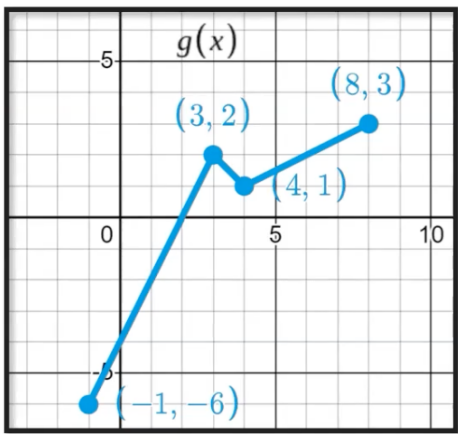


Topic 2.7 Composition of Functions (Daily Video 1)

AP Precalculus

In this video, we will explore the composition of functions with multiple representations.

Let's look at an **EXAMPLE!**

| Composition means _____ the results of one function to another function. | NOTATION: $(f \circ g)(x) = f(g(x))$ | | | | | | | | | | | | | | | | |
|--|---|-----|--------|----|---|---|---|---|---|---|----|---|---|---|---|---|----|
| Using the information provided numerically and graphically about functions f and g , answer the composition question below. For $x = 3$, find the value of $f(g(x))$. 1) Find $g(3)$. 2) Use that result in $f(x)$. $g(3) = 2$ and $f(2) = \underline{\hspace{2cm}}$ So, $f(g(3)) = \underline{\hspace{2cm}}$. For $x = 3$, find the value of $g(f(x))$. 1) Find $f(3)$ 2) Use that result in $g(x)$. $f(3) = \underline{\hspace{1cm}}$ and $g(1) = \underline{\hspace{1cm}}$ So, $g(f(3)) = \underline{\hspace{2cm}}$. | <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  <p>Powered by Desmos</p> </div> <table border="1" style="border-collapse: collapse;"> <thead> <tr style="background-color: #fff9c4;"> <th style="padding: 5px;">x</th> <th style="padding: 5px;">$f(x)$</th> </tr> </thead> <tbody> <tr><td style="padding: 5px;">-1</td><td style="padding: 5px;">0</td></tr> <tr><td style="padding: 5px;">0</td><td style="padding: 5px;">4</td></tr> <tr><td style="padding: 5px;">1</td><td style="padding: 5px;">2</td></tr> <tr><td style="padding: 5px;">2</td><td style="padding: 5px;">-6</td></tr> <tr><td style="padding: 5px;">3</td><td style="padding: 5px;">1</td></tr> <tr><td style="padding: 5px;">4</td><td style="padding: 5px;">5</td></tr> <tr><td style="padding: 5px;">6</td><td style="padding: 5px;">-3</td></tr> </tbody> </table> </div> <p style="margin-top: 10px;">NOTES: 1) The inner function is evaluated first. 2) The output of the inner function becomes the input of the outer function. 3) Since $f(g(3)) \neq g(f(3))$, function composition is NOT _____.</p> <p>* The video has the notation reversed in the second example.</p> | x | $f(x)$ | -1 | 0 | 0 | 4 | 1 | 2 | 2 | -6 | 3 | 1 | 4 | 5 | 6 | -3 |
| x | $f(x)$ | | | | | | | | | | | | | | | | |
| -1 | 0 | | | | | | | | | | | | | | | | |
| 0 | 4 | | | | | | | | | | | | | | | | |
| 1 | 2 | | | | | | | | | | | | | | | | |
| 2 | -6 | | | | | | | | | | | | | | | | |
| 3 | 1 | | | | | | | | | | | | | | | | |
| 4 | 5 | | | | | | | | | | | | | | | | |
| 6 | -3 | | | | | | | | | | | | | | | | |

Let's PRACTICE!

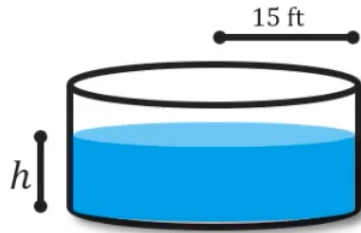
Use the previous table and graph to answer the following problems.

Question 1: For $x = 2$, find the value of $f(g(x))$.

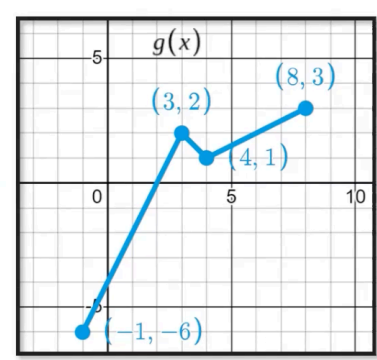
Question 2: Find the value of $2g(f(x))$ when $x = -1$.

Question 3: For $x = 4$, find the value of $f(g(x) + f(x))$.

