Who's Who on Halloween?

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Amari, Francisca, Jamirea, Micaiah, and Senad live in the same neighborhood. On Halloween, they each go out trick-or-treating, but they each start at slightly different times, wear a different costume, and have different favorite candies they like to collect. For each derivative problem you solve, you'll receive a clue from one of the neighbors about what they saw that night. Can you figure out who's who on Halloween, including what time each person started trick-or-treating, what their favorite candy is, and what costume they were in?

		First Names					Favorite Candy					Costume				
		Amari	Francisca	Jamirea	Micaiah	Senad	M&Ms	Starbursts	Twix	Air Heads	Mounds	Chef	Banana	Ghost	Lumberjack	Pirate
Time	5:45	X	■	X	X	X	X	X	X	■	X	■	X	X	X	X
	6:00	X	X	X	X	■	X	X	■	X	X	X	X	X	■	X
	6:30	X	X	X	■	X	X	■	X	X	X	X	■	X	X	X
	6:45	■	X	X	X	X	■	X	X	X	X	X	X	X	X	■
	7:00	X	X	■	X	X	X	X	X	X	■	X	X	■	X	X
Costume	Chef	X	■	X	X	X	X	X	X	■	X					
	Banana	X	X	X	■	X	X	■	X	X	X					
	Ghost	X	X	■	X	X	X	X	X	X	■					
	Lumberjack	X	X	X	X	■	X	X	■	X	X					
	Pirate	■	X	X	X	X	■	X	X	X	X					
Favorite Candy	M&Ms	■	X	X	X	X										
	Starbursts	X	X	X	■	X										
	Twix	X	X	X	X	■										
	Air Heads	X	■	X	X	X										
	Mounds	X	X	■	X	X										

1. The person whose favorite candy is Air Heads started trick-or-treating before Senad.

2. The 5 people are the person whose favorite candy is Air Heads, the pirate, Micaiah, the person who started trick-or-treating at 7 PM and the person whose favorite candy is Twix.

3. Of the chef and Senad, one started trick-or-treating at 5:45 PM and the other has Twix as their favorite candy.

4. Francisca started trick-or-treating at 5:45 PM.

5. Jamirea is either the ghost or the lumberjack.

6. The ghost started trick-or-treating after the banana.

7. The person whose favorite candy is M&Ms is neither the ghost nor the banana.

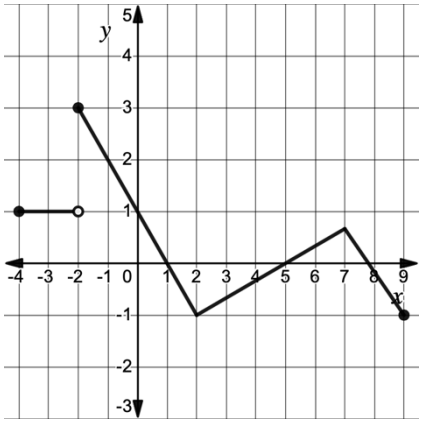
8. The person who started trick-or-treating at 6:30 has Starbursts as their favorite candy.

9. The person who started trick-or-treating at 6:45 was not the banana.

10. The banana started trick-or-treating after Senad.

Key

1. The graph of $y = f(x)$ is shown.



Evaluate $\lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$

This means find $f'(3)$

$\frac{1}{3}$

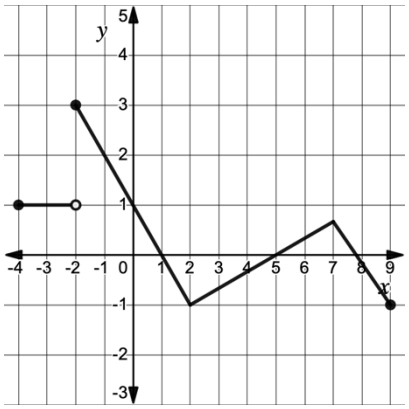
2. Find the derivative of each function.

