Topic 2.1 Change in Arithmetic and Geometric Sequences (Daily Video 1)

AP Precalculus

In this video, we will explore two different methods to express both arithmetic and geometric sequences and discuss which method is best in a given situation.

Let's REVIEW!

Arithmetic Sequences	Geometric Sequences
• A sequence of numbers in which the **difference** between any two consecutive numbers is a constant d. • The n^{th} term, a_n, in the sequence can be expressed by: $a_n = a_1 + d(n-1)$.	• A sequence of numbers in which the **ratio** between any two consecutive numbers is a constant r. • The n^{th} term, a_n, in the sequence can be expressed by: $a_n = a_1(r^{n-1})$.
Observe: $2, 7, 12, 17, 22, 27, \ldots$ The value of $d =$____. The first term $a_1 =$ _____ To get from the first term to the fourth term, we add _______ to the first term _______ times. Write an equation, in terms of d, for the fourth term. $a_4 = 2 +$ ________________	**Observe:** $2, 6, 18, 54, 162, \ldots$ The value of $r =$____. The first term $a_1 =$ _____ To get from the first term to the fifth term, we multiply _______ times the first term _______ times. Write an equation, in terms of r, for the fifth term. $a_5 = 2 \cdot$ ________________
The general (or generic) term a_n of an arithmetic sequence with a common difference d is given by $a_n = a_k +$_____________, where a_k is the k^{th} term.	The general (or generic) term a_n of a geometric sequence with a common ratio r is given by $a_n = a_k \cdot$ _____________, where a_k is the k^{th} term.

Let's look at an EXAMPLE! *Note: The presenter incorrectly wrote a sum instead of a sequence.

Is the sequence $-5, -\frac{9}{2}, -4, -\frac{7}{2}, -3, \ldots$ arithmetic or geometric? Justify your answer. Use the general equation for $a_n = a_k + d \cdot (n-k)$ to find the eighth term of the sequence. Show how you arrived at your answer.	Is the sequence $1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \ldots$ arithmetic or geometric? Justify your answer. Use the general equation for $a_n = a_k \cdot r^{n-k}$ to find the eighth term of the sequence. Show how you arrived at your answer.

What should we take away?

We should be able to recognize whether a sequence is arithmetic or geometric and write an equation for a general term of both.

Topic 2.1 Change in Arithmetic and Geometric Sequences (Daily Video 2)

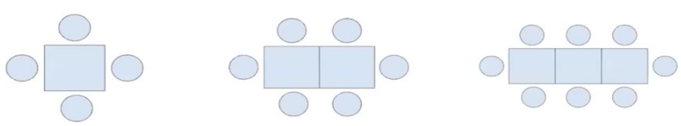
AP Precalculus

In this video, we will analyze functions that represent arithmetic and geometric sequences.

Let's REVIEW!

Arithmetic Sequences	Geometric Sequences
• A sequence of numbers in which the ____________ between any two consecutive terms is a constant. • A constant, often denoted d, can be _______ to any term to get the next term in the sequence. • Can be expressed, in general, as an equation: ________________________	• A sequence of numbers in which the ____________ of any two consecutive terms is a constant. • A constant, often denoted r, can be _____________ by any term to get the next term in the sequence. • Can be expressed, in general, as an equation: ________________________

Let's look at an EXAMPLE!

Example 1: Is the sequence arithmetic or geometric? Four people can sit around a square table. If two tables are put together, 6 people can sit around them. If three tables are put together, 8 people can sit around them.	
If we count the number of people around the table, we get the first few terms of the sequence: 4, 6, 8… 	This sequence is ________________ with a_1 = _____ and d = ______. Use the equation $a_n = a_k + (n - k)d$ to find the 8th term, a_8.

Example 2: Is the sequence arithmetic or geometric? A ball is dropped from the roof of a house that is 27 feet above the ground. Each time the ball bounces, it reaches a height that is $\frac{1}{3}$ of the height from which it last fell.	
The first few heights of the ball are 27, 9, 3… Use these terms to explain why this <u>cannot</u> be an arithmetic sequence.	This is a geometric sequence with a first term of a_0 = _____ and r = _____. Using $a_n = a_k(r^{n-k})$ find how high the ball will be after its 6th bounce.

What should we take away?

Equations can be written for arithmetic and geometric sequences by using a starting term and either a common difference (if __________________) or the common ratio (if ________________).

Topic 2.2 Change in Linear and Exponential Functions (Daily Video 1)

AP Precalculus

In this video, we will compare linear and exponential growth and practice representing these growth patterns with function formulas and graphs.

Let's WARM UP!

Imagine two fast growing vines that sprouted from Jack's magical beans. When you begin measuring the vines, they are both 10 feet long. For the next 4 days, Vine A's length increases 50 feet each day, while Vine B's length doubles each day. The tables that follow show the changes in length for both vines.

Δt	Number of days since Vine A was 10 feet long. t	Vine A's length (in feet) $f(t)$	$\Delta f(t)$
	0	10	
1			50
	1	60	
1			50
	2	110	
1			50
	3	160	
1			50
	4	210	

Number of days since Vine B was 10 feet long t	Vine B's length (in feet) $g(t)$
0	10
1	20
2	40
3	80
4	160

Vine A's length on day t is $f(t) =$ __________
Whenever t changes by 1 day, Vine A's length changes by ____ feet.
Whenever t changes by Δt days, Vine A's length changes by ________ feet.

Vine B's length on day t is $g(t) =$ __________
Whenever t changes by 1 day, Vine B's length becomes ____ times as long.
Whenever t days have passed, Vine B's length is ___ times as long as its starting length.

Let's look at another EXAMPLE!

Which vine will be longer 3 days after the vines are 10 feet long ($t = 3$)?

Vine A	Vine B	Vine Length Comparison
$f(3) =$	$g(3) =$	Vine A is 160 – 80 = 80* feet longer * The video has an error here.

Which vine will be longer 30 days after the vines are 10 feet long ($t = 30$)?

Vine A	Vine B	Vine Length Comparison
$f(30) =$	$g(30) =$	

Anytime the growth factor is greater than 1, _______________ growth will always outgrow linear or polynomial growth.

Let's PRACTICE!

If Vine B's length grows by only 5% each day (instead of 100% or doubling) will Vine B ever become larger than Vine A? Let's compare their lengths after 180 days (about 6 months).	
Linear growth (Vine A) $f(180) =$_____________ = __________ After 180 days, Vine B is about _________ feet longer than Vine A.	Exponential Growth (Vine B) The growth factor is ________ $g(t) =$ _______________ $g(180) =$ ______________ ≈ ________

What should we take away?

- Linear functions have a constant ________ of change.
- Exponential functions have a constant __________ change.
- If a function is growing exponentially (growth factor > 1), equal changes in the independent variable produce larger and larger changes in the dependent variable. In the long run, exponential growth (growth factor > 1), will always outpace _________ and other _________________ functions.

Topic 2.2. Change in Linear and Exponential Functions (Daily Video 2)

AP Precalculus

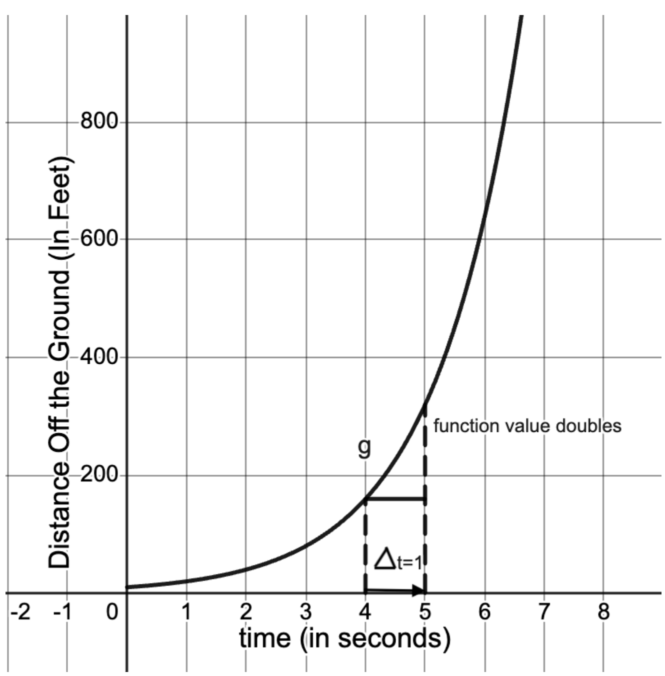
In this video, we will explore patterns in linear and exponential growth by contrasting ideas of a constant percent change with a constant amount of change.

Let's WARM-UP!