Let's look at an EXAMPLE!

| | |
|--|--|
| <p>An above-ground circular pool with radius 15 feet was installed in a yard. Water began filling the pool at $t = 0$, with the height of the water increasing at a constant rate of 0.08 ft/hour. The volume of the water, as a function height, is modeled by $V(h) = 225\pi h$.</p> |  |
| <p>Question 1: Given the equation that models the height of the water as a function of time, t, with t measured in hours.</p> <p>$h(t)$ is a _____ function.</p> <p>$h(t) =$ _____</p> | <p>Question 2: Using $V(h) = 225\pi h$, identify the meaning of $V(h(t))$.</p> <p>$h(t)$ is the water's _____ as a function of _____</p> <p>$V(h)$ is the water's _____ as a function of _____</p> <p>$V(h(t))$ is the water's _____ as a function of _____</p> |
| <p>Question 3: Find the value of $V(h(t))$ at $t = 5$ hours (filling began at $t = 0$ hours)</p> <p>Evaluate inner function: $h(5) =$ _____</p> <p>Evaluate outer function: $V(0.4) =$ _____</p> <p>Therefore, $V(h(5)) =$ _____</p> | |

Let's REVIEW!

| | |
|--|--|
| <p>Error Analysis: Using the information provided about functions g and p, analyze the work shown below and identify the error.</p> <p>When $x = 0$, find the value of $2p(g(x))$.</p> <p>Work: First find $g(0) = -4$</p> <p>Then find $p(-4) = -(-4)^2 + 9$</p> $= 16 + 9$ $= 25$ <p>Then $2p(g(0)) = 2(25) = 50$</p> | <p>$p(x) = -x^2 + 9$</p>  <p>Powered by Desmos</p> |
|--|--|

What should we take away?

- Composed functions involve layers. The output for a layer becomes the input for the next layer.

Topic 2.7 Composition of Functions (Daily Video 2)

AP Precalculus

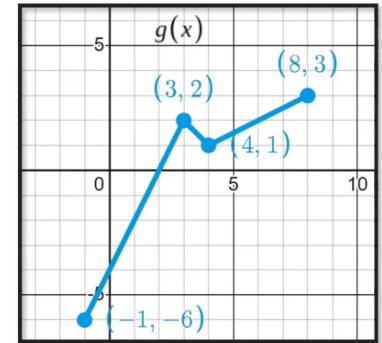
In this video, we will explore rewriting a function by decomposing it into less complex functions.

Let's REVIEW!

Error Analysis: Using the information provided about functions g and p , answer the composition question.

When $x = \sqrt{6}$, find the value of $g(p(x) - 4)$.

$$p(x) = -x^2 + 9$$



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Let's look at an EXAMPLE!

Let's analyze the composition of two functions defined by $h(x) = e^{x^3}$.

Decompose means:

Given $h(x) = f(g(x))$, decompose $h(x)$ by identifying a valid f and g .

How does the composition graph combine the characteristics of both f and g ?

$g(x) = \underline{\hspace{2cm}}$ $f(x) = \underline{\hspace{2cm}}$

Let's PRACTICE!

Question 1: Decompose $h(x) = \frac{1}{x+4}$ using $h(x) = f(g(x))$.

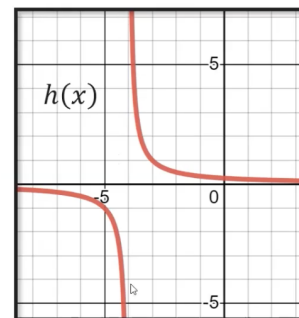
Inner function: Outer function:

Question 2: Give the equation of the vertical asymptote for $h(x)$.

How does our decomposition help us identify this information?

Vertical asymptote:

Since $f(x) = \frac{1}{x'}$ has a vertical asymptote of $\underline{\hspace{2cm}}$, $g(x)$ is a translation $\underline{\hspace{1cm}}$ units to the $\underline{\hspace{1cm}}$.

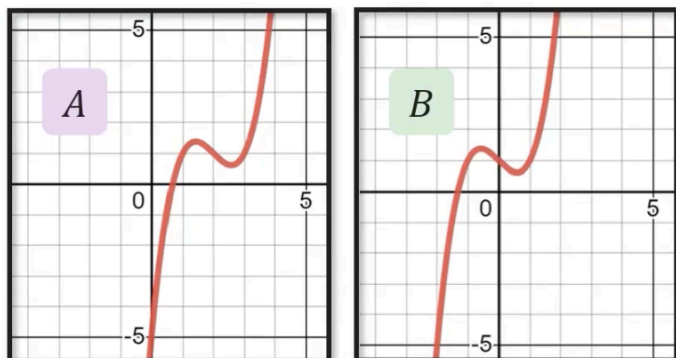


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Question 3: A stone tossed into a pond creates a circular ripple that grows for $t \geq 0$. The circular area formed by the ripple can be modeled by $A(t) = \frac{\pi}{4}t^2$, with time, t , measured in minutes. Using the formula for the area of a circle ($A = \pi r^2$), decompose $A(t) = \frac{\pi}{4}t^2$ using $A(r(t))$.

Question 4: Part 1: Correctly match the graphs of h and f to their equations, given below.

Part 2: What function is composed with f to obtain h ? What does this reveal about the graph of h as compared to the graph of f ?



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Equations

$$f(x) = x^3 - x + 1$$

$$h(x) = (x - 2)^3 - (x - 2) + 1$$

Determine which functions are composed together to get $f(g(x))$.

Composition Groups



$f(x) = e^{3x} + 1$

$g(x) = x^2 + 1$

$f(x) = x^2$

$f(g(x))$ gives $e^{3x^2} + 1$


$g(x) = e^{3x}$

$g(x) = x^2$

$f(x) = e^{3x}$

$f(g(x))$ gives e^{6x}

$f(g(x))$ gives e^{3x^2+3}



What should we take away?

- A function presented analytically can often be decomposed into less complex functions.
- A composition can be used to describe a transformation.
- Analyzing the functions of a decomposition can help reveal information about the composed function.

