How Many Bottle Flips to Go Viral?

THE Talent SHOW

Scene: The senior talent show is scheduled to take place this afternoon. Michael Senatore is in stats class practicing his act - bottle flipping. He tells his friends that the probability of successfully landing the water bottle right side up is 20%. Assume that each water bottle flip is independent.

1. If he flips the bottle ten times, how many do you expect to land right side up? Justify.

$\mu = np$ $10(0.20) = 2$ flips

BINS ✓✓✓✓ Binomial $n=10$ $p=0.20$

2. What is the probability that he makes exactly two flips?

binompdf(n, p, x) = binompdf(10, 0.20, 2) $= {}_{10}C_2(0.20)^2(0.80)^8 = 0.302$

3. What is the probability that he makes less than two flips?

binomcdf(n, p, x) = binomcdf(10, 0.20, 1) $= {}_{10}C_0(0.20)^0(0.80)^{10} + {}_{10}C_1(0.20)^1(0.80)^9 = 0.3758$

4. What is the probability that he makes two or more flips?

$1 - 0.3758 = 0.6242$

STOP After the first ten flips, Michael wants to do more, but his teacher is getting annoyed with all the noise and tells him he must stop flipping **once the bottle lands right side up**.

5. Find the probability that he lands it on the first try.

0.20

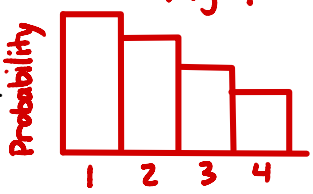
6. Find the probability that he fails on the first try and lands it on the second try.

$0.80 \times 0.20 = 0.16$

7. Find the probability that he fails the first two and lands it on the third try.

$0.80 \times 0.80 \times 0.20 = 0.128$

skewed right



8. Find the probability that he fails the first three and lands it on the fourth try.

$0.80 \times 0.80 \times 0.80 \times 0.20 = 0.1024 \rightarrow 0.80^3 \times 0.20^1$

Let X = the number of flips it takes for Michael to land the bottle right side up. Fill in the table below.

"until first success" → Geometric

X	1	2	3	4	5	6
P(X)	0.20	0.16	0.128	0.1024	0.0819	0.0655

×0.8 ×0.8 ×0.8 ×0.8 ×0.8 Geometric sequence

9. Find the probability that he lands it on the tenth try. Write a generic rule for finding the probability that he first lands the bottle on the k^{th} flip.

$(0.80)^9 \times 0.20$ $P(X=k) = (1-p)^{k-1} \times p$

$\mu = \frac{1}{p}$ 10. How many flips do you expect it to take for the bottle to land right side up? Why?

$\mu = \frac{1}{0.20} = 5$ 20% probability, so we expect 1 out of 5

Math Medic

Lesson 6.8 – The Geometric Distribution

QuickNotes

LT*1 Geometric distribution

- Binary: success or failure
- Independent trials
- Trials until success
- Same probability of success

LT*2 Geometric formula

$$P(X=k)=(1-p)^{k-1}\times p$$

(k − 1: # of failures; $1-p$: probability failure; p: probability success)

LT*3

- Shape: skewed right
- center: $\mu=\frac{1}{p}$
- variability: $\sigma=\frac{\sqrt{1-p}}{p}$

Check Your Understanding

Mason never has a pencil when test day rolls around. Because his classmates are tired of having to supply pencils for Mason, only 15% of students will give Mason a pencil when asked. Today is test day and Mason begins asking randomly selected students for a pencil. Let Y = the number of students Mason asks until he finds someone who will give him a pencil.

a. Does the random variable Y follow a geometric distribution? Be sure to check the appropriate conditions.

Yes. ✓ Binary. success → gets pencil, failure → no pencil
✓ Independent trials (assume)
✓ Trials until Mason gets a pencil
✓ Same probability of success ($p=0.15$)

b. What is the probability that the third person asked is the first person who gives him a pencil?

geometpdf(0.15, 3) $P(Y=3)=0.85^2\times 0.15=0.1084$

c. What is the probability that Mason gets a pencil by the third person he would ask?

geometcdf(0.15, 3) $P(Y\le 3)=0.15+0.85\times 0.15+0.85^2\times 0.15=0.3859$

d. How many people should Mason expect to ask before getting a pencil?

$\mu=\frac{1}{0.15}=6.67$ people

e. Should Mason be surprised if he doesn't receive a pencil until he asked at least 10 people? Calculate a probability to justify your answer.

1 − geometcdf(0.15, 9) $P(Y\ge 10)=1-P(Y\le 9)=0.2316$

No, this result is not unlikely.

f. Find the standard deviation for Y.

$\sigma=\frac{\sqrt{1-0.15}}{0.15}=6.15$ people