Recall that: If two quantities are changing together at a constant rate of change, ______ changes in one quantity results in a _____________ change in the other.

Let's REVIEW!

Let's examine the graph of g, which models the distance of a rocket from the ground (initially 10 feet off the ground), if its distance off the ground is doubling every second since liftoff.



What is the function for the distance the rocket is off the ground after t seconds?

$g(t) =$_______________

When t days have passed, the rocket's distance from the ground is _____ times as far as its starting length.

The distance off the ground for another rocket is modeled by the function $f(t) = 15(4^t)$.

The rocket's distance from the ground ______________ or increases by _____% every second.

Describe the distance off the ground in terms of the number of seconds, t, since liftoff.

$g(t) = 5(1.3)^t$	$h(t) = 12(1.07)^t$
The rocket is initially ____ feet off the ground and its distance from the ground increases by ___% or grows by a factor of _____ every second.	The rocket is initially ____ feet off the ground and its distance from the ground increases by ___% or grows by a factor of _____ every second.

Let's look at an EXAMPLE!

Another rocket is 30 feet off the ground when it lifts from its launching site. Its distance from the ground $g(t)$ doubles every 3 seconds.

Show how to find the 1-unit growth factor, *b*.

What is the function that gives the height of the rocket after t seconds?

*Note: Video states t years, instead of seconds.

	Number of seconds since the rocket lifted off t	Distance off the ground $g(t)$	
	0	30	
+ 1	1	?	x b
+ 1	2	?	x b
+ 1	3	60	x b (x 2)
+ 1	6	120	x 2

Let's PRACTICE!

Exploring Growth Patterns: Water levels in lakes in the western United States are dropping. If the recorded water level of a lake in this region was 1215 feet in 2015 and 1042 in 2020, what methods might you use to project what the water level will be in 2035?

Let t represent the number of years since 2015.
Let l represent the water level of the lake in feet.

Using the points (0, 1215) and (5, 1042) show how to find the linear and exponential growth rates.

Δt	Number of years, t, since 2015	Water level l (in feet)	Δl
	0	1215	
1			
1			
1			
1			
1	5	1042	
15	20	?	

What should we take away?

- If two quantities are related _________ the changes in the two quantities are ___________ and the corresponding changes in the two quantities' values remain _________ as the two quantities' values vary together.
- If two quantities are related ______________ the value of the dependent quantity changes by a constant factor b for every one unit increase in the value of x.
  The value of the dependent quantity gets larger if __________ and the value of the dependent quantity gets smaller if ______________.

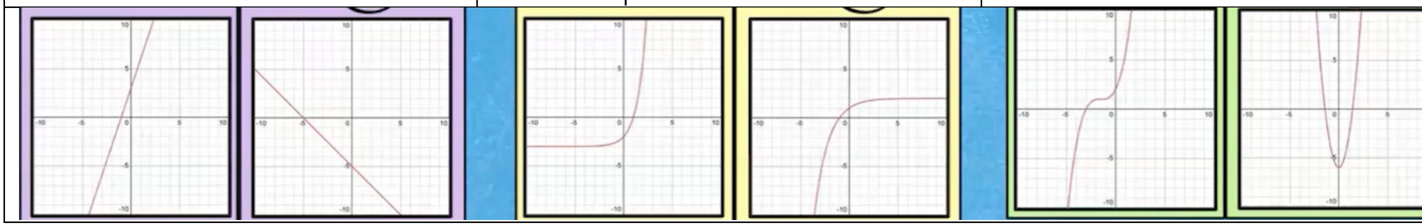
Topic 2.3 Exponential Functions (Daily Video 1)

AP Precalculus

In this video, we will learn the key characteristics of an exponential function, including general form, growth/decay, and domain/range.

Let's WARM-UP!

Linear Function	Exponential Function	Neither



Based on the pictures, write a sentence or two to explain how exponential functions look compared to non-exponential functions.

Let's REVIEW!

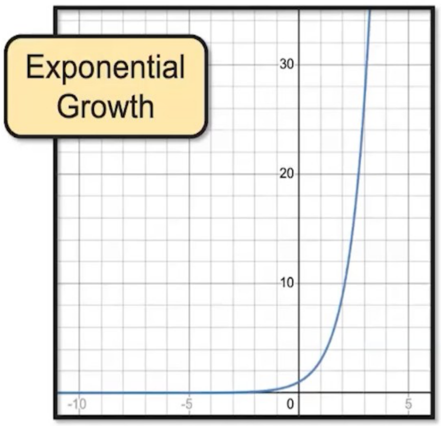
An exponential function has the form ______________, where __________ and ________.	
What is the base of each of the following functions? $f(x) = 2^x$ base = _____ $f(x) = \left(\frac{1}{2}\right)^x$ base = _____ $f(x) = 3e^{2x}$ base = _____ e is called ________________and is about _______.	Science and economics applications often involve exponential functions. Example: If bacterial doubles every hour, you would have _____ bacterial after x hours, written as $f(x) =$___________.

Let's REVIEW!

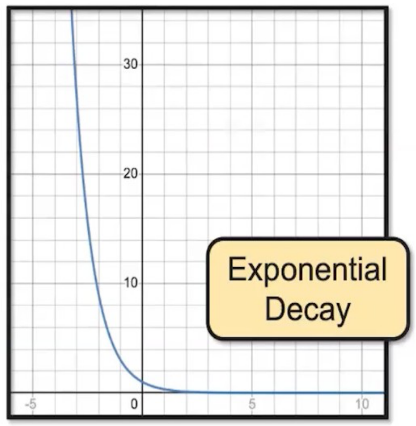
Exponential Growth	Exponential Decay
• A quantity __________slowly in the beginning, then there is ________ increase. • Used to model population growth, compound interest, doubling time, etc. • The graph is always ________________. • The base, b, is always greater than _____. • The domain is __________. • The range is ________.	• A quantity ___________ rapidly in the beginning and then there is a decrease. • Used to model population decay, find half-life, etc. • The graph is always ________________. • The base, b, is between _____ and _____. • The domain is _________. • The range is _______

Both exponential growth and exponential decay can be written in the form $f(x) = ab^x$, where a is the initial value.

Graph $f(x) = 3^x$



Graph $g(x) = \left(\frac{1}{3}\right)^x$ or $g(x) = (3)^{-x}$



Let's PRACTICE!

A function that has exponential growth has a base (multiplier) that is ____________.

A) Equal to 1 B) between 0 and 1 C) greater than 1 D) less than 0

Given $f(x) = 3^x$, choose the best representation of the domain.

A) $(0, \infty)$ B) $(-\infty, \infty)$ C) $(-\infty, 0)$ D) (-3, 3)

Given $f(x) = 2^x$, choose the best representation of the range.

A) $(0, \infty)$ B) $(-\infty, \infty)$ C) $(-\infty, 0)$ D) (-2, 2)

What should we take away?

General form

- $f(x) = b^x$, where $b > 0$ and $b \neq 1$

Growth and decay

- **Growth**: $f(x) = a \cdot b^x$, where $b > 1$
- **Decay**: $f(x) = a \cdot b^x$, where $0 < b < 1$

Domain and range

- **Domain**: $(-\infty, \infty)$
- **Range**: $(0, \infty)$

Topic 2.4 Exponential Function Manipulation (Daily Video 1)

AP Precalculus

In this video, we will learn the properties of exponential expressions and how to use them to rewrite the expressions in equivalent forms.

Let's WARM-UP!

Explain why $(-7)^4$ is not equivalent to $-(7)^4$. Use the words base and power in your response.

Let's Look at an EXAMPLE!

Complete each table and then write the general rules for exponents in the final row of the table.

Original Product	Exponential Form
$2^2 \cdot 2^3$	
$3^4 \cdot 3^2$	
$4^6 \cdot 4^2$	
$x^2 \cdot x^4$	
$x^a \cdot x^b$	

Original Product	Exponential Form
$(2^2)^3$	
$(4^5)^3$	
$(x^3)^4$	
$(2 \cdot 3)^3$	
$(3 \cdot 4)^3$	
$(xy)^3$	
$(x^a)^b$	
$(xy)^b$	

Original Product	Exponential Form
$\frac{3^5}{3^2}$	
$\frac{2^4}{2^1}$	
$\frac{5^{12}}{5^8}$	
$\frac{4^7}{4^5}$	
$\frac{x^4}{x^2}$	
$\frac{x^a}{x^b}$	

Let's PRACTICE!

Use the properties of exponents to rewrite each expression in simplified form. Show how you determined your answer.

