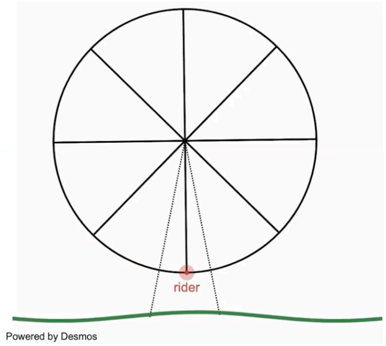
Topic 1.1 Change in Tandem (Daily Video 1)

AP Precalculus

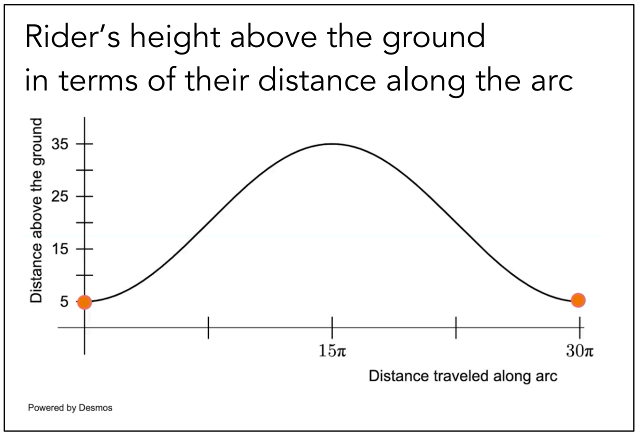
In this video, we will explore how a graph allows us to track how the values of two quantities change together.



Goal: Track one rider as they complete one full trip around the Ferris wheel with a diameter of 30 feet (Circumference 30π)

1. What quantities are we tracking? Sketch them on the diagram.

2. How are these two quantities changing?



Interpretation of the *y*-intercept (0,5): The rider's height above the ground is ______feet when they get on the Ferris wheel and have traveled _____feet along the arc.

Minimum height: What is the rider's minimum height above the ground? How many times is that height reached?
Express your answers to the above question as ordered pairs.

Maximum height: Locate a point on the graph where the rider is the maximum height above the ground. What are the coordinates of this point? _______ Write an interpretation of the coordinates of this point in the context of a Ferris wheel ride?

How is the height of the rider above the ground changing before reaching the maximum height? Circle one.

increasing   decreasing

How is the height of the rider above the ground changing after reaching the maximum height? Circle one.

increasing   decreasing

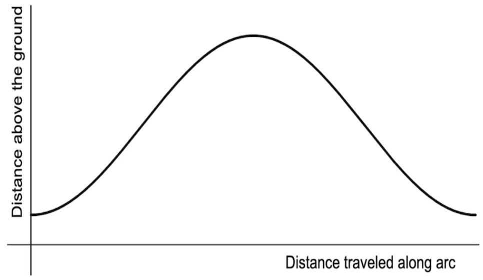
What should we take away?

- Graphs track how ____.
- **When we describe a graph, we should talk about:**
  - ____-intercept(s) and/or ____-intercept(s)
  - Intervals over which the function is _______________ or _______________
  - _______________ and/or ________________ points

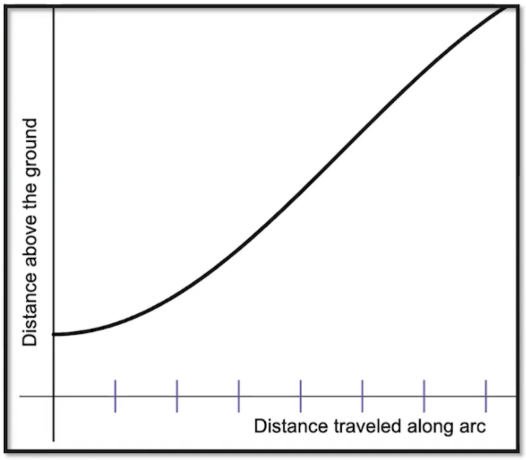
Topic 1.1 Change in Tandem (Daily Video 2)
AP Precalculus

In this video, we will investigate how two quantities change together, how to calculate that rate of change, and how to visualize this through the concavity of the graph.