Topic 2.8 Inverse Functions (Daily Video 1)

AP Precalculus

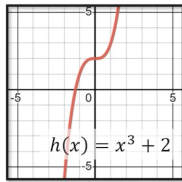
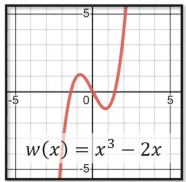
In this video, we will examine input-output pairs of a function's inverse, relevant notation, and what it means for a function to be invertible.

Let's WARMUP!

Notation: Given function: $f(x)$ Inverse Function: $f^{-1}(x)$ The inverse function is a _____ mapping of the function $f(x)$, meaning the inverse function swaps input and output pairs.

Terminology: *Invertible* (adjective): able to have position or order _____ and create a function.

* Need every input mapped to one _____ and every output generated by one _____.



Which function(s), $w(x)$ and/or $h(x)$, are invertible? Justify your answer.

| x | $f(x)$ |
|-----|--------|
| 1 | 3 |
| 3 | 9 |
| 7 | 10 |
| 9 | 12 |

Selected values for the increasing function f are given in the table.

$$f(3) = 9 \text{ then } f^{-1}(9) = \underline{\hspace{2cm}}$$

Let's look at an EXAMPLE! An invertible function, h , is known to have the given information indicated in each question. Using this, answer the questions below.

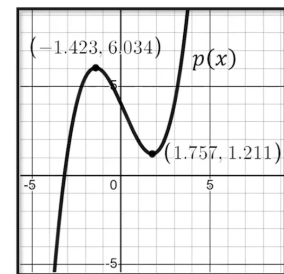
| <p>Question 1: If $h(2.6) = 0$ then $h^{-1}(\quad) = \underline{\hspace{2cm}}$</p> | <p>Question 2: If $h^{-1}(\pi) = -\frac{1}{5}$ then $h(\quad) = \underline{\hspace{2cm}}$</p> | | | | | | | | | | | | |
|---|--|-----|--------|------|---|---|---|---|---|---|---|---|------|
| <p>Question 3: An invertible function, w, has an x-intercept of $(-3,0)$ and its graph included the ordered pair $(2,7)$. List two ordered pairs that are points on the graph of $w^{-1}(x)$.</p> | <p>Question 4: For $a > b$, it is known that $f(a) < f(b)$. Selected values are given in the table.</p> <table border="1" data-bbox="1328 1348 1507 1575"> <thead> <tr> <th>x</th> <th>$f(x)$</th> </tr> </thead> <tbody> <tr> <td>-2.4</td> <td>6</td> </tr> <tr> <td>2</td> <td>5</td> </tr> <tr> <td>3</td> <td>2</td> </tr> <tr> <td>6</td> <td>1</td> </tr> <tr> <td>8</td> <td>-2.4</td> </tr> </tbody> </table> <p>$f^{-1}(6) = \underline{\hspace{2cm}}$</p> | x | $f(x)$ | -2.4 | 6 | 2 | 5 | 3 | 2 | 6 | 1 | 8 | -2.4 |
| x | $f(x)$ | | | | | | | | | | | | |
| -2.4 | 6 | | | | | | | | | | | | |
| 2 | 5 | | | | | | | | | | | | |
| 3 | 2 | | | | | | | | | | | | |
| 6 | 1 | | | | | | | | | | | | |
| 8 | -2.4 | | | | | | | | | | | | |

Let's PRACTICE! Use the information and table from Question 4 above.

What is the value of $f^{-1}(2) + 4f(6)$? Show all work.

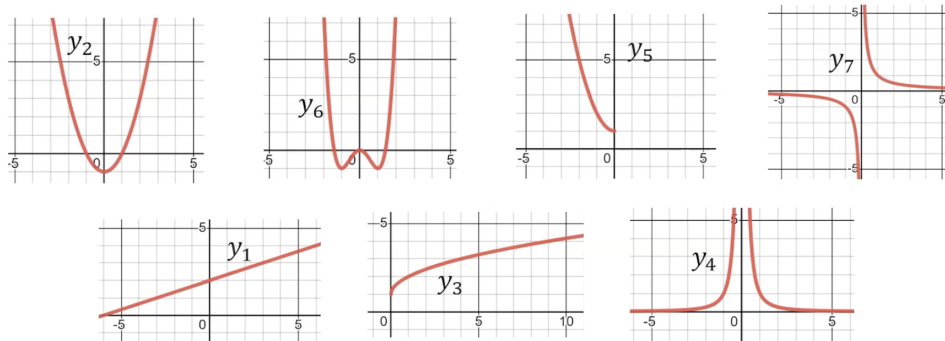
Consider a noninvertible function, p , whose graph is shown. Circle the domain restrictions that represent invertible pieces of the noninvertible function p .

- $(-\infty, -1.423]$ $[-1.423, \infty)$ $(-\infty, \infty)$
 $(-\infty, 0]$ $[-1.423, 1.757]$ $[1.757, \infty)$



Place each of the following functions, given by their graphs, into the indicated groups.

| <p style="text-align: center;">Group 1: Not Invertible</p> <p>Write the function names.</p> | <p style="text-align: center;">Group 2: Invertible</p> <p>Write the function names.</p> |
|--|--|
|--|--|



Odd One Out

Thinking about invertible functions, identify which of the four listed functions is the “odd one out.” Give a reason for your answer.

$$y = -x^5 \quad y = e^x \quad y = x^4 \quad y = \frac{1}{x}$$

What should we take away?