1. $2m^4n^2 \cdot 4nm^2$

2. $\frac{4x^3y^4}{3xy^3}$

3. $2x(x^4y^4)^4$

4. $x^2y^4 \cdot xy^2$

5. $\frac{xy^3}{4xy}$

6. $\left(\frac{(2x)^3}{x^3}\right)^2$

7. $(2u^3v^4)^2$

8. $\frac{2a^2b^2a^7}{(ba^4)^2}$

9. $\left(\frac{2ba^7 \cdot 2b^4}{ba^2 \cdot 3a^3b^4}\right)$

What should we take away?

Exponential Properties are used to rewrite equivalent expressions:

Product:

Power:

Quotient:

Topic 2.4 Exponential Function Manipulation (Daily Video 2)
AP Precalculus
In this video, we will learn equivalent representations of exponential expressions, including negative and fractional exponents.

Let's Look at an EXAMPLE!
Zero Property of Exponents:
Explain why $x^0 = 1$. Note: 0^0 is ________.

A negative exponent conveys the number of times to multiply the _____ of the base.

Example: $7^{-2} =$ ___________

Write each expression with a single, positive exponent.

a. 2^{-1}	b. $\left(\frac{1}{3}\right)^{-1}$	c. x^{-3}	d. $(2+4x)^{-2}$

Rational (Fractional) Exponents

$$x^{\frac{1}{n}} = \_\_\_\_\_\_\_\_$$

a. $8^{\frac{2}{3}}$	b. $16^{\frac{1}{4}}$	c. $4^{\frac{3}{2}}$	d. $100^{-\frac{3}{2}}$

Let's PRACTICE!

Use the properties of exponents to rewrite each expression in simplified form. Note: Answers should contain only positive exponents.

1. $\left(a^{\frac{1}{2}}b^{\frac{1}{2}}\right)^{-1}$	2. $\left(x^{\frac{5}{3}}y^{-2}\right)^{0}$	3. $\left(\frac{a^2b^0}{3a^4}\right)$
4. $\frac{2x^{\frac{1}{2}}y^{\frac{1}{3}}}{2x^{\frac{4}{3}}y^{-\frac{7}{4}}}$	5. $\frac{y^0}{\left(x^{\frac{4}{3}}y^{-1}\right)^{\frac{1}{3}}}$	6. $\left(\frac{y^{\frac{1}{3}}y^{-2}}{\left(x^{\frac{5}{3}}y^{3}\right)^{-\frac{3}{2}}}\right)^{\frac{3}{2}}$

What should we take away?

- Zero/negative exponents

- Fractional exponents

Topic 2.5 Exponential Function Context and Data Modeling (Daily Video 1)

AP Precalculus

In this video, we will explore how proportional growth patterns suggest an exponential model.

Let's REVIEW!

The general form of an exponential equation is: ________________

- For every one-unit increase in the x-value, the y-value increases by a common __________ or proportion.
- A ____________sequence can be modeled by an exponential equation with only integer values of x.

Interpreting the Parts of an Exponential Equation: $y = a \cdot b^x$	What is e?
• a is the ______value and graphically it is the same as the y-value when x is ____________ • b describes how the graph increases (or decreases). b is the growth rate if _______and it is the decay rate if____________. • x describes the number of times the growth or decay factor is applied.	• It is also known as Euler's number. • It is ______________ (like π) and it is approximately _____________ • e is used to represent _____________ growth.

Let's Look at an EXAMPLE!

A small business is growing at a fast rate, but it is unclear whether its growth follows an exponential model. The business's revenue in its first year of operation was \$150,000. In year 2, its revenue was \$223,000. In year 3, its revenue was \$310,000. Is the business's growth over its first three years exponential?

Compare the proportional change in the y-values over equal-width intervals of x-values.	The ratios are ______________ and __________________. Are the growth rates equal? YES or NO (circle one) Is the growth rate exponential? YES or NO (circle one)

Let's look at an EXAMPLE!

<table>
<tr><td colspan="2">The population in Mali was 7.09 million in 1980. Since that time, the population has increased, on average, 2.07% per year. Write an exponential equation that models the population in Mali since 1980.</td></tr>
<tr><td>$a =$ ________ million
$b =$ ________</td><td>The equation is ________________, where y is _______________(in millions) and x is __________.</td></tr>
<tr><td colspan="2">Based on this model, what would we expect the population to be in 1994?

Why must $x = 14$?

How close was the prediction?</td></tr>
</table>

Let's look at an EXAMPLE!

You invested $11,235 five years ago. Interest is compounded continuously at a rate of 3%. What is this investment worth today?

What should we take away?

- The parts of an exponential equation have meanings in terms of the graph and in terms of the real-world context.
- e is an irrational number and it is about 2.71828.

Topic 2.5 Exponential Function Context and Data Modeling (Daily Video 2)

AP Precalculus

In this video, we will explore how different forms of equivalent exponential functions reveal different characteristics about particular exponential patterns.

Let's REVIEW!

Interpreting the Parts of an Exponential Equation: $y = a \cdot b^x$
• a is the initial value and graphically it is the same as the y-value when x is zero. • b describes how the graph increases (or decreases). b is the growth rate if $b > 1$ and it is the decay rate if $0 < b < 1$. • x describes the number of times the growth or decay factor is applied.

Let's look at an EXAMPLE!

Two Ways to Describe a Growth Rate
• To compare two growth rates, they need to be on the same __________ of the dependent variable. • Compare 1% interest compounded each month to 12% compounded each year. 1% per month for 12 months for x years   12% per year for x years Circle the greater growth rate.
Convert the Following Growth Rates (Round to the nearest ten-thousandth.) • A certain type of bacteria is growing at a rate of 1% per day. What is the equivalent *weekly* rate of growth? • A student's childcare business grew 30% over the course of a year. What is the equivalent *monthly* rate? Why is the exponent for this problem a fractional exponent? • A nation's GDP is forecasted to grow at a rate of 2.5% per week over the next year. What is the equivalent annual rate? Why do we subtract 1 from each of the above calculations?

Rewriting a Function with a Different Growth Rate

The population of a certain type of bacteria can be modeled by the equation $f(n) = 300 \cdot 2^n$, where $f(n)$ represents the number of bacteria after n days.

- What does the 300 represent in this context?
- How many bacteria are present after 7 days?
- Rewrite the function so the growth rate is weekly (w) instead of daily (d).

What should we take away?

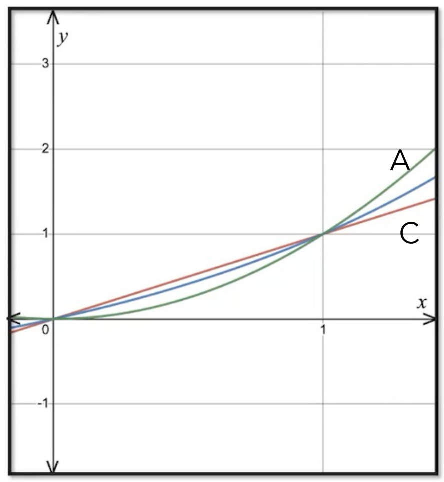
- There are multiple ways to write the same exponential model.
- Given growth rates can be written over different time intervals.

Topic 2.6 Competing Function Model Validation (Daily Video 1)

AP Precalculus

In this video, we will explore patterns in data that reflect how linear, exponential, and quadratic models differ.

Let's WARM UP!

<table>
<tr>
<td>
</td>
<td>Three functions are graphed on the left. They are:

$f(x) = x$
$f(x) = 2^x - 1$
$f(x) = x^2$

Which graph goes with which function?

What is making it difficult to tell these graphs apart?</td>
</tr>
</table>

- It may be very difficult to tell what type of function best models a set of data with two points.
- The minimum number of points needed to assess which model is best is ______.
- Why can't we always rely on a negative value for x to identify which function is best for a graph?

Rates of Growth for Different Functions

Linear: A ______________means the y-values increase by the same amount for equal values of x.	Exponential: The y-values increase_________________.	Quadratic: The y-values can ___________ and the difference between the differences of the y-values is ______________.

Let's look at an EXAMPLE!

What type of model is represented by the data? Explain.

<table>
<tr>
<td>Show the calculation of the differences.

t	$f(t)$
2	6.4
4	4.2
6	2
8	–0.2

</td>
<td>Show the calculation of the ratios.

x	y
0	0.5
1	2
2	8
3	32

</td>
</tr>
</table>

Let's look at an EXAMPLE!
What type of model is represented by the data? Explain.

x	y
–6	5
–4	–1
–2	–3
0	–1

What y-values repeat? __________

Compute the first differences and second differences for the table to the left.

Based on the data in the table, why is the quadratic model the best choice?

Let's look at an EXAMPLE!
What type of model is represented by the data?

