AP Precalculus Unit 1: Exploring Rates of Change

Lesson	Learning Targets
1.1 Functions and Function Notation	• Understand that a function describes the relationship between an independent variable and a dependent variable where each input value is mapped to exactly one output value. Functions can be expressed with an equation, table, graph, or verbal description. • Describe the set of inputs of a function (the domain) and the set of outputs (the range). • Use and interpret function notation.
1.2 Interpreting Graphs of Functions	• Describe how two quantities vary with respect to each other from a graph in a contextual scenario. • Determine when a function is increasing or decreasing. • Interpret key points and graph behavior in context.
1.3 Concavity	• Connect the sign of a graph's slope to the increasing or decreasing behavior of a function and the value of the slope to the function's rate of change. • Use the concavity of a function's graph to describe the change in the function's rate of change and vice versa.
1.4 Rates of Change	• Find and interpret a function's average rate of change over an interval. • Estimate and interpret a function's rate of change at a point. • Compare rates of change at different intervals or values of a function's domain.
1.5 Change in Linear Functions	• Understand that a linear function has a constant rate of change over any interval of its domain. • Explain why the rate of change of the average rates of change of a linear function is zero. • Interpret the slope of a linear function in terms of a rate of change.

1.6 Change in Quadratic Functions	• Understand that for quadratic functions, the change in output values over equal intervals of the domain grows linearly. • Explain why the rate of change of the average rates of change of a quadratic function is constant. • Connect the concavity of a parabola to whether the average rates of change of the quadratic function are increasing (concave up) or decreasing (concave down).

AP Precalculus Unit 2: Polynomial and Rational Functions

Lesson	Learning Targets
2.1 Polynomial Functions and Rates of Change	• Identify key characteristics of a polynomial function including its degree, leading coefficient, relative and absolute extrema, and points of inflection. • Determine the degree of a polynomial using first, second, third, …nth differences.
2.2 Zeros of Polynomial Functions	• Use a root's multiplicity to describe the polynomial graph's behavior at an x-intercept. • Understand that a polynomial of degree n has exactly n complex zeros and can be written as a product of n linear factors. • Find all zeros of a polynomial function when given in factored form; identify when zeros will be imaginary based on the polynomial's graph or equation in factored form.
2.3 Even and Odd Polynomials	• Understand the properties of even and odd functions. • Algebraically prove whether a polynomial function is even, odd, or neither.
2.4 Polynomial Functions and End Behavior	• Determine the end behavior of a polynomial from its degree and leading coefficient. • Explain why the end behavior of a polynomial function is determined by its leading term. • Use limit notation to describe the end behavior of a polynomial function.
2.5 Rational Functions and End Behavior	• Interpret the behavior of a rational function in context, specifically its horizontal asymptote. • Determine the end behavior of a rational function by comparing the dominance of the polynomials in the numerator and denominator. • Explain why the end behavior of a rational function is determined by the quotient of the leading terms in the numerator and denominator.

2.6 Graphs of Rational Functions	• Identify key features of a rational function including its domain, intercepts, holes, and vertical asymptotes from its graph and equation in factored form. • Use one-sided limit notation to describe the behavior of a rational function near a vertical asymptote. • Determine the y-value of a hole by examining function outputs at input values sufficiently close to the x-value of the hole.
2.7 Factored and Standard Forms of Polynomials	• Describe the advantages of writing a polynomial in factored form versus standard form. • Convert polynomials from factored to standard forms and vice versa. • Explain why when (x-k) is a factor of a polynomial, x=k is a zero of the polynomial • Find all zeros of a polynomial function by hand or using technology.
2.8 Equivalent Representations of Rational Functions	• Expand the ideas of factors, dividends, divisors, quotients, and remainders from numbers to functions. • Divide polynomials using an area model. • Explain why rewriting a rational function in equivalent ways can reveal different characteristics of the function, including slant asymptotes.
2.9 The Binomial Theorem	• Generalize patterns for the expansion of binomials and explain the connection to the entries of Pascal's triangle. • Expand binomial expressions using the Binomial Theorem.

AP Precalculus Unit 3: Constructing Functions

Lesson	Learning Targets
3.1 A Library of Parent Functions	• Understand that the parent function represents the most basic function in a family of functions. • Describe the key features of six parent functions: identity, absolute value, square root, quadratic, cubic, and reciprocal. • Analyze and compare the key features of parent functions.
3.2 Transformations of Functions	• Construct a new function by applying translations, dilations, and reflections to a parent function. • Given an equation or graph of a transformed function, describe the transformations that occurred from the parent function. • Determine the domain and range of a transformed function.
3.3 Piecewise Functions	• Interpret and evaluate functions that have different rules for certain intervals of the domain. • Graph piecewise defined functions. • Write equations for piecewise-defined functions given a graph or from a context.
3.4 Selecting a Function Model	• Identify an appropriate function type to construct a function model based on key observations about how the quantities in a scenario are changing. • Describe the assumptions and restrictions related to a particular function model.
3.5 Constructing a Function Model	• Construct a function model based on the constraints of a mathematical or contextual scenario. • Construct a function model using transformations from a parent function. • Use rational functions to model quantities that are inversely proportional. • Apply a function model to answer questions about a data set or contextual scenario.

