Practice Question 4: Symbolic Manipulations

Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.

(A) The functions g and h are given by

$$g(x) = \frac{\tan x\,(1 - \sin^2 x)}{\cos x}$$

$$h(x) = \log_3(x^2) + 10\log_9 x$$

(i) Rewrite $g(x)$ as a single expression including only one trigonometric function.

(ii) Rewrite $h(x)$ as a constant multiple of $\log_3 x$.

(B) The functions j and k are given by

$$j(x) = \frac{e^4}{\sqrt{e^x}}$$

$$k(x) = \cos^{-1}(5x)$$

(i) Solve $j(x) = e$ for values of x in the domain of j.

(ii) Solve $k(x) = \frac{2\pi}{3}$ for values of x in the domain of k.

(C) The function m is given by

$$m(x) = 6\csc\left(x - \frac{\pi}{6}\right)$$

Find all values in the domain of m that yield an output of 12.