Show the calculation of the differences of the differences.

x	y
0	1.5
1	16.6
2	21.9
3	17.4
4	3.1

What should we take away?

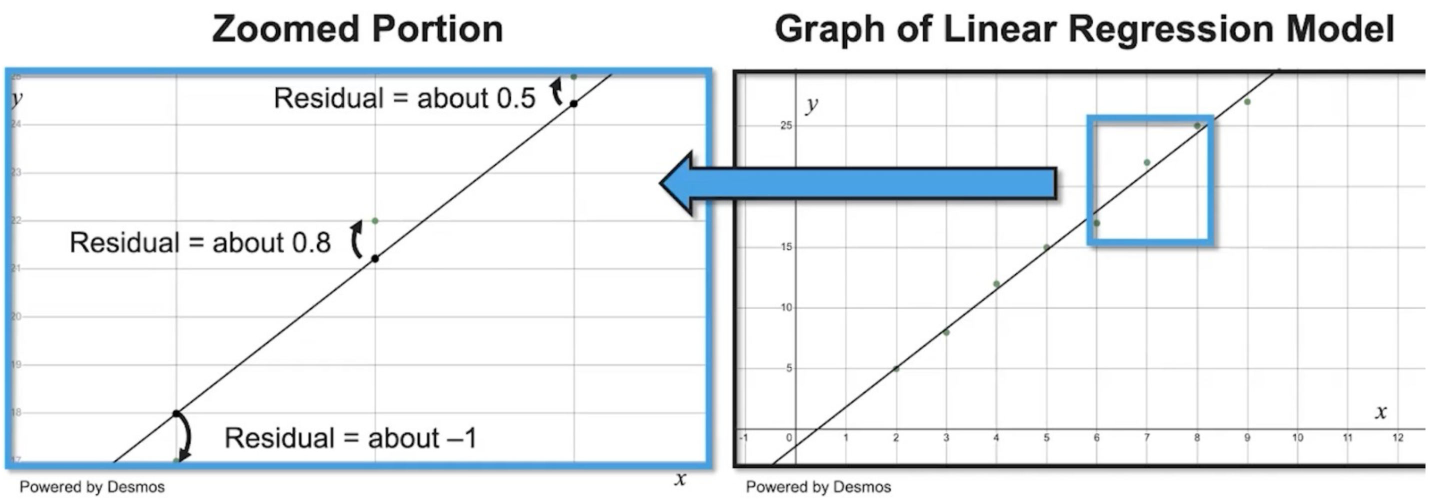
- At least _____ points are needed to assess which model is best.
- If the y-values are repeated, a quadratic model <u>may</u> be best.
- A ______________ model is best if the differences between y-values over equal intervals of x is a constant.
- A ______________ model is best if the differences between the differences of the y-values over equal intervals of x is a constant.
- An ______________ model is best if the proportions between the y-values over equal intervals of x is a constant.

Topic 2.6 Competing Function Model Validation (Daily Video 2)

AP Precalculus

In this video, we will explore what a residual plot reveals about a given model.

What is a residual?



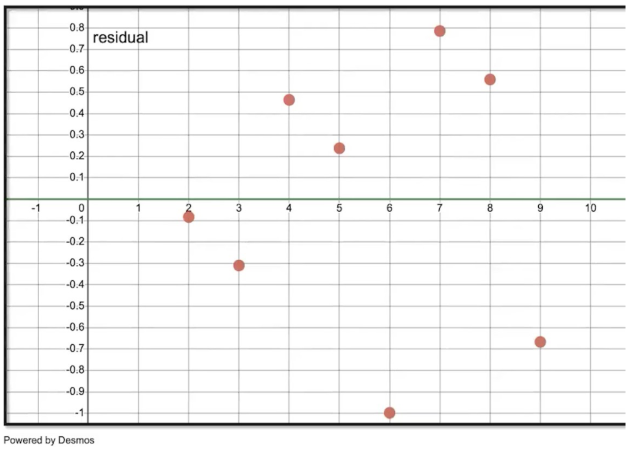
Residual = y-value from the data point – y-value from the ____________.

If the residual is negative, is the point above or below the line? __________

If the residual is positive, is the point above or below the line? ______________

Table of Some Residual Values and a Residual Plot

x	y	y-value on regression line	residual	point on residual plot
2	5	5.08333	5 − 5.08333 = **−0.08333**	(2, −0.08333)
3	8	8.30952	8 − 8.30952 = **−0.30952**	(3, −0.30952)
4	12	11.53571	12 − 11.53571 = **0.46429**	(4, 0.46429)



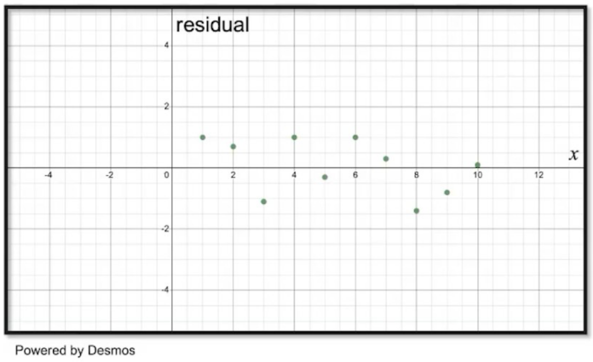
point on residual plot
(2, −0.08333)
(3, −0.30952)
(4, 0.46429)
(5, 0.2381)
(6, −0.98809)
(7, 0.78572)
(8, 0.55953)
(9, −0.66666)

The ordered pair for a point on a residual plot is (x, __________).

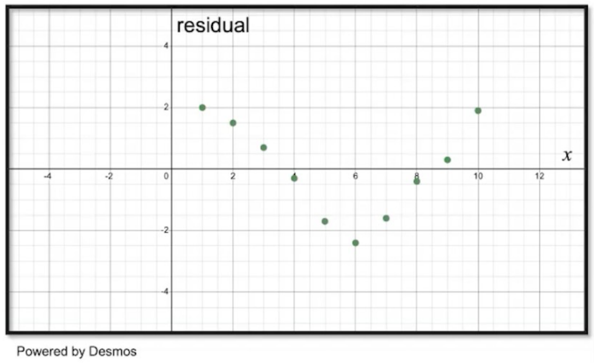
Is the model appropriate?

- If there is a pattern, the given model ________ appropriate.
- If there is not a pattern, the given model ________ appropriate.

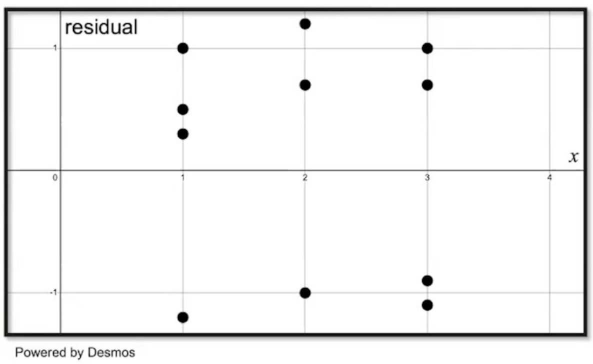
The given model is appropriate



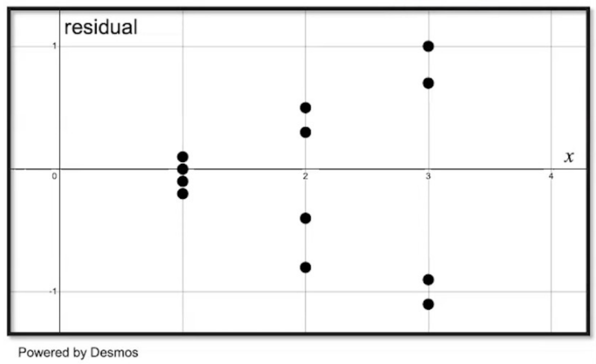
The given model is not appropriate



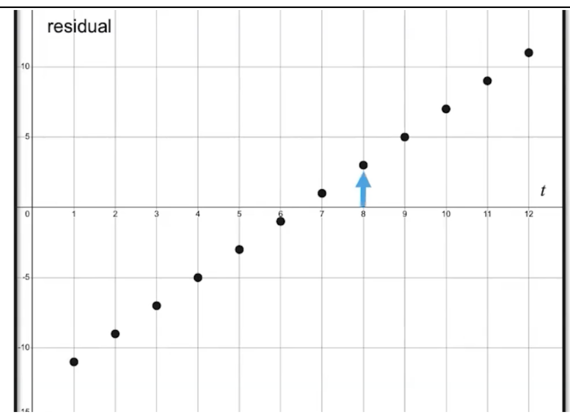
The given model is appropriate



The given model is not appropriate



Let's look at an EXAMPLE!