$f(x) = \ln(4x)$ $\quad f'(x) = \frac{4}{4x} = \frac{1}{x}$

$g(x) = (3x-5)^2$ $\quad g'(x) = 3 \cdot 2(3x-5) = 6(3x-5) = 18x-30$

$h(x) = \sec x$ $\quad h'(x) = \sec x \tan x$

3. The graph of $y = f(x)$ is shown below for $-4 \le x \le 9$. For which value(s) of x is f continuous but not differentiable?



$x = 2$ and $x = 7$

4. Selected values of f, g, and their derivatives are given in the table.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-3	10	1	-1	3
-1	4	-2	-3	0
2	1	-5	0	-2
5	-2	-3	2	8
8	-5	11	7	-3.5
11	5	8	13	1

Let $h(x) = f(x) \cdot g(x)$. Find $h'(8)$.

$h'(8) = F(8) \cdot g'(8) + g(8) \cdot f'(x)$

$= -5 \cdot (-3.5) + 11(7)$

$= 94.5$

5. Let $j(x) = 5x^3 - kx^2 + 10x + m$ for some constants k and m. If $j(2) = -10$ and $j'(2) = 6$, find the values of k and m.

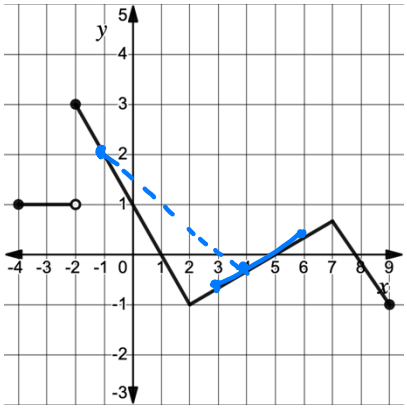
$j'(x) = 15x^2 - 2kx + 10$

$j'(2) = 15(2)^2 - 2k(2) + 10 = 70 - 4k = 6 \Rightarrow k = 16$

$j(2) = 5(2)^3 - 16(2)^2 + 10(2) + m = -10$

$= 40 - 64 + 20 + m = -10 \quad m = -6$

6. The graph of $y = f(x)$ is shown below. Order the following from least=1 to greatest=4.



1 $\lim_{x \to -1} \frac{f(x) - f(-1)}{x+1} = -1$

4 The average rate of change of f on the interval [3,6] > 0

2 $\frac{f(4) - f(-1)}{5}$ neg but less steep

3 $f'(-3)$ $= 0$

7. Selected values of f, g, and their derivatives are given in the table.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-3	10	1	-1	3
-1	4	-2	-3	0
2	1	-5	0	-2
5	-2	-3	2	8
8	-5	11	7	-3.5
11	5	8	13	1

Let $k(x) = \frac{g(x)}{f(x)}$. Find $k'(-1)$.

$k'(-1) = \frac{f(-1) \cdot g'(-1) - f'(-1) \cdot g(-1)}{(f(-1))^2} = \frac{4(0) - (-3)(-2)}{4^2} = \frac{-6}{16} = \boxed{\frac{-3}{8}}$

8. If $f(x) = \frac{\cos^2 x}{\sin x}$, find $f'(x)$.

$f(x) = \frac{1-\sin^2 x}{\sin x} = \csc x - \sin x$

$f'(x) = -\csc x \cot x - \cos x$

9. Selected values of f, g, and their derivatives are given in the table.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-3	10	1	-1	3
-1	4	-2	-3	0
2	1	-5	0	-2
5	-2	-3	2	8
8	-5	11	7	-3.5
11	5	8	13	1

Let $j(x) = f(g(x))$. Find $j'(5)$.

$j'(5) = g'(5) \cdot f'(g(5)) = 8 \cdot f'(-3) = 8 \cdot (-1) = -8$

10. Let $f(x) = x^2 + 5x$ and let g be a function so that $g'(x) = 3$ for all x.
If $h(x) = f(g(x))$ and $h'(x) = 18x + 57$, write an equation for $g(x)$.

$g(x)$ is linear with a slope of 3. $g(x) = 3x + b$

$f'(x) = 2x + 5$

$h'(x) = g'(x) \cdot f'(g(x)) = 3 \cdot [2(3x+b) + 5] = 18x + 57$

$= 18x + 6b + 15 = 18x + 57$

$6b = 42$

$b = 7$

$g(x) = 3x + 7$