivatives of cos(x), sin(x), e^x, and ln(x) (Skill 1.E)

- Lesson 2.5: Toothpick Tangents (Part 2)
- Lesson 2.6: How is Netflix's Revenue Changing?

CED Topic 2.8 The Product Rule (Skill 1.E)

• Lesson 2.7: Is There a Rule for Products?

CED Topic 2.9 The Quotient Rule (Skill 1.E)

• Lesson 2.8: Is There a Rule for Quotients

CED Topic 2.10 Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions (Skill 1.E)

• Lesson 2.9: Are There More Trig Derivatives?

#### Math Medic Unit 3: Differentiating Composite, Implicit, and Inverse Functions

Big Ideas: FUN

CED Topic 3.1 The Chain Rule (Skill 1.C)

• Lesson 3.1: How is Lindt Chocolate Made?

CED Topic 3.2 Implicit Differentiation (Skill 1.E)

- Lesson 3.2: The Tangent Line Problem
- CED Topic 3.3 Differentiating Inverse Functions (Skill 3.G)
  - Lesson 3.3: What's Your Slope?
- CED Topic 3.4 Differentiating Inverse Trigonometric Functions (Skill 1.E)
  - Lesson 3.4: Getting Triggy With It
- CED Topic 3.5 Selecting Procedures for Calculating Derivatives (Skill 1.C)
  - Unit 3 Review: Stinky Feet

CED Topic 3.6 Calculating Higher-Order Derivatives (Skill 1.E)

• Lesson 3.5: Can We Try Again?

# Math Medic Unit 4: Contextual Applications of Differentiation

Big Ideas: CHA, LIM

CED Topic 4.1 Interpreting the Meaning of the Derivative in Context (Skill 1.D)

• Lesson 4.1: The Burger Blitz Business

CED Topic 4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration (Skill 1.E)

• Lesson 4.2: The Lovely Lady Bug

CED Topic 4.3 Rates of Change in Applied Contexts Other Than Motion (Skill 2.A)

• Lesson 4.3: How Many Shoppers on Black Friday?

CED Topic 4.4 Introduction to Related Rates (Skill 1.E)

• Lesson 4.4: How Fast is the Balloon Growing?

CED Topic 4.5 Solving Related Rates Problems (Skill 3.F)

• Lesson 4.5: How Fast is the Water Level Changing?

CED Topic 4.6 Approximating Values of a Function Using Local Linearity and Linearization (Skill 1.F)

• Lesson 4.6: Can a Straight Line Outsmart a Curve?

CED Topic 4.7 Using L'Hospital's Rule for Determining Limits of Indeterminate Forms (Skill 3.D)

• Lesson 4.7: How Do We Decipher Mixed Messages?

### Math Medic Unit 5: Analytical Applications of Differentiation

Big Ideas: FUN

CED Topic 5.1 Using the Mean Value Theorem (Skill 3.E)

• Lesson 5.1: Can Calculus Get You Fired?

CED Topic 5.2 Extreme Value Theorem, Global vs. Local Extrema, and Critical Points (Skill 3.E)

• Lesson 5.2: How Much for a Carton of Eggs?

CED Topic 5.3 Determining Intervals on Which a Function is Increasing or Decreasing (Skill 2.E)

- Lesson 5.3: When Should I Exit the Market?
- CED Topic 5.4 Using the First Derivative Test to Determine Relative (Local) Extrema (Skill 3.D)
  - Lesson 5.3: When Should I Exit the Market?

CED Topic 5.5 Using the Candidates Test to Determine Absolute (Global) Extrema (Skill 1.E)

• Lesson 5.4: How Fast is Victoria Running Her Marathon?

CED Topic 5.6 Determining Concavity of Functions over Their Domains (Skill 2.E)

• Lesson 5.5: How Fast Does a Video Go Viral?

CED Topic 5.7 Using the Second Derivative Test to Determine Extrema (Skill 3.D)

• Lesson 5.6: What Clues Can I Use?

CED Topic 5.8 Sketching Graphs of Functions and Their Derivatives (Skill 2.D)

• Lesson 5.7: How Fast is the Line Moving?

CED Topic 5.9 Connecting a Function, Its First Derivative, and Its Second Derivative (Skill 2.D)

• Lesson 5.7: How Fast is the Line Moving?

CED Topic 5.10 Introducing to Optimization Problems (Skill 2.A)

• Lesson 5.8: Can Good Design Minimize Costs?

CED Topic 5.11 Solving Optimization Problems (Skill 3.F)

• Lesson 5.8: Can Good Design Minimize Costs?

Topic 5.12 Exploring Behaviors of Implicit Relations (Skill 1.E, 3.E)

• Lesson 5.9: What About Us?