Is the model appropriate? A social media expert is studying how a particular meme is spreading. They can see how many times the meme has been viewed each hour on their platform. After creating an exponential function, $f(t)$, to model the number of views after t hours, they look at 12 hours of data and see the residual plot shown here. What can be said of the exponential model based on this residual plot?	

What should we take away?

- A residual plot shows how different the actual y-values are from what the model predicts.
- A residual plot has x-values for the x -axis and the value of the residuals for the y-axis.
- A residual plot shows which values are most different from what the model predicts.
- If the residuals are randomly distributed, a given model is appropriate.
- If the residuals follow a pattern, a given model is not appropriate.

Topic 2.7 Composition of Functions (Daily Video 1)

AP Precalculus

In this video, we will explore the composition of functions with multiple representations.

Let's look at an EXAMPLE!

Composition means _______________ the results of one function to another function.

NOTATION: $(f \circ g)(x) = f(g(x))$

Using the information provided numerically and graphically about functions f and g answer the composition question below.

For $x = 3$, find the value of $f(g(x))$.

1) Find $g(3)$.
2) Use that result in $f(x)$.

$g(3) = 2$ and $f(2) =$______

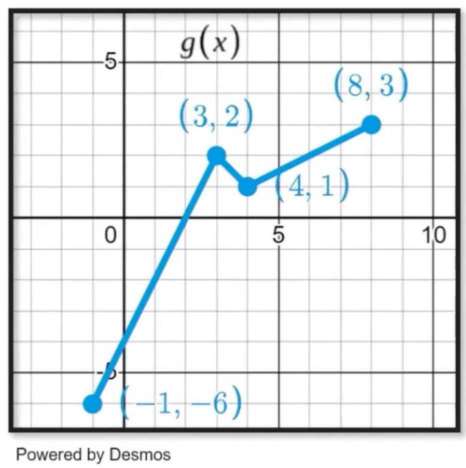
So, $f(g(3)) =$ ________.

For $x = 3$, find the value of $g(f(x))$.

1) Find $f(3)$.
2) Use that result in $g(x)$.

$f(3) =$ ________ and $g(1) =$ ______

So, *$g(f(3)) =$ ________.



x	$f(x)$
–1	0
0	4
1	2
2	–6
3	1
4	5
6	–3

NOTES:

1) The inner function is evaluated first.
2) The output of the inner function becomes the input of the outer function.
3) Since $f(g(3)) \neq g(f(3))$ function composition is NOT ___________________________.

* The video has the notation reversed in the second example.

Let's PRACTICE!

Use the previous table and graph to answer the following problems.

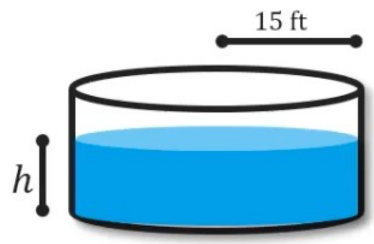
Question 1: For $x = 2$, find the value of $f(g(x))$.

Question 2: Find the value of $2g(f(x))$ when $x = -1$.

Question 3: For $x = 4$, find the value of $f(g(x) + f(x))$.

Let's look at an EXAMPLE!

An above-ground circular pool with radius 15 feet was installed in a yard. Water began filling the pool at $t = 0$, with the height of the water increasing at a constant rate of 0.08 ft/hour. The volume of the water, as a function height, is modeled by $V(h) = 225\pi h$.



Question 1: Given the equation that models the height of the water as a function of time, t, with t measured in hours.

$h(t)$ is a ________________ function.

$h(t)$= ______________

Question 2: Using $V(h) = 225\pi h$, identify the meaning of $V(h(t))$.

$h(t)$ is the water's ________ as a function of ________

$V(h)$ is the water's ___________ as a function of _______

$V(h(t))$ is the water's ________ as a function of _____

Question 3: Find the value of $V(h(t))$ at $t =$ 5 hours (filling began at $t = 0$ hours)

Evaluate inner function:

$h(5) =$ ________________

Evaluate outer function:

$V(0.4) =$ _______________

Therefore, $V(h(5)) =$ ______________

Let's REVIEW!

Error Analysis: Using the information provided about functions g and p, analyze the work shown below and identify the error.

When $x = 0$, find the value of $2p(g(x))$.

Work: First find $g(0) = -4$

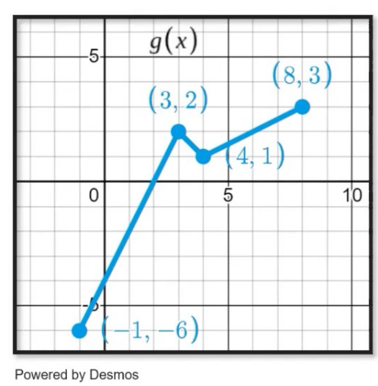
Then find $p(-4) = -(-4)^2 + 9$

$= 16 + 9$

$= 25$

Then $2p(g(0)) = 2(25) = 50$

$p(x) = -x^2 + 9$



What should we take away?

- Composed functions involve layers. The output for a layer becomes the input for the next layer.

Topic 2.8 Inverse Functions (Daily Video 1)

AP Precalculus

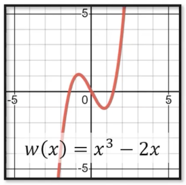
In this video, we will examine input-output pairs of a function's inverse, relevant notation, and what it means for a function to be invertible.

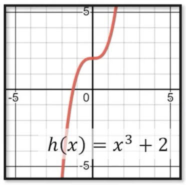
Let's WARMUP!

Notation: Given function: $f(x)$ Inverse Function: $f^{-1}(x)$ The inverse function is a ___________ mapping of the function $f(x)$, swaps input and output pairs.

Terminology: ***Invertible*** **(adjective):** able to have position or order __________ and create a function.

* Need every input mapped to one _________ and every output generated by one __________.





Which function(s), $w(x)$ and/or $h(x)$, are invertible? Justify your answer.

x	$f(x)$
1	3
3	9
7	10
9	12

Selected values for the increasing function f are given in the table.

$f(3) = 9$ then $f^{-1}(9) =$______

Let's look at an EXAMPLE! An invertible function, h, is known to have the given information indicated in each question. Using this, answer the questions below.

Question 1: If $h(2.6) = 0$ then $h^{-1}(\quad) =$_____

Question 2: If $h^{-1}(\pi) = -\frac{1}{5}$ then $h(\quad) =$_____

Question 3: An invertible function, w, has an x-intercept of $(-3,0)$ and its graph included the ordered pair $(2,7)$. List two ordered pairs that are points on the graph of $w^{-1}(x)$.

Question 4: For $a > b$, it is known that $f(a) < f(b)$. Selected values are given in the table.

$f^{-1}(6) =$_________

x	$f(x)$
–2.4	6
2	5
3	2
6	1
8	–2.4

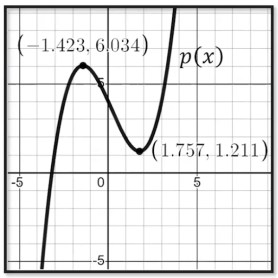
Let's PRACTICE! Use the information and table from Question 4 above.

$f^{-1}(2) + 4f(6) =$_______ Show all work.

Consider a noninvertible function, p, whose graph is shown. Circle the domain restrictions that represent invertible pieces of the noninvertible function p.

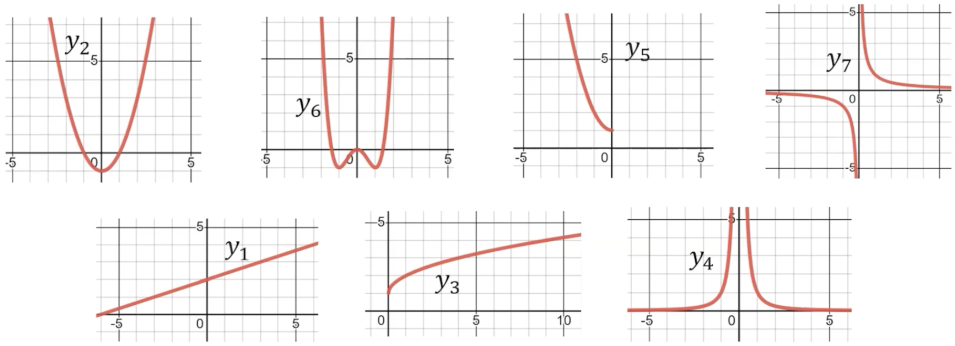
$(-\infty, -1.423]$ $[-1.423, \infty)$ $(-\infty, \infty)$

$(-\infty, 0]$ $[-1.423, 1.757]$ $[1.757, \infty)$



Place each of the following functions, given by their graphs, into the indicated groups.