AP Precalculus Unit 4: Exponential Functions

Lesson	Learning Targets
4.1 Change in Arithmetic Sequences	• Understand that sequences are a special type of function whose domain is the positive integers. • Write an explicit rule for arithmetic sequences using the common difference and any term in the sequence. • Apply understanding of how arithmetic sequences grow to determine the common difference, find missing terms and reason about arithmetic sums.
4.2 Change in Geometric Sequences	• Write an explicit rule for geometric sequences using the common ratio and any term in the sequence. • Apply understanding of how geometric sequences grow and knowledge of exponents and roots to determine the common ratio and find missing terms. • Compare arithmetic and geometric sequences.
4.3 Change in Linear and Exponential Functions	• Create linear and exponential functions using constant rates of change and constant proportions. • Interpret the parameters of a linear and exponential function in context and to describe their growth patterns. • Describe similarities and differences between linear and exponential functions.
4.4 Exponential Functions	• Recognize scenarios that depict exponential growth or decay by identifying a fixed percent change or common ratio. • Write equations of the form $y=ab^x$ to model scenarios that grow or decay by a fixed percent or factor.

4.5 Graphing and Manipulating Exponential Functions	• Graph functions of the form $y=b^x$ and identify key characteristics including end behavior, concavity, domain and range, and key points. • Determine the growth factor of an exponential function from its graph, including when the function has been transformed. • Apply knowledge of transformations to exponential functions. • Explain using exponent properties and transformations why two exponential functions are equivalent.
4.6 Modeling with the Natural Base, "e"	• Describe the effects of compounding interest quarterly, monthly, weekly, daily, and continually and make use of structure to arrive at the compound interest formula. • Use an exponential model to make predictions about the dependent variable. • Understand "e" as the base rate of growth for all continually growing processes.
4.7 Constructing Exponential Models	• Construct exponential models from an initial value and ratio or from two input-output pairs. • Use an exponential model to make predictions about the dependent variable. • Understand how equivalent forms of an exponential function can reveal different properties about its growth rate.
4.8 Using Regression Models	• Use the characteristics of a data set to decide whether a linear, quadratic, or exponential model is most appropriate. • Create a regression model for a scenario using technology. • Use a residual plot to validate whether a given model was appropriate.

AP Precalculus Unit 5: Logarithmic Functions

Lesson	Learning Targets
5.1 Compositions of Functions	• Understand that when two functions are composed, the output of one function becomes the input of the second function. • Write equations for compositions of functions. • Decompose a complicated function into a composite of two or more functions. • Reason about the domain of a composition of functions.
5.2 Intro to Inverse Functions	• Repeatedly solve equations of the form f(x)=c to recognize the need for a function that "undoes" the original function, i.e. to find the value in the domain that generates a certain output. • Understand the relationship between the inputs and outputs of a function and its inverse and use this to evaluate inverse functions. • Find an inverse function algebraically. • Verify by composition that one function is the inverse of another.
5.3 Graphs of Inverse Functions	• Understand why a function must be one-to-one, or invertible, in order for the inverse mapping to be a function. • Explore relationships between the graph of a function and its inverse, including their domains and ranges.

5.4 Inverses of Exponential Functions	• Understand that a logarithm represents the exponent to which the base must be raised in order to attain the input value; use this understanding to evaluate logarithmic expressions. • Use exponential and logarithmic forms to write equivalent statements about powers. • Understand the inverse relationship between how inputs and outputs change in exponential versus logarithmic functions. • Understand the inverse relationship between exponential and logarithmic functions of the same base, including the natural base, e.
5.5 Graphs of Logarithmic Functions	• Describe key features (domain, range, asymptotes, concavity, and end behavior) of the graph of a parent logarithmic function, $y=\log_b(x)$. • Sketch parent logarithmic functions and their transformations. • Connect key features on the graphs of exponential and logarithmic functions.
5.6 Logarithm Properties	• Discover the sum, difference, and power properties of logarithms and use them to rewrite logarithmic expressions. • Explain using logarithm properties and transformations why two logarithmic functions are equivalent.
5.7 Solving Exponential and Logarithmic Equations	• Understand that a function describes the relationship between an independent variable and a dependent variable where each input value is mapped to exactly one output value. Functions can be expressed with an equation, table, graph, or verbal description. • Describe the set of inputs of a function (the domain) and the set of outputs (the range). • Use and interpret function notation.

5.8 Modeling with Logarithmic Functions	• Understand that a logarithmic model takes quantities that grow proportionally and assigns them output values that grow linearly. • Identify situations that could be modeled with a logarithmic function. • Construct logarithmic models using input-output pairs or transformations. • Use logarithmic function models to predict values of the dependent variable.
5.9 Semi-log Plots	• Understand that quantities exhibiting exponential growth or decay can be linearized using a log transformation. • Interpret the parameters of exponential regression models and their associated linear regression models after a log transformation.