#### Math Medic Unit 6: Integration and Accumulation of Change Big Ideas: CHA, LIM, FUN

CED Topic 6.1 Exploring Accumulations of Change (Skill 4.B)

• Lesson 6.1: How Many Scones Are in Supply?

CED Topic 6.2 Approximating Areas with Riemann Sums (Skill 1.F)

- Lesson 6.2: Fast and Curious
- Lesson 6.3: Fast and Curious (Part 2)

CED Topic 6.3 Riemann Sums, Summation Notation, and Definite Integral Notation (Skill 2.C)

- Lesson 6.4: How Confident Are You?
- Lesson 6.12: Back to the Basics

CED Topic 6.4 The Fundamental Theorem of Calculus and Accumulation Functions (Skill 1.D)

• Lesson 6.5: Under Cover

CED Topic 6.5 Interpreting the Behavior of Accumulation Functions Involving Area (Skill 2.D)

• Lesson 6.6: How Many Canoes Are Available?

CED Topic 6.6 Applying Properties of Definite Integrals (Skill 3.D)

• Lesson 6.7: How Much Trash Has Been Removed?

CED Topic 6.7 The Fundamental Theorem of Calculus and Definite Integrals (Skill 3.D)

- Lesson 6.8: How Far Did Nadine Skate?
- CED Topic 6.8 Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation (Skill 4.C)
  - Lesson 6.9: What Came Before?

CED Topic 6.9 Integrating Using Substitution (Skill 1.E)

• Lesson 6.10: Which One Doesn't Belong?

CED Topic 6.10 Integrating Functions Using Long Division and Completing the Square (Skill 1.E)

• Lesson 6.11: Sneaky Antiderivatives

CED Topic 6.14 Selecting Techniques for Antidifferentiation (Skill 1.C)

- Lesson 6.11: Sneaky Antiderivatives
- Unit 6 Review

### Math Medic Unit 7: Differential Equations

#### Big Ideas: FUN

CED Topic 7.1 Modeling Situations with Differential Equations (Skill 2.C)

- Lesson 7.1: How Long Does Coffee Stay How?
- CED Topic 7.2 Verifying Solutions for Differential Equations (Skill 3.G)
  - Lesson 7.1: How Long Does Coffee Stay How?

CED Topic 7.3 Sketching Slope Fields (Skill 2.C)

• Lesson 7.2: Can Tangents Tell a Story?

CED Topic 7.4 Reasoning Using Slope Fields (Skill 4.D)

• Lesson 7.2: Can Tangents Tell a Story?

CED Topic 7.6 Finding General Solutions Using Separation of Variables (Skill 1.E)

• Lesson 7.3: Are You a Solution Seeker?

CED Topic 7.7 Finding Particular Solutions Using Initial Conditions and Separation of Variables (Skill 1.E)

• Lesson 7.4: How Many Sea Lions Are on Elliott Bay?

CED Topic 7.8 Exponential Models with Differential Equations (Skill 3.G)

• Lesson 7.5: Are Electric Vehicles Taking Off?

#### Math Medic Unit 8: Applications of Integration

#### Big Ideas: CHA

CED Topic 8.1 Finding the Average Value of a Function on an Interval (Skill 1.E)

• Lesson 8.1: How Many M&Ms?

CED Topic 8.2 Connecting Position, Velocity, and Acceleration of Functions Using Integrals (Skill 1.D)

• Lesson 8.2: How Long is Whitney's Bike Ride?

CED Topic 8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts (Skill 3.D)

• Lesson 8.3: How Many People Are at the Met?

CED Topic 8.4 Finding the Area Between Curves Expressed as Functions of x (Skill 4.C)

• Lesson 8.4: How Rich Are the Top 1%?

CED Topic 8.5 Finding the Area Between Curves Expressed as Functions of y (Skill 1.E)

• Lesson 8.5: How Do You Build a Deck?

CED Topic 8.6 Finding the Area Between Curves that Intersect at. More Than Two Points (Skill 2.B)

- Lesson 8.4: How Rich Are the Top 1%?
- Lesson 8.5: How Do You Build a Deck?

CED Topic 8.7 Volumes with Cross Sections: Squares and Rectangles (Skill 3.D)

• Lesson 8.6: What's the Volume of a Loaf of Bread?

CED Topic 8.8 Volumes with Cross Sections; Triangles and Semicircles (Skill 3.D)

• Lesson 8.6: What's the Volume of a Loaf of Bread?

CED Topic 8.9 Volume with Disc Method: Revolving Around the x- or y-axis (Skill 3.D)

• Lesson 8.7: What's the Volume of a Pear?