Group 1: Not Invertible	Group 2: Invertible
Write the function names.	Write the function names.



Odd One Out

Thinking about invertible functions, identify which of the four listed functions is the "odd one out." Give a reason for your answer.

$$y = -x^5 \qquad y = e^x \qquad y = x^4 \qquad y = \frac{1}{x}$$

What should we take away?

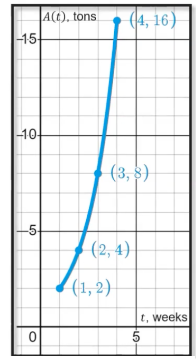
- An invertible function has an ________________ that is a function.
- An inverse maps _____________ values of f to the corresponding input values.
- The ______________ of a function may need to be restricted to achieve invertibility.

Topic 2.8 Inverse Functions (Daily Video 2)

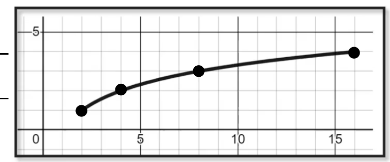
AP Precalculus

In this video, we will examine the characteristics of a function's inverse on its invertible domain.

Let's WARMUP! The graph of A models recycling material collected and reported at the end of each week, with $A(t)$ measured in tons and t measured in weeks.

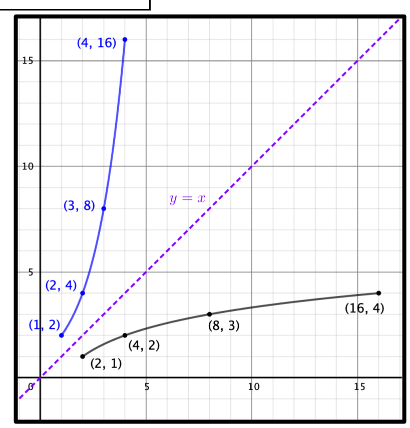


Question 1:	**Question 2:**
What is the domain and range of $A(t)$? Domain: __________ Range: __________	Label the **axes** and **coordinate pairs** on the graph of A^{-1} below. Give the domain and range of $A^{-1}(x)$. Domain: __________ Range: __________



Question 3:	**Question 4:**
The domain of A becomes the _________ of A^{-1} and the range of A becomes the _________ of A^{-1}.	$A(3) = 8$ means that at the end of the ______________, ____________ of collected recycling was reported. $A^{-1}(8) = 3$ means that ____________ of collected recycling was reported at the end of the ______________,

Question 5: What transformation is applied to A to obtain A^{-1}?



Analytically establish the following facts.

$A^{-1}\big(A(x)\big) = x$	$A\big(A^{-1}(x)\big) = x$

Composing a function and its inverse produces the ____________________ function.

Let's PRACTICE!

$g(x) = \frac{1}{x-4}, x > 4$

Characteristics of $g(x)$	Corresponding fact about $g^{-1}(x)$
g has a domain of $(4, \infty)$	
g has a range of $(0, \infty)$	
Graph of g has a vertical asymptote at $x = 4$.	

Let's look at an EXAMPLE!	**Let's REVIEW! Two Truths and a Lie**
Consider the invertible function given by $h(x) = x^2 + 4 \text{ for } x \leq 0$. Find the analytical representation of the inverse function, $h^{-1}(x)$.	Consider f, defined as $f(x) = x^3 + 2$. Two statements below are true, and one is a lie. Identify the lie and transform the lie into a truth. f is invertible and an increasing function. f has a *y*-intercept $(0,2)$, which indicates $f^{-1}(2) = 0$. The analytical form of $f^{-1}(x) = (x-2)^3$.

What should we take away?

- For an invertible function, the domain and ___________ of f ___________ to become the __________ and ___________of f^{-1}.
- Composing a function and its inverse produces the _____________________ function.

Topic 2.9 Logarithmic Expressions (Daily Video 1)

AP Precalculus

In this video, we will learn to evaluate logarithmic expressions, using arithmetic to obtain exact values (if possible) or technology to estimate values.

Let's WARMUP!

Definition: The logarithmic expression $\log_b c$ is equal to, or represents, the value that the base b must be exponentially raised to in order to obtain the value c.

Let's look an EXAMPLE!

Example 1: Evaluate $n = \log_2 8$ means we need to think __________ $n = \log_2 8 =$______

Example 2: Evaluate $n = \log_{10} 100$ means we need to think __________ $n = \log_{10} 100 =$______

Example 3: Evaluate $n = \log_3 \left(\frac{1}{3}\right)$ means we need to think __________ $n = \log_3 \left(\frac{1}{3}\right) =$______

Let's PRACTICE!

Example 4: Evaluate $\log_2 32$

Example 5: Evaluate $\log_3 81$

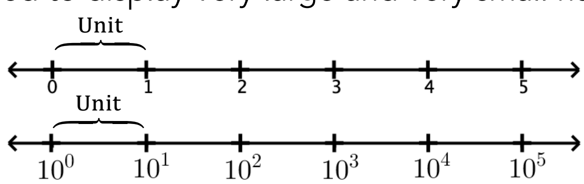
Example 6: Evaluate $\log_5(-25)$

Example 7: Evaluate $\log_2 7$

Logarithmic scales: Used to display very large and very small numbers.

Standard scale:

Log scale:



Give three examples where log scales are used:

1. ____
2. ____
3. ____

What should we take away?

$n = \log_b c$ is equal to the ______________ of base b in order to obtain the value c or $b^{(\_\_\_)} = c$.

Topic 2.10 Inverses of Exponential Functions Expressions (Daily Video 1)

AP Precalculus

In this video, we will learn to evaluate the general form of a logarithmic function and explore the relationship between input and output values.

Let's REVIEW!

Evaluate $n = \log_5 125$ means we need to think ____________ $n = \log_5 125 =$______

The definition of logarithm expresses a relationship between logs and exponents.

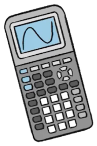
$$\log_b c = a \text{ if and only if } b^a = c \text{ where } b > 0 \text{ and } b \neq 1$$

Let's look at an EXAMPLE!

Complete the table for $f(x) = \log_3 x$ using arithmetic and technology.

x	$f(x)$	Notes
0	$f(0) = \log_3 0$	
1	$f(1) = \log_3 1$	
2	$f(2) = \log_3 2$	
3	$f(3) = \log_3 3$	
4	$f(4) = \log_3 4$	
9	$f(9) = \log_3 9$	
27	$f(27) = \log_3 27$	

*Note: on the AP Precalculus exam all decimal answers need to be to 3 decimal places.



What do you notice?

Table for $g(x) = 3^x$

x	$g(x)$
0	
1	
2	
3	
4	

Table for $f(x) = \log_3 x$

x	$f(x)$
	0
	1
	2
	3
	4

Let's PRACTICE!

Table for $f(x) = 2^x$

x	$f(x)$
0	
1	
2	
3	
4	

Table for $g(x) = \log_2 x$

x	$g(x)$
	0
	1
	2
	3
	4

What should we take away?

Exponential and logarithmic functions' input and output values are ________________.

Topic 2.10 Inverses of Exponential Functions Expressions (Daily Video 2)

AP Precalculus

In this video, we will explore the inverse relationship between logarithmic and exponential functions.

Let's REVIEW!

Topic 2.9: Evaluate $n = \log_2 \frac{1}{16}$ means we need to think ____________ $n = \log_2 \frac{1}{16} =$______

Topic 2.10: $f(x) = \log_b x$ and $g(x) = b^x$ where $b > 0$ and $b \neq 1$ are ________________ functions

Topic 2.8: Composition of a function and its inverse will result in the ____________function, x.

The graphs of inverse functions are ______________ over the graph of the $h(x) = x$.

A point (s, t) on a function becomes the point ________ on the inverse function.

Let's look at an EXAMPLE!

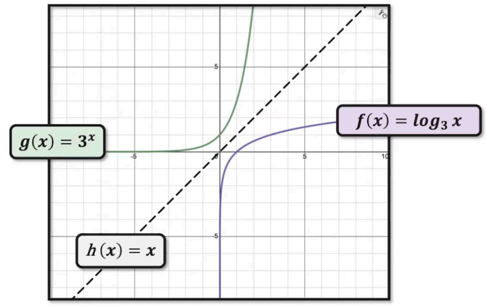
Let $f(x) = \log_3 x$ and its inverse $g(x) = 3^x$.
Show that $f(g(x)) = x$.