Example 1: So, let's explore why the graph from Topic 1.1 was curved and what that curvature tells us about HOW these quantities change together.



Coordinating Amounts of Change: Let's systematically explore how the output changes for equal changes in the input.



As you watch the video use red and blue to fill in the picture.

1. Partition the horizontal axis into equal-sized chunks
2. Focus on one of these intervals
3. Identify corresponding points on the graph
4. Determine corresponding change in the output
5. Compare how the output changes for equal changes in the input.

What does the "red" segment represent? ____

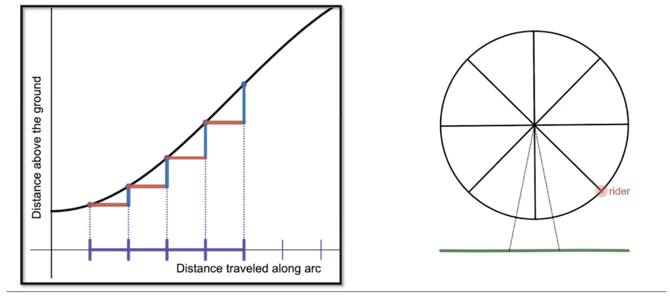
What does the "blue" segment represent? ____

This type of diagram, that shows both the change in the ______________ and the change in the ______________ is sometimes called? ____

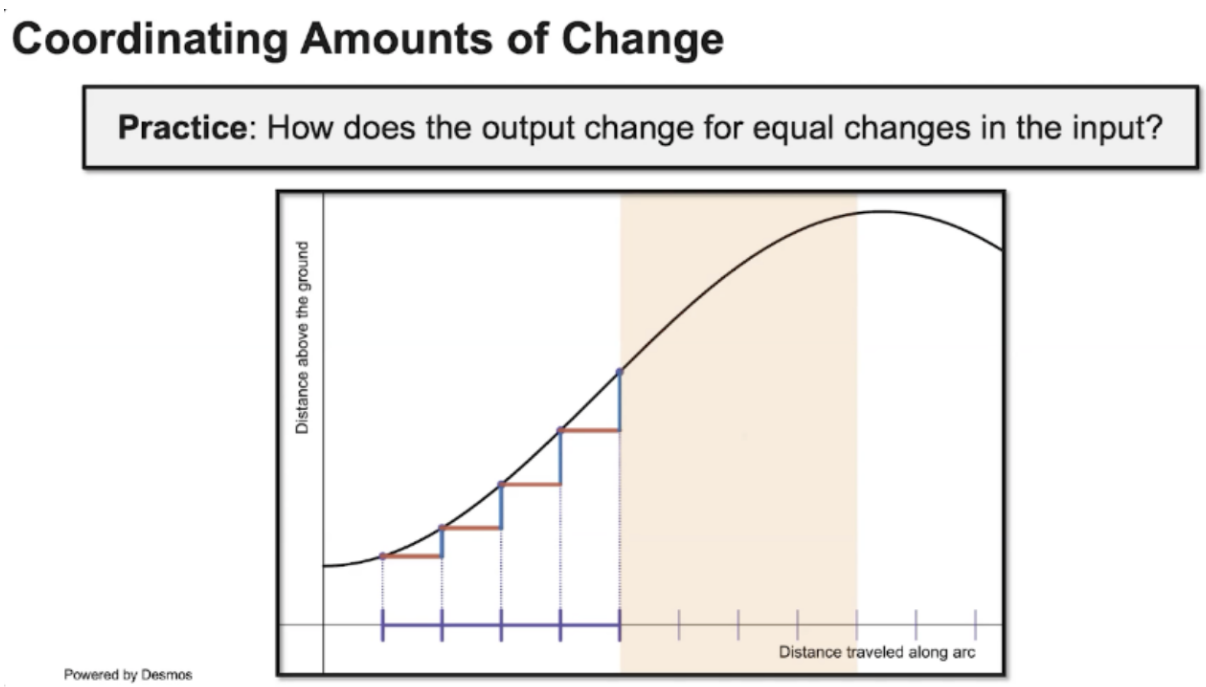
If, for equal changes in the __________, the corresponding change in the __________ is increasing, then the graph is _________________________.