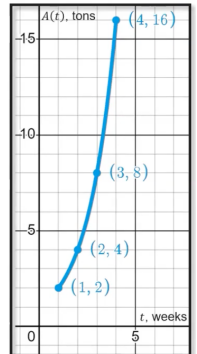
- An invertible function has an _____ that is a function.
- An inverse maps _____ values of f to the corresponding input values.
- The _____ of a function may need to be restricted to achieve invertibility.

Topic 2.8 Inverse Functions (Daily Video 2)

AP Precalculus

In this video, we will examine the characteristics of a function's inverse on its invertible domain.

Let's WARMUP! The graph of A models recycling material collected and reported at the end of each week, with $A(t)$ measured in tons and t measured in weeks.



Question 1:

What is the domain and range of $A(t)$?

Domain: _____

Range: _____

Question 2:

Label the axes and coordinate pairs on the graph of A^{-1} below. Give the domain and range of $A^{-1}(x)$.

Domain: _____

Range: _____



Question 3:

The domain of A becomes the _____ of A^{-1} and the range of A becomes the _____ of A^{-1} .

Question 4:

$A(3) = 8$ means that at the end of the _____, _____ of collected recycling was reported.

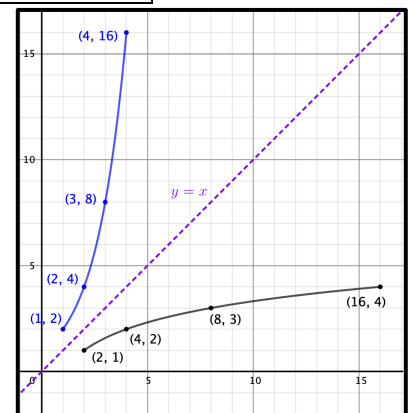
$A^{-1}(8) = 3$ means that _____ of collected recycling was reported at the end of the _____,

Question 5: What transformation is applied to A to obtain A^{-1} ?

Analytically establish the following facts.

$$A^{-1}(A(x)) = x$$

$$A(A^{-1}(x)) = x$$



Composing a function and its inverse produces the _____ function.

Let's PRACTICE!

$$g(x) = \frac{1}{x-4}, x > 4$$

| Characteristics of $g(x)$ | Corresponding fact about $g^{-1}(x)$ |
|--|--------------------------------------|
| g has a domain of $(4, \infty)$ | |
| g has a range of $(0, \infty)$ | |
| Graph of g has a vertical asymptote at $x = 4$. | |

Let's look at an EXAMPLE!

Consider the invertible function given by $h(x) = x^2 + 4$ for $x \leq 0$. Find the analytical representation of the inverse function, $h^{-1}(x)$.

Let's REVIEW! Two Truths and a Lie

Consider f , defined as $f(x) = x^3 + 2$. Two statements below are true, and one is a lie. Identify the lie and transform the lie into a truth.

f is invertible and an increasing function.

f has a y -intercept $(0,2)$, which indicates $f^{-1}(2) = 0$.

The analytical form of $f^{-1}(x) = (x - 2)^3$.

What should we take away?

- For an invertible function, the domain and _____ of f _____ to become the _____ and _____ of f^{-1} .
- Composing a function and its inverse produces the _____ function.

Topic 2.9 Logarithmic Expressions (Daily Video 1)

AP Precalculus

In this video, we will learn to evaluate logarithmic expressions, using arithmetic to obtain exact values (if possible) or technology to estimate values.

Let's WARMUP!

Definition: The logarithmic expression $\log_b c$ is equal to, or represents, the value that the base b must be exponentially raised to in order to obtain the value c .

Let's look an EXAMPLE!

Example 1: Evaluate $n = \log_2 8$ means we need to think _____ $n = \log_2 8 =$ _____

Example 2: Evaluate $n = \log_{10} 100$ means we need to think _____ $n = \log_{10} 100 =$ _____

Example 3: Evaluate $n = \log_3 \left(\frac{1}{3}\right)$ means we need to think _____ $n = \log_3 \left(\frac{1}{3}\right) =$ _____

Let's PRACTICE!

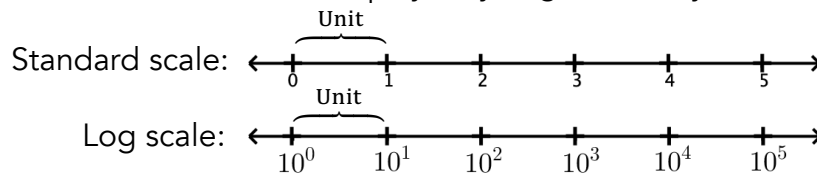
Example 4: Evaluate $\log_2 32$

Example 5: Evaluate $\log_3 81$

Example 6: Evaluate $\log_5(-25)$

Example 7: Evaluate $\log_2 7$

Logarithmic scales: Used to display very large and very small numbers.



Give three examples where log scales are used:

1. _____
2. _____
3. _____

What should we take away?

$n = \log_b c$ is equal to the _____ of base b in order to obtain the value c or $b^{(\text{---})} = c$.

Topic 2.10 Inverses of Exponential Functions Expressions (Daily Video 1)

AP Precalculus

In this video, we will learn to evaluate the general form of a logarithmic function and explore the relationship between input and output values.

Let's REVIEW!