Show that the point (s, t) on $f(x)$ is reversed on $g(x)$.

x	$f(x)$
	0
	1
	2
	3
	4

x	$g(x)$
0	
1	
2	
3	
4	

Explain how the graphs of $f(x)$ and $g(x)$ show that $f(x)$ and $g(x)$ are inverses.



What should we take away?

$f(x) = \log_b x$ and $g(x) = b^x$ are ________________ functions.

- $g(f(x)) = f(g(x)) =$______
- The graphs of $f(x)$ and $g(x)$ are ____________ over the graph of the _________ function.
- The ordered pair (s, t) on the graph of $g(x)$ is the ordered pair ______ on the graph of $f(x)$.

Topic 2.11 Logarithmic Functions (Daily Video 1)

AP Precalculus

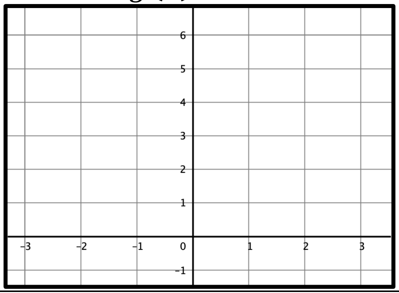
In this video, we will learn the key features of the logarithmic function, including domain, range, translations, and end behavior.

Let's WARMUP!

Complete the table for $g(x) = 2^x$ and plot the points on the grid.

x	-2	-1	0	1	2
$g(x)$					

$g(x) = 2^x$

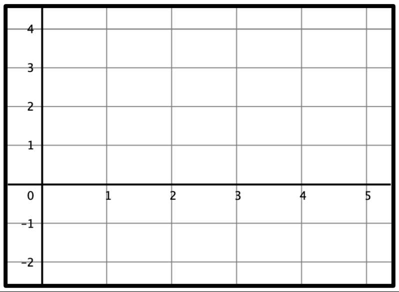


Complete the table for the inverse of $g(x) = 2^x$ and plot the points on the grid.

x					
$g^{-1}(x)$					

Inverse equation: ______________________________

inverse of $g(x) = 2^x$



Let's look at an EXAMPLE!

Complete the table for $g(x) = \log_5 x$ and plot the points on the grid.

Domain: ______________ Range: ______________

x-intercept: __________ y-intercept: __________

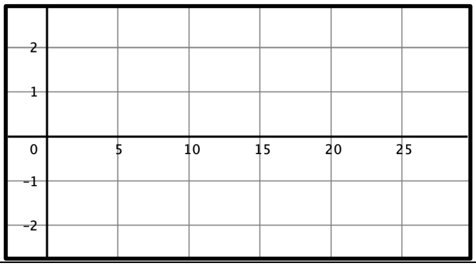
Asymptote: _______________

Shape of graph: $\underbrace{\hspace{3cm}}_{\text{inc/dec}}$ and $\underbrace{\hspace{4cm}}_{\text{concave up/down}}$

End behavior: ________________________________

$g(x) = \log_5 x$

x	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25
$g(x)$					



Let's PRACTICE!

What is the equation of the asymptote of the graph of the function $h(x) = 2 + \log_3(x - 5)$?

A $y = 2$

B $x = 3$

C $y = 0$

D $x = 5$

What should we take away?

The important features of $f(x) = \log_b x$, $b > 1$ Domain: __________ Range: ______________

x –intercept ________ Asymptote: ________ increasing and _____________ As $x \to \infty, f(x) \to$ ____

Math Medic

Topic 2.12 Logarithmic Function Manipulation (Daily Video 1)

AP Precalculus

In this video, we will learn the properties of logarithms and how to use them to manipulate expressions.

Let's WARMUP!

Evaluate: $\log_2 8 + \log_2 4 - \log_2 32$. Show your work.

A -20 B 0 C 1 D 2

Let's look at an EXAMPLE!

What happens to the graph of the parent function?	Related Exponent Rule
$h(x) = \log_3(9x) = \underbrace{\hspace{4cm}}_{\text{Log Rule Rewrite}}$ Transformation:	$x^2 \cdot x^7 =$ Multiplying then __________ exponents
$h(x) = \log_4(x^5) = \underbrace{\hspace{4cm}}_{\text{Log Rule Rewrite}}$ Transformation:	$(x^2)^7 =$ Power to a power then __________ exponents
$h(x) = \log_5\left(\frac{x}{2}\right) = \underbrace{\hspace{4cm}}_{\text{Log Rule Rewrite}}$ Transformation:	$\frac{x^7}{x^2} =$ Dividing then __________ exponents

Let's PRACTICE!

How does the graph of $f(x) = \log_5(3x)$ compare the graph of $g(x) = \log_5 x$?

A The graph of $f(x)$ is a horizontal translation to the left of $g(x)$.

B The graph of $f(x)$ is a vertical translation upward of $g(x)$.

C The graph of $f(x)$ is a vertical dilation of $g(x)$.

D The graph of $f(x)$ is a horizontal reflection of $g(x)$.

What should we take away?

Exponent Rules	Properties of Logs
$b^x \cdot b^y =$ $\frac{b^x}{b^y} =$ $(b^x)^y =$	$\log_b(\qquad) = \log_b(x) + \log_b(y)$ $\log_b(\qquad) = \log_b(x) - \log_b(y)$ $\log_b(\qquad) = y\log_b(x)$

Topic 2.12 Logarithmic Function Manipulation (Daily Video 2)

AP Precalculus

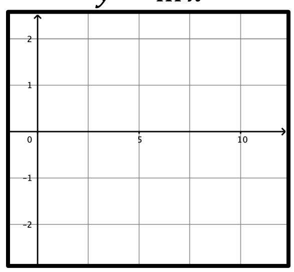
In this video, we will learn about the natural logarithmic function, $\boldsymbol{f(x) = \ln x}$, and apply the properties of logarithms to it.

Let's WARMUP!

$\log_b b =$ $\qquad$ $\log_b b^3 =$ $\qquad$ $\log_b 1 =$ $\qquad$ $\log 1000 =$ $\qquad$ $\log_5 \sqrt{\frac{1}{125}} =$

The Natural Logarithm has a base of $\boldsymbol{e}$, an irrational number, and instead of writing $\log_e x$, we write ___________.

Let's PRACTICE!

Complete the table for $y = \ln x$ and plot the points on the grid. | x | $y = \ln x$ | | | | | | | | | | | | | | | | | | |	$y = \ln x$ 
What happens to the graph of the parent function $y = \ln x$?	$y = \ln x^3 = \underbrace{\qquad\qquad\qquad}_{\text{Ln Rule Rewrite}}$ Transformation:
$y = \ln(e^2 x) = \underbrace{\qquad\qquad\qquad}_{\text{Ln Rule Rewrite}}$ Transformation:	$y = \ln\left(\frac{e}{x}\right) = \underbrace{\qquad\qquad\qquad}_{\text{Ln Rule Rewrite}}$ Transformation:

Let's PRACTICE! How does the graph of $g(x) = \ln x^3$ compare the graph of $f(x) = \ln x$?

A The graph of $g(x)$ is a horizontal translation to the left of $f(x)$.

B The graph of $g(x)$ is a vertical translation upward of $f(x)$.

C The graph of $g(x)$ is a vertical dilation of $f(x)$.

D The graph of $g(x)$ is a horizontal reflection of $f(x)$.

What should we take away?

Base b	Base e
$\log_b x + \log_b y = \log_b(\qquad)$ $\log_b x - \log_b y = \log_b(\qquad)$ $\log_b(\qquad) = y \cdot \log_b x$ $\log_b(\quad) = 1$ $\qquad$ $\log_b 1 =$	$\ln(xy) =$ ______________ $\ln\left(\frac{x}{y}\right) =$ ______________ $\ln(x^y) =$ ______________ $\ln e =$ ____ $\qquad$ $\ln 1 =$ ____

Topic 2.13 Exponential and Logarithmic Equations and Inequalities (Daily Video 1)

AP Precalculus

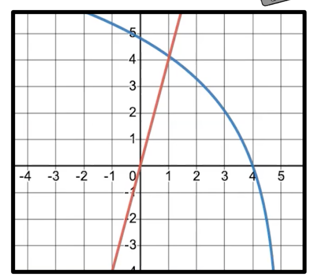
In this video, we will explore strategies for solving exponential and logarithmic equations and inequalities and assess the reasonableness of the solution(s) found.

Let's WARMUP! If $2 \cdot 3^x = 54$, what is the value of x ? Show how you arrived at the answer.

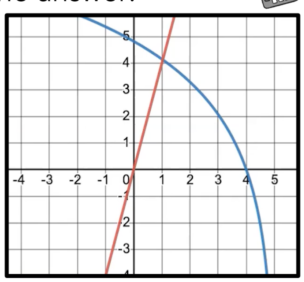
Let's look at an EXAMPLE!