So, how does the graph describe aspects of the Ferris Wheel phenomena? Complete the diagram in red and blue as you watch the video.

As the rider travels equal distances along the __________, the distance above the ground increases by _________________________.



Let's Practice:



If, for equal changes in the ____________, the corresponding change in the _____________ is decreasing, then the graph is _________________________.

What should we take away?

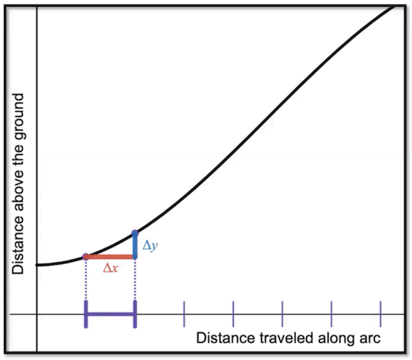
- _______________________________ describes how two quantities change together.
- A graph is ______________________ if, for equal changes in the _____________, the corresponding change in ____________ is ______________________________
- A graph is ______________________ if, for equal changes in the _____________, the corresponding change in the ______________ is _________________________.

Topic 1.2 Rates of Change (Daily Video 1)

AP Precalculus

In this video, we will define average rate of change and explore how to use average rate of change to solve problems.

Let's Review!



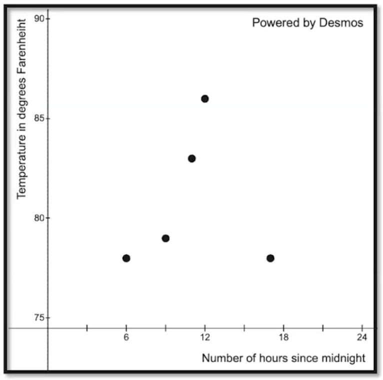
The **Rate of change** describes how the independent and dependent variables change together.

We can visualize how variables change together on the graph by looking at the corresponding change in x (Δx) and the change in y (Δy) on the graph.

Example: In What Way Does the Temperature Change?

The table below gives the temperature in Baltimore, MD, on July 5, 2022. The independent variable is the number of hours since midnight and the dependent variable is the temperature in degrees Fahrenheit.

x	$f(x)$
6	78
9	79
11	83
12	86
17	78



In what way does the temperature change between 6 a.m. and 9 a.m.?

What is the change in time? $\Delta x =$ ______

What is the change in temperature?
$\Delta f(x) =$ ______

Draw the slope triangle on the graph.

Average Rate of Change over the interval $[x_1, x_2]$	Average Rate of Change over the interval $[6, 9]$
The ratio of the change in output values to the change in input values over the specified interval of the domain. $$\frac{\Delta f(x)}{\Delta x} = \frac{\qquad\qquad}{x_2 - x_1}$$	$$\frac{\Delta f(x)}{\Delta x} = \frac{f(9) - f(6)}{9 - 6} = \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_$$ **Interpretation:** If the temperature changed by the same amount each hour between 6 a.m. and 9 a.m., the temperature would have increased by ________ degrees Fahrenheit per hour.

Practice Computing Average Rate of Change

The table below gives the temperature in Baltimore, MD, on July 5, 2022. The independent variable is the number of hours since midnight and the dependent variable is the temperature in degrees Fahrenheit.

x	$f(x)$
6	78
9	79
11	83
12	86
17	78

Compute the average rate of change for the following intervals of the domain, then interpret that average rate of change. Show your work.