Evaluate $n = \log_5 125$ means we need to think _____ $n = \log_5 125 =$ _____

The definition of logarithm expresses a relationship between logs and exponents.

$$\log_b c = a \text{ if and only if } b^a = c \text{ where } b > 0 \text{ and } b \neq 1$$

Let's look at an EXAMPLE!

Complete the table for $f(x) = \log_3 x$ using arithmetic and technology.

| x | $f(x)$ | Notes |
|-----|---------------------|-------|
| 0 | $f(0) = \log_3 0$ | |
| 1 | $f(1) = \log_3 1$ | |
| 2 | $f(2) = \log_3 2$ | |
| 3 | $f(3) = \log_3 3$ | |
| 4 | $f(4) = \log_3 4$ | |
| 9 | $f(9) = \log_3 9$ | - |
| 27 | $f(27) = \log_3 27$ | - |

*Note: on the AP Precalculus exam all decimal answers need to be to 3 decimal places.



What do you notice?

| Table for $g(x) = 3^x$ | | Table for $f(x) = \log_3 x$ | |
|------------------------|--------|-----------------------------|--------|
| x | $g(x)$ | x | $f(x)$ |
| 0 | | | 0 |
| 1 | | | 1 |
| 2 | | | 2 |
| 3 | | | 3 |
| 4 | | | 4 |

Let's PRACTICE!

| Table for $f(x) = 2^x$ | | Table for $g(x) = \log_2 x$ | |
|------------------------|--------|-----------------------------|--------|
| x | $f(x)$ | x | $g(x)$ |
| 0 | | | 0 |
| 1 | | | 1 |
| 2 | | | 2 |
| 3 | | | 3 |
| 4 | | | 4 |

What should we take away?

Exponential and logarithmic functions' input and output values are _____.

Topic 2.10 Inverses of Exponential Functions Expressions (Daily Video 2)

AP Precalculus

In this video, we will explore the inverse relationship between logarithmic and exponential functions.

Let's REVIEW!

Topic 2.9: Evaluate $n = \log_2 \frac{1}{16}$ means we need to think _____ $n = \log_2 \frac{1}{16} =$ _____

Topic 2.10: $f(x) = \log_b x$ and $g(x) = b^x$ where $b > 0$ and $b \neq 1$ are _____ functions

Topic 2.8: Composition of a function and its inverse will result in the _____ function, x .

The graphs of inverse functions are _____ over the graph of the $h(x) = x$.

A point (s, t) on a function becomes the point _____ on the inverse function.

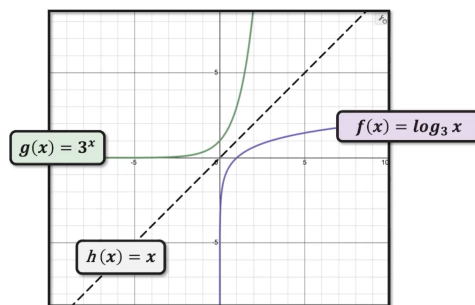
Let's look at an EXAMPLE!

Let $f(x) = \log_3 x$ and its inverse $g(x) = 3^x$.
Show that $f(g(x)) = x$.

Show that the point (s, t) on $f(x)$ is reversed on $g(x)$.

| x | $f(x)$ | x | $g(x)$ |
|-----|--------|-----|--------|
| | 0 | 0 | |
| | 1 | 1 | |
| | 2 | 2 | |
| | 3 | 3 | |
| | 4 | 4 | |

Explain how the graphs of $f(x)$ and $g(x)$ show that $f(x)$ and $g(x)$ are inverses.



What should we take away?

$f(x) = \log_b x$ and $g(x) = b^x$ are _____ functions.

- $g(f(x)) = f(g(x)) =$ _____
- The graphs of $f(x)$ and $g(x)$ are _____ over the graph of the _____ function.
- The ordered pair (s, t) on the graph of $g(x)$ is the ordered pair _____ on the graph of $f(x)$.

Topic 2.11 Logarithmic Functions (Daily Video 1)

AP Precalculus

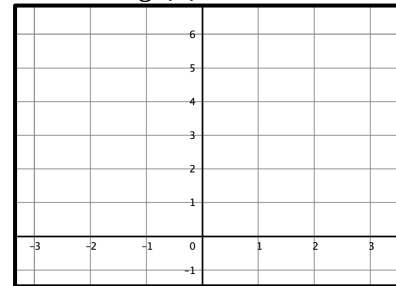
In this video, we will learn the key features of the logarithmic function, including domain, range, translations, and end behavior.

Let's WARMUP!

Complete the table for $g(x) = 2^x$ and plot the points on the grid.

| x | -2 | -1 | 0 | 1 | 2 |
|--------|----|----|---|---|---|
| $g(x)$ | | | | | |

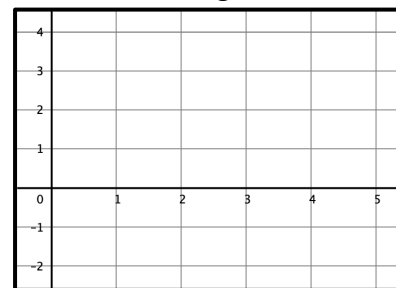
$$g(x) = 2^x$$



Complete the table for the inverse of $g(x) = 2^x$ and plot the points on the grid.

| x | | | | | |
|-------------|--|--|--|--|--|
| $g^{-1}(x)$ | | | | | |

$$\text{inverse of } g(x) = 2^x$$



Inverse equation: _____

Let's look at an EXAMPLE!

Complete the table for $g(x) = \log_5 x$ and plot the points on the grid.