CED Topic 8.10: Volume with Disc Method: Revolving Around Other Axes (Skill 2.D)

• Lesson 8.7: What's the Volume of a Pear?

CED Topic 8.11 Volume with Washer Method: Revolving Around the x- or y-Axis (Skill 4.E)

• Lesson 8.8: Solid or Hollow?

CED Topic 8.12 Volume with Washer Method: Revolving Around Other Axes (Skill 2.D)

• Lesson 8.8: Solid or Hollow?

### Developing MP1: Implementing Mathematical Practices [33]

In Lesson 6.5 ("Under Cover"), students are given the rate at which umbrellas are being produced at a factory and determine that an integral is needed to calculate the number of umbrellas that have been produced over a certain time interval. They then learn by intuition that when finding the rate at which those umbrellas are being accumulated (the derivative of their accumulation function), they can just look back at the original rate function given, which is the integrand function! This is an example of identifying an appropriate mathematical rule or procedure based on the relationship between concepts (Skill 1.D) and helps students learn the Fundamental Theorem of Calculus.

In <u>Lesson 2.8</u> ("Is There a Rule for Quotients?") students must calculate derivatives without technology and first **determine whether the product rule, quotient rule, or neither is required** to find the derivative. This is an example of applying appropriate mathematical rules or procedures, without technology (Skill 1.E)

In Lesson 2.3 ("Can We Find a Slope There?") students use Desmos to explore the slope of a function by moving a tangent line along and curve and observing its slope. By **calculating numerical derivatives** and examining their results, including when those values are undefined, students learn the rules for what makes a function differentiable. This is an example of applying appropriate mathematical rules or procedures, with technology (Skill 1.E).

In <u>Lesson 4.6</u> ("Can a Straight Line Outsmart a Curve?") students use tangent lines to approximate function values, then determine if the tangent line approximation is an **over- or under-estimate** and why. This is an example of explaining how an approximated value relates to the actual value (Skill 1.F.)

### Developing MP2: Connecting Representations

In a <u>limit card sort</u>, students must match **graphs** of functions, **equations**, limit statements, and **verbal descriptions** of the function's continuity. This is an example of identifying how mathematical characteristics or properties of functions are related in different representations (Skill 2.D.)

In <u>Lesson 5.6</u> ("What Clues Can I Use?"), students are given sets of **ordered pairs** from a function, and its first and second derivative (**numerical representation**). They must

then determine the location of x-intercepts and whether the function is increasing or decreasing at these values and also analyze the function's concavity. Finally, they are given a **graph** and asked whether that function could represent the function whose values they are given. In the Check Your Understanding Questions, students apply the Second Derivative Test from an **equation**, from a **table of values**, and from a **graph**. This is an example of identifying a re-expression of mathematical information presented in a given representation (Skill 2.C) as well as identifying mathematical information from graphical, numerical, analytical, and/or verbal representations (Skill 2.B.)

### Developing MP3: Justification CR5

In Lesson 1.9 ("Are You a 5-Star Uber Driver?") students must determine whether the Intermediate Value Theorem applies by analyzing a function's continuity. They first do this contextually by being asked whether a certain output is guaranteed and thinking intuitively about that function's continuity (e.g. "Must the taxi have been 5 miles away from MOMA at one point?" "Yes, because distance is a continuous function, and can't jump values") and later they do this by justifying responses to IVT questions more formally and abstractly (e.g. "Must there exist a c, for 0 < c < 3, such that f(x)=2.5? Justify your answer"). This is an example of confirming whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied (Skill 3.C).

In Lesson 5.7 ("How Fast is the Line Moving?") students are given the graph of a derivative f' and must answer questions about the behavior of f, such as where it has relative extrema, and on which intervals it is concave up or down. Students must **justify their answer** using information from the graph of f'. This is an example of providing reasons or rationales for solutions and conclusions (Skill 3.E)

In Lesson 6.6 (How Many Canoes Are Available?), students are given a scenario about a canoe rental business and are provided with the equation of the accumulation function that can be used to determine the number of canoes in the livery at any given time. They are asked to evaluate this function and interpret the meaning of their answer. They are then asked to evaluate the derivative of that function and again interpret their answer in the context of the problem. This is an example of explaining the meaning of mathematical solutions in context (Skill 3.F).

Furthermore, each lesson is conducted in the Experience First, Formalize Later model, so students have daily opportunities to reason about and discuss a mathematical task

in small groups. Prompts such as "How do you know?" "Give a convincing reason" "Provide a rationale" and "Justify your response" are used in almost every lesson.