1. If $5 \cdot 2^x - 3 = 112$, what is the value of x ? Show how you arrived at the answer.

2. Solve $3\ln(5 - x) = 4x$. Show how you arrived at the answer.



3. For which values of x is $3\ln(5 - x) > 4x$. Show how you arrived at the answer.



Let's PRACTICE!

Solve $\log(x - 6) + \log(x + 3) = 1$. Show how you arrived at the answer.

What should we take away?

Exponential and logarithmic equations inequalities cab be solved in a variety of ways including graphically. Also make sure your answers are in the domain and discard extraneous solutions.

Topic 2.13 Exponential and Logarithmic Equations and Inequalities (Daily Video 2)

AP Precalculus

In this video, we will explore how rewriting exponential and logarithmic expressions in equivalent forms can reveal relationships that make solving equations easier.

Let's WARMUP! Equivalent or Not?

16^{m-1}	2^{4x-4}
$\log(3x) - \log 100$	$\dfrac{\log(3x)}{2}$
$\log_5 w$	$\log_{25} w^2$

Let's REVIEW! Exponential and Logarithm Properties

$x^a \cdot x^b =$ $\dfrac{x^a}{x^b} =$ $(x^a)^b =$	One-to-one property If $b^x = b^y$, then __________
$\log a + \log b =$ $\log a - \log b =$ $\log(x^a) =$	If $\log x = \log y$, then __________

Let's look at an EXAMPLE!

Solve $2^x = 8^{3x-2}$. Show how you arrived at the answer.	Find all solutions to $\log_9(8x-15) = \log_3 x$. Use the rule $\log_9 w = \frac{1}{2}\log_3 w$. Show how you arrived at the answer.

What should we take away?

- Understanding equivalence helps us notice two expressions and these relationships allow us to simplify an expression or solve an equation.
- Simply memorizing rules is not sufficient, you must truly understand how exponents and logarithms work.

Topic 2.13 Exponential and Logarithmic Equations and Inequalities (Daily Video 3)

AP Precalculus

In this video, we will apply the strategy of inverse operations to write the equation for the inverse of a transformed function.

Let's REVIEW!

Consider the function $f(x) = 2\ln(x-7) + 5$. What is the parent function? ________

What transformations occurred? Describe any horizontal and/or vertical shift.	Describe any horizontal and/or vertical dilation (stretch).
Write an equation for the inverse of $f(x) = 2\ln(x-7) + 5$. Show how you arrived at the answer. $f^{-1}(x) =$	**Let's Practice!** Let $f(x) = 4^{(3x-8)}$. Write an equation for f^{-1}. Show how you arrived at the answer.

What should we take away?

- All exponential and logarithmic functions are transformations of a ____________ function.
- To find an inverse funcrion, use _____________ operations to __________ the other variable.

Topic 2.14 Logarithmic Function Context and Data Modeling (Daily Video 1)

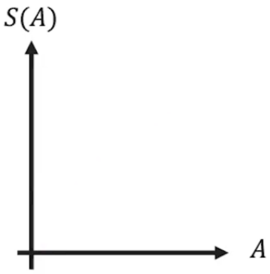
AP Precalculus

In this video, we will interpret logarithmic functions in context, explore their growth rates, and use an algebraic model to make predictions.

Let's look at an EXAMPLE!

The number of unique plant species in an area of California can be modeled by the function $S(A) = 77 + 147\log(A)$. where $S(A)$ is the number of species in a region with an area of A square miles.

a) Describe how the number of unique plant species changes as the area of the region increases. Sketch a logarithmic graph.



The number of species ________________, but at a __________________ rate.

b) Predict how many unique plant species exist in a 110 square-mile region of California.

$S(110) =$

Let's PRACTICE!

The number of words that a child knows can be modeled by the function V, where $V(t)$ is the number of words a child knows, in thousands, when they are t months old. An equation $V(t)$ is given by $V(t) = 10\log t - 13$.

a) Predict the number of words a child knows when they are 2 years old ($t = 24$).

$V(24) =$

When the child is _______ years old, the model predicts they will know approximately ________________ words.

b) Find $V(48)$ and interpret your results in the context of this problem.

$V(48) =$

When the child is _______ years old, the model predicts they will know approximately ________________ words.

c) Explain why vocabulary growth can be reasonably modeled by a logarithmic function.

d) How many years does the model predict it will take to learn 15,000 words? Show how you arrived at your answer.

What should we take away?

Logarithmic models describe relationships where the dependent variable increases (or decreases), but the rate slows down over time.

Topic 2.14 Logarithmic Function Context and Data Modeling (Daily Video 2)

AP Precalculus

In this video, we will construct logarithmic models from given data, with and without technology.

Let's look at an EXAMPLE!

Selected values of a logarithmic function, f, are given in the table.
If $f(x) = a\log_3 x + b$ for some parameters a and b, find the values of a and b.

x	$f(x)$
1	−1
3	3
9	7
27	11

What is the parent function? $y =$ ________

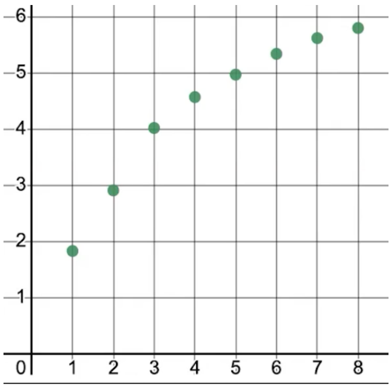
The graph passes through points ________ and ________

Find the values of a and b. Show all your work.

Let's PRACTICE!

Information about the age and height of a tree is given in the table. Construct a natural log regression model to predict the tree's height, in meters, after t years.

Age (years)	Height (meters)
1	1.83
2	2.91
3	4.02
4	4.57
5	4.97
6	5.34
7	5.62
8	5.80



Explain why a logarithmic model makes sense?

Use your calculator to find a regression equation. Directions vary by calculator.

The height of the tree, in meters, after t years can be modeled by the equation

$H(t) =$ ______________________________

What should we take away?

- Using **Key points** of a logarithmic function, ________ and ________, can help determine the parameters of the logarithmic equation that passes through the given points
- Logarithmic regression is a tool used to construct a logarithmic model for given real world data.

Topic 2.15 Semi-log Plots (Daily Video 1)

AP Precalculus

In this video, we will review how to read a semi-log plot.

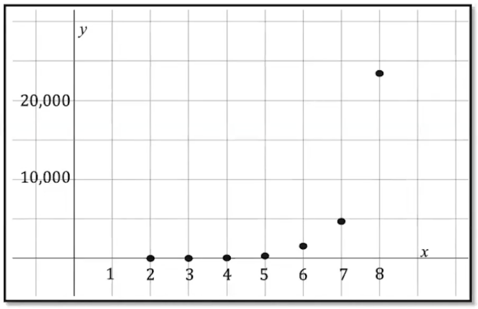
What is a semi-log plot?

It is a graph where one axis is scaled ______________ and the other is scaled ________________.

Why would we need to do that?

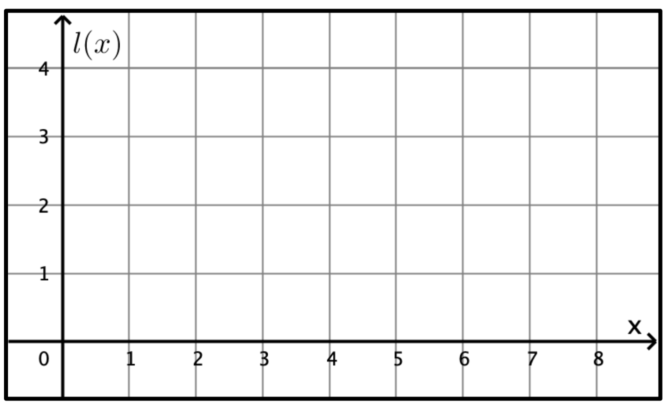
x	$f(x)$
2	2.5
3	12.5
4	62.5
5	312.5
6	1,562.5
7	4,687.5
8	23,437.5

Data on a standard x-y plane



Plot the data for $l(x) = \log f(x)$

x	$l(x) = \log f(x)$
2	$\log(2.5) = 0.3979$
3	$\log(12.5) = 1.097$
4	$\log(62.5) = 1.796$
5	$\log(312.5) = 2.495$
6	$\log(1{,}562.5) = 3.194$
7	$\log(4{,}687.5) = 3.671$
8	$\log(23{,}437.5) = 4.370$



What should we take away?

If a semi-log plot is drawn of a graph for whick an exponential model os appropriate, the semi-log plot will appear linear.