AP Precalculus Unit 6: Exploring Sine and Cosine Functions

Lesson	Learning Targets
6.1 Periodic Phenomena	• Identify when two variables share a periodic relationship and construct their graph. • Describe the key features of a periodic function based on a verbal description or graph.
6.2 Angles on the Coordinate Plane	• Understand how to measure angles in standard position on the coordinate plane and their properties. • Understand that a radian is an angle measure with an arc length of one radius. • Label the angles on the unit circle in radians using proportional reasoning (i.e. partitions of semicircles).
6.3 Defining Sine, Cosine, and Tangent for Any Angle	• Extend the definition of sine, cosine, and tangent ratios to angles greater than 90˚ using the coordinate plane and horizontal and vertical displacement. • Understand why in a unit circle, the sine and cosine ratios correspond with the y-value and x-value, respectively, of the point where the terminal ray intersects the circle. • Understand that in a unit circle, the tangent of an angle is the ratio of the y-coordinate to the x-coordinate of the point where the terminal ray intersects the circle. Alternately, the tangent ratio is the slope of the terminal ray. • Use symmetry to identify relationships between the sine, cosine, and tangent values of angles in all four quadrants.
6.4 Coordinates on the Unit Circle	• Use special right triangles to determine the coordinates at key points on the unit circle. • Evaluate sine, cosine, and tangent for key angles on the unit circle. • Find coordinates of points on circles where $r \neq 1$.

6.5 Graphs of Sine and Cosine	• Construct graphs of the sine and cosine functions using values from the unit circle. • Identify key characteristics for the parent functions y=sin x and y=cos x including domain, amplitude, midline, period, and symmetry.
6.6 Transformations of Sine and Cosine	• Determine how the amplitude, period, domain, range, and midline of sinusoidal functions are affected by transformations. • Graph transformed sine and cosine functions given an equation.
6.7 Modeling with Trigonometric Functions	• Interpret a sinusoidal function's period, amplitude, midline, and range in context. • Construct a trigonometric model based on data points and key features.

AP Precalculus Unit 7: Working with Trigonometric Functions

Lesson	Learning Targets
7.1 The Tangent Function	• Understand how the tangent of an angle is determined by the slope of the terminal ray of the angle and use this to understand the behavior of the tangent function. • Describe the key features of the graph of the tangent function, including its domain, range, x-intercepts, and period. • Identify how the graph of the parent tangent function is affected by transformations.
7.2 Inverse Trig Functions	• Understand that inverse trigonometric functions input ratios and output angles. The input and output values are switched from their corresponding trigonometric functions. • Explain why and how the domains of sine, cosine, and tangent must be restricted to create an inverse function. • Evaluate inverse trig expressions.
7.3 Trigonometric Equations and Inequalities	• Extend the process of inverse operations to trigonometric equations and inequalities. • Understand that using the unit circle will give infinite solutions to a trigonometric equation which may need to be restricted based on context and that an inverse trig function gives only one solution that may need to be expanded using symmetry.
7.4 The Secant, Cosecant, and Cotangent Functions	• Define the secant, cosecant, and cotangent functions as the reciprocal of the cosine, sine, and tangent functions, respectively. • Understand how the zeros, vertical asymptotes, and range are related for a trigonometric function and its reciprocal function.
7.5 Trigonometric Relationships	• Explore relationships between all six trigonometric functions, including the Pythagorean identities. • Use identities to establish and verify other trigonometric relationships and solve trigonometric equations.

7.6 Angle Sum Identities	• Find exact values for the sine and cosine of angles not on the unit circle by writing the angle as a sum or difference of known angles. • Use equivalent trigonometric expressions arising from the angle sum and double angle identities to solve equations.

AP Precalculus Unit 8: Polar Functions

Lesson	Learning Targets
8.1 Polar Coordinates	• Understand that polar coordinates give an alternate method for locating points using a distance from the origin and an angle from the positive x-axis. • Use coterminal angles and reflected radii to name polar points in multiple ways. • Convert between polar and rectangular coordinates.
8.2 Complex Numbers	• Represent complex numbers on the complex plane in rectangular and polar form. • Given a complex number in rectangular or polar form, identify its real and imaginary component.
8.3 Polar Graphs: Circles and Roses	• Understand that polar functions input angle measures and output radii and point-by-point graphing can be used to construct their graphs. • Identify the number, length, and location of petals of a polar rose from the values of the parameters, a and n. • Describe key features of the polar graphs of circles and roses including their symmetry, domain and range.
8.4 Polar Graphs: Limacons	• Identify special types of limacons by comparing values of the parameters, a and b. • Describe key features of the graphs of limacons including symmetry, intercepts, domain, and range, and maximum and minimum values.
8.5 Rates of Change of Polar Functions	• Analyze and interpret key features of polar functions including intervals of increasing/decreasing and extrema. • Find and interpret the average rate of change of a polar function.