Domain: _____ Range: _____

x -intercept: _____ y -intercept: _____

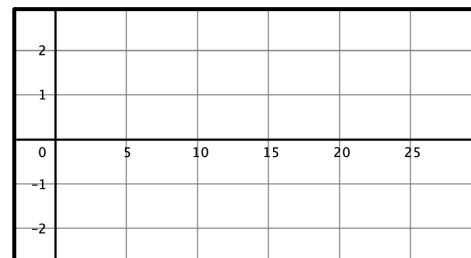
Asymptote: _____

Shape of graph: inc/dec and concave up/down

End behavior: _____

$$g(x) = \log_5 x$$

| x | $\frac{1}{25}$ | $\frac{1}{5}$ | 1 | 5 | 25 |
|--------|----------------|---------------|---|---|----|
| $g(x)$ | | | | | |



Let's PRACTICE!

What is the equation of the asymptote of the graph of the function $h(x) = 2 + \log_3(x - 5)$?

- A) $y = 2$ C) $y = 0$
 B) $x = 3$ D) $x = 5$

What should we take away?

The important features of $f(x) = \log_b x$, $b > 1$

Domain: _____ Range: _____ x -intercept _____

Asymptote: _____ Increasing and _____ As $x \rightarrow \infty$, $f(x) \rightarrow$ _____

Topic 2.12 Logarithmic Function Manipulation (Daily Video 1)

AP Precalculus

In this video, we will learn the properties of logarithms and how to use them to manipulate expressions.

Let's WARMUP!

Evaluate: $\log_2 8 + \log_2 4 - \log_2 32$. Show your work.

- A) -20 B) 0 C) 1 D) 2

Let's look at an EXAMPLE!

| | |
|---|---|
| What happens to the graph of the parent function? | Related Exponent Rule |
| $h(x) = \log_3(9x) = \underbrace{\hspace{10em}}_{\text{Log Rule Rewrite}}$ <p>Transformation:</p> | $x^2 \cdot x^7 =$ <p>Multiplying then _____ exponents</p> |
| $h(x) = \log_4(x^5) = \underbrace{\hspace{10em}}_{\text{Log Rule Rewrite}}$ <p>Transformation:</p> | $(x^2)^7 =$ <p>Power to a power then _____ exponents</p> |
| $h(x) = \log_5\left(\frac{x}{2}\right) = \underbrace{\hspace{10em}}_{\text{Log Rule Rewrite}}$ <p>Transformation:</p> | $\frac{x^7}{x^2} =$ <p>Dividing then _____ exponents</p> |

Let's PRACTICE!

How does the graph of $f(x) = \log_5(3x)$ compare the graph of $g(x) = \log_5 x$?

- A) The graph of $f(x)$ is a horizontal translation to the left of $g(x)$.
 B) The graph of $f(x)$ is a vertical translation upward of $g(x)$.
 C) The graph of $f(x)$ is a vertical dilation of $g(x)$.
 D) The graph of $f(x)$ is a horizontal reflection of $g(x)$.

What should we take away?

| Exponent Rules | Properties of Logs |
|---|--|
| $b^x \cdot b^y =$ $\frac{b^x}{b^y} =$ $(b^x)^y =$ | $\log_b(\quad) = \log_b x + \log_b y$ $\log_b(\quad) = \log_b x - \log_b y$ $\log_b(\quad) = y \log_b x$ |

Topic 2.12 Logarithmic Function Manipulation (Daily Video 2)

AP Precalculus

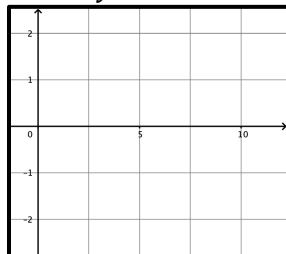
In this video, we will learn about the natural logarithmic function, $f(x) = \ln x$, and apply the properties of logarithms to it.

Let's WARMUP!

$$\log_b b = \quad \log_b b^3 = \quad \log_b 1 = \quad \log 1000 = \quad \log_5 \sqrt{\frac{1}{125}} =$$

The **Natural Logarithm** has a base of e , an irrational number, and instead of writing $\log_e x$, we write _____.

Let's PRACTICE!

| <p>Complete the table for $y = \ln x$ and plot the points on the grid.</p> <p>Note: $e \approx$ _____</p> | <table border="1"> <thead> <tr> <th>x</th> <th>$y = \ln x$</th> </tr> </thead> <tbody> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> <tr><td> </td><td> </td></tr> </tbody> </table> | x | $y = \ln x$ | | | | | | | | | | | | | | | <p style="text-align: center;">$y = \ln x$</p>  |
|---|--|---|-------------|--|--|--|--|--|--|--|--|--|--|--|--|--|--|---|
| x | $y = \ln x$ | | | | | | | | | | | | | | | | | |
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| <p>What happens to the graph compared to the parent function $y = \ln x$ for each of these examples?</p> | | <p>$y = \ln x^3 =$ _____ <small>Rewrite using logarithm properties</small></p> <p>Transformation:</p> | | | | | | | | | | | | | | | | |
| <p>$y = \ln(e^2 x) =$ _____ <small>Rewrite using logarithm properties</small></p> <p>Transformation:</p> | | <p>$y = \ln\left(\frac{e}{x}\right) =$ _____ <small>Rewrite using logarithm properties</small></p> <p>Transformation:</p> | | | | | | | | | | | | | | | | |

Let's PRACTICE! How does the graph of $g(x) = \ln x^3$ compare the graph of $f(x) = \ln x$?