#### Developing MP4: Communication and Notation **CR6**

In a limit <u>Quiz, Quiz, Trade</u> activity, each student is given a card with a graph on it and a limit. Students evaluate the limit on their card, then find a partner. The partner must read the limit statement out loud from their partner's card using correct language ("the limit as x approaches 3 from the right of f(x) is...") and then evaluate the limit. Their partner is responsible for correcting both the language of the notation and the value of the limit. This is then repeated with the other partner on the other graph. Partners then switch cards, find a new partner and repeat the process. The teacher is monitoring the room, checking to make sure that students are using appropriate mathematical language. This is an example of using precise mathematical language (Skill 4A).

In the Check Your Understanding questions of <u>Lesson 6.12</u> ("Back to the Basics") students must **rewrite** limits of infinite Riemann sums as definite integrals and, given a definite integral, rewrite an equivalent limit of an infinite Riemann sum. Students must **explain to their group members how they identified** the limits of integration and the integrand function, and conversely, how they determined the expression for the width of each rectangle and the height of each rectangle. This is an example of using appropriate mathematical symbols and notation (Skill 4.C).

In Lesson 2.2 ("The Making of a Slopes Graph") each student calculates the instantaneous rate of change of a function f at a different x-value and plots their value on the class "slopes graph" using a dot sticker. The teacher then annotates the graph and introduces this new slopes graph as representing the *derivative* of f, or f'. Students record the new notation on their paper and several students are asked to answer the question "What does this dot represent?" for a variety of dot stickers. The student **answers out loud using precise vocabulary** their interpretation in a way that refers to both the x-value, the function f, and the slope. The teacher **evaluates the student's language** and makes **corrections to their vocabulary** as needed, then asks more students to practice saying these interpretations out loud. This process is repeated with two more functions g and h, giving all students the opportunity to **use the new notation and speak the language out loud**. At the end of the lesson, **alternate notations** for derivatives are introduced, and the Check Your Understanding questions feature y',  $\frac{dy}{dx'}$ , and f' notation. This is an example of using precise

mathematical language and using appropriate mathematical symbols and notation (Skills 4.A and 4.C.)

#### Technology Requirement CR7

Each student will purchase or be able to borrow a graphing calculator for individual use. Students will practice solving problems with and without a calculator throughout the year. Classroom sets of calculators are available for students. Students will also be using Desmos on their devices at times to prepare for the built-in calculator on the AP Calculus Exam.

In Lesson 8.3 ("How Many People Are at the Met?") students are given complicated functions representing the rate at which people enter and leave the Metropolitan Museum of Art (Met). They are asked to find the average value of the rate in, the number of people that entered over a specific time interval, and the number of people at the Met at a particular time using their calculators to **compute a numerical integral**. They are also asked to find the time at which the number of visitors is at a maximum by defining a new function as the difference between the rate in and rate out (Y3=Y1-Y2) and using the calculator to **graph the function** and determine the x-intercept of this graph.

In <u>Lesson 2.3</u> ("Can We Find a Slope There?") students use Desmos to **explore the slope of a function** by moving a tangent line along and curve and observing its slope. By **calculating numerical derivatives** and examining their results, including when those values are undefined, students learn the rules for what makes a function differentiable.

In Lesson 8.7 ("What's the Volume of a Pear?") students are introduced to the disk method for finding volume. In the Check Your Understanding questions, students must use their graphing calculator to **solve equations** to determine the boundaries of their region by finding the intersection between two curves. Students are encouraged to store these values on their calculator to reduce copy errors when setting up and evaluating their integral. They then use their calculator to evaluate the integral expression for the volume of the solid obtained by revolving the region.

To review all the calculator skills of the course, students use the <u>Math Medic Ultimate</u> <u>Calculator Guide</u> at the end of the year to go over the 5 main calculator skills. They are then given a circuit of all calculator questions to apply these skills.

#### Solving Real World Problems CR8

In Lesson 5.8 ("Can Good Design Minimize Costs?") students are tasked with working on the research and design team of the distribution company of LaCroix® Sparkling Water. They analyze possible dimensions of cans that would hold the same amount of volume. They then are given information about the cost of aluminum for the bottom and sides and write an equation for the total cost of the can that depends on the radius and height. They then rewrite the equation using only one variable and use Calculus to determine the relative minimum of this function by finding its derivative and applying the First Derivative Test. This is an example of students using optimization in a realworld context.

In Lesson 2.1 ("Can a Human Break the Sound Barrier?") students study Felix Baumgartner's historic free-fall in 2012. They analyze a quadratic function that represents his height above sea-level t seconds after stepping out of the balloon. They then analyze his velocity at various points during the free fall by calculating average rates of change over smaller and smaller intervals. Then then use the limit definition of the derivative to determine Baumgartner's speed at exactly t = 30 seconds. Finally, they analyze their result to decide if he did in fact go "supersonic".