- A) The graph of $g(x)$ is a horizontal translation to the left of $f(x)$.
- B) The graph of $g(x)$ is a vertical translation upward of $f(x)$.
- C) The graph of $g(x)$ is a vertical dilation of $f(x)$.
- D) The graph of $g(x)$ is a horizontal reflection of $f(x)$.

What should we take away?

| Base b | Base e |
|--|---|
| $\log_b x + \log_b y = \log_b (\quad)$ | $\ln(xy) = \underline{\hspace{2cm}}$ |
| $\log_b x - \log_b y = \log_b (\quad)$ | $\ln\left(\frac{x}{y}\right) = \underline{\hspace{2cm}}$ |
| $\log_b (\quad) = y \cdot \log_b x$ | $\ln(x^y) = \underline{\hspace{2cm}}$ |
| $\log_b (\quad) = 1 \quad \log_b 1 =$ | $\ln e = \underline{\hspace{1cm}} \quad \ln 1 = \underline{\hspace{1cm}}$ |

Topic 2.13 Exponential and Logarithmic Equations and Inequalities (Daily Video 1)

AP Precalculus

In this video, we will explore strategies for solving exponential and logarithmic equations and inequalities and assess the reasonableness of the solution(s) found.

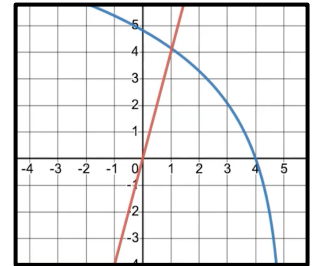
Let's WARMUP! If $2 \cdot 3^x = 54$, what is the value of x ? Show how you arrived at the answer.

Let's look at an EXAMPLE!

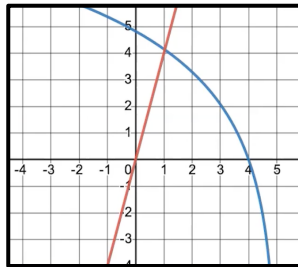
1. If $5 \cdot 2^x - 3 = 112$, what is the value of x ?
Show how you arrived at the answer.



2. Solve $3 \ln(5 - x) = 4x$. Show how you arrived at the answer.



3. For which values of x is $3 \ln(5 - x) > 4x$?
Show how you arrived at the answer.



Let's PRACTICE!

Solve $\log(x - 6) + \log(x + 3) = 1$. Show how you arrived at the answer.

What should we take away?

Exponential and logarithmic equations and inequalities can be solved in a variety of ways, including graphically. Make sure your answers are in the domain of the function and discard extraneous solutions.

Topic 2.13 Exponential and Logarithmic Equations and Inequalities (Daily Video 2)

AP Precalculus

In this video, we will explore how rewriting exponential and logarithmic expressions in equivalent forms can reveal relationships that make solving equations easier.

Let's WARMUP! Equivalent or Not?

| | |
|-----------------------|----------------------|
| 16^{m-1} | 2^{4m-4} |
| $\log(3x) - \log 100$ | $\frac{\log(3x)}{2}$ |
| $\log_5 w$ | $\log_{25} w^2$ |

Let's REVIEW! Exponential and Logarithm Properties

| | |
|---------------------|-----------------------------|
| $x^a \cdot x^b =$ | One-to-one property |
| $\frac{x^a}{x^b} =$ | If $b^x = b^y$, then ____. |
| $(x^a)^b =$ | |
| $\log a + \log b =$ | If $\log x = \log y$, then |
| $\log a - \log b =$ | _____. |
| $\log(x^a) =$ | |

Let's look at an EXAMPLE!

| | |
|--|---|
| Solve $2^x = 8^{3x-2}$. Show how you arrived at the answer. | Find all solutions to $\log_9(8x - 15) = \log_3 x$. Use the rule $\log_9 w = \frac{1}{2} \log_3 w$. Show how you arrived at the answer. |
|--|---|

What should we take away?

- Understanding equivalence helps us notice relationships between two expressions and these relationships allow us to simplify an expression or solve an equation.
- Simply memorizing rules is not sufficient; you must truly understand how exponents and logarithms work.

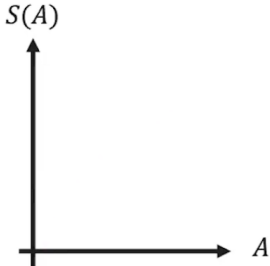

Topic 2.14 Logarithmic Function Context and Data Modeling (Daily Video 1)

AP Precalculus


In this video, we will interpret logarithmic functions in context, explore their growth rates, and use an algebraic model to make predictions.

Let's look at an **EXAMPLE!**

The number of unique plant species in an area of California can be modeled by the function $S(A) = 77 + 147 \log(A)$, where $S(A)$ is the number of species in a region with an area of A square miles.

| | |
|---|---|
| <p>a) Describe how the number of unique plant species changes as the area of the region increases. Sketch a logarithmic graph.</p>  <p>The number of species _____, but at a _____ rate.</p> | <p>b) Predict how many unique plant species exist in a 110 square-mile region of California.</p> <p>$S(110) =$ </p> |
|---|---|

Let's **PRACTICE!**

The number of words that a child knows can be modeled by the function V , where $V(t)$ is the number of words a child knows, in thousands, when they are t months old. An equation $V(t)$ is given by $V(t) = 10 \log t - 13$. 

| | |
|---|---|
| <p>a) Predict the number of words a child knows when they are 2 years old ($t = 24$).</p> <p>$V(24) =$</p> <p>When the child is _____ years old, the model predicts they will know approximately _____ words.</p> | <p>b) Find $V(48)$ and interpret your results in the context of this problem.</p> <p>$V(48) =$</p> <p>When the child is _____ years old, the model predicts they will know approximately _____ words.</p> |
| <p>c) Explain why vocabulary growth can be reasonably modeled by a logarithmic function.</p> | <p>d) How many years does the model predict it will take to learn 15,000 words? Show how you arrived at your answer.</p> |

What should we take away?

Logarithmic models describe relationships where the dependent variable increases (or decreases), but the rate slows down over time.

Topic 2.14 Logarithmic Function Context and Data Modeling (Daily Video 2)

AP Precalculus

In this video, we will construct logarithmic models from given data, with and without technology.

Let's look at an EXAMPLE!

Selected values of a logarithmic function, f , are given in the table.

If $f(x) = a \log_3 x + b$ for some parameters a and b , find the values of a and b .

What is the parent function? $y =$ _____

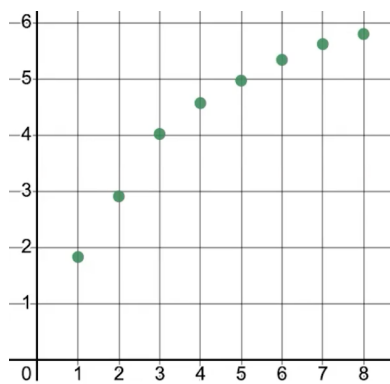
The graph passes through points _____ and _____

Find the values of a and b . Show all your work.

| x | $f(x)$ |
|-----|--------|
| 1 | -1 |
| 3 | 3 |
| 9 | 7 |
| 27 | 11 |

Let's PRACTICE!

Information about the age and height of a tree is given in the table. Construct a natural log regression model to predict the tree's height, in meters, after t years.



Explain why a logarithmic model makes sense?

Use your calculator to find a regression equation. Directions vary by calculator.

The height of the tree, in meters, after t years can be modeled by the equation

$$H(t) = \underline{\hspace{10em}}$$

| Age (years) | Height (meters) |
|-------------|-----------------|
| 1 | 1.83 |
| 2 | 2.91 |
| 3 | 4.02 |
| 4 | 4.57 |
| 5 | 4.97 |
| 6 | 5.34 |
| 7 | 5.62 |
| 8 | 5.80 |

What should we take away?

- Using **key points** of a logarithmic function, _____ and _____, can help determine the parameters of the logarithmic equation that passes through the given points.
- Logarithmic regression is a tool used to construct a logarithmic model for given real world data.

Topic 2.15 Semi-log Plots (Daily Video 1)

AP Precalculus

In this video, we will review how to read a semi-log plot.

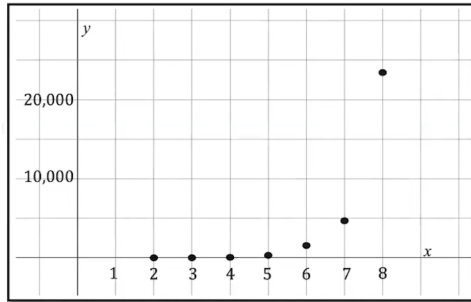
What is a semi-log plot?

It is a graph where one axis is scaled _____ and the other is scaled _____.

Why would we need to do that?

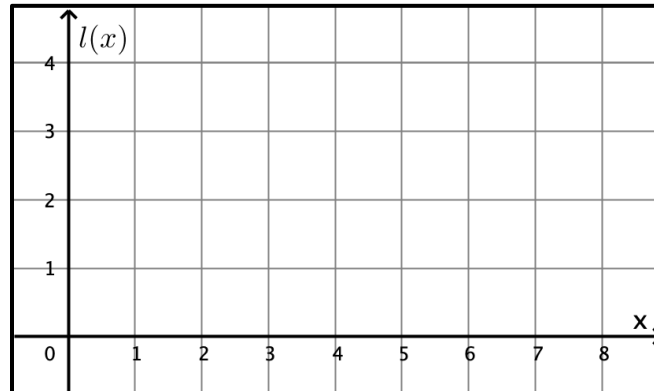
| x | $f(x)$ |
|-----|----------|
| 2 | 2.5 |
| 3 | 12.5 |
| 4 | 62.5 |
| 5 | 312.5 |
| 6 | 1,562.5 |
| 7 | 4,687.5 |
| 8 | 23,437.5 |

Data on a standard x - y plane



Plot the data for $l(x) = \log f(x)$

| x | $l(x) = \log f(x)$ |
|-----|--------------------------|
| 2 | $\log(2.5) = 0.3979$ |
| 3 | $\log(12.5) = 1.097$ |
| 4 | $\log(62.5) = 1.796$ |
| 5 | $\log(312.5) = 2.495$ |
| 6 | $\log(1,562.5) = 3.194$ |
| 7 | $\log(4,687.5) = 3.671$ |
| 8 | $\log(23,437.5) = 4.370$ |



What should we take away?

- A semi-log plot is a graph whose y -axis is drawn with equal-sized intervals between powers of _____.
- If a semi-log plot is drawn of a graph for which an exponential model is appropriate, the semi-log plot will appear _____.