A. [9,11] $\dfrac{\Delta f(x)}{\Delta x} =$

If the temperature changed by the same amount each hour between _____ a.m. and ______ a.m., the temperature would have increased by _________ degrees Fahrenheit per hour.

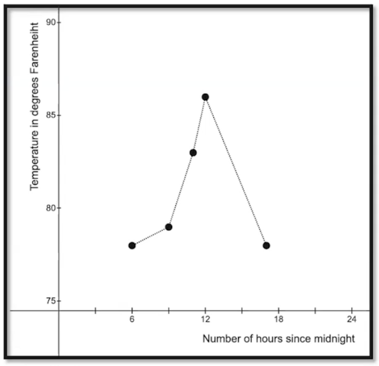
B. [11,12] $\dfrac{\Delta f(x)}{\Delta x} =$

If the temperature changed by the same amount each hour between _____ a.m. and ______ a.m., the temperature would have increased by _________ degrees Fahrenheit per hour.

C. [12,17] $\dfrac{\Delta f(x)}{\Delta x} =$

If the temperature changed by the same amount each hour between _____ a.m. and ______ a.m., the temperature would have decreased by _________ degrees Fahrenheit per hour.

Assumptions of Average Rate of Change



Average rate of change assumes constant rate of change— the same rate of change over the entire interval of the domain.

What should we take away?

- Average rate of change is the ratio of the change in _________ _________ to the change in input values over the specified interval of the domain.
- Average rate of change describes how two quantities would have changed together if the output consistently changed by the same amount over a specified interval of the domain.

Topic 1.2 Rates of Change (Daily Video 2)

AP Precalculus

In this video, we will attempt to improve our estimate of a function's rate of change by working with average rate of change over various intervals.

Example!

In 2008, Usain Bolt set a world-record time running the 100-meter sprint; he ran 100 meters in 9.69 seconds. What was Bolt's average speed over the entire race?

$$\frac{\Delta\text{distance}}{\Delta\text{time}} = \frac{100\text{ m}}{9.69\text{ s}} = \_\_\_\_\_\_\_\_\_\_\_\_$$

Average speed is a type of average rate of change.

Interpreting Average Speed

Did Bolt run 10.32 meters every second?

$$\frac{\Delta\text{distance}}{\Delta\text{time}} = \frac{50-0}{5.50-0} = \_\_\_\_\_\_\_\_$$

$$\frac{\Delta\text{distance}}{\Delta\text{time}} = \frac{100-50}{9.69-5.50} = \_\_\_\_\_\_\_\_$$

Was 10.32 meters per second the fastest that Bolt ran?

The table below gives the time recorder every 50 meters during Bolt's 2008 race.

Time (seconds)	0	5.50	9.69
Distance (meters)	0	50	100
Average Speed (m/s)			

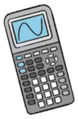
Is a speed of 10.32 meters per second a good approximation for Bolt's speed 4 seconds into the race?

The table below gives the time recorded every 10 meters during Bolt's 2008 race.

Time (seconds)	0	1.85	2.87	3.78	4.65	5.50	6.32	7.14	7.96	8.79	9.69
Distance (meters)	0	10	20	30	40	50	60	70	80	90	100
Average Speed (m/sec)											

Fill in the third row of the table by calculating the average speed over each of these 10 m intervals.

Workspace:



Was Bolt running the fastest as he crossed the finish line? ________________

What was Bolt's fastest speed? ______________

What should we take away?

- Computing average speed over a long period of time gives an estimate of bolt's actual speed. Looking at the average speed over shorter intervals of time gives a _________ approximation of Bolt's actual speed throughout the race.
- In general, we can better describe ______ ____ ________ by determining the average rate of change over smaller and smaller intervals of the domain.

Topic 1.3 Rates of Change in Linear and Quadratic Functions (Daily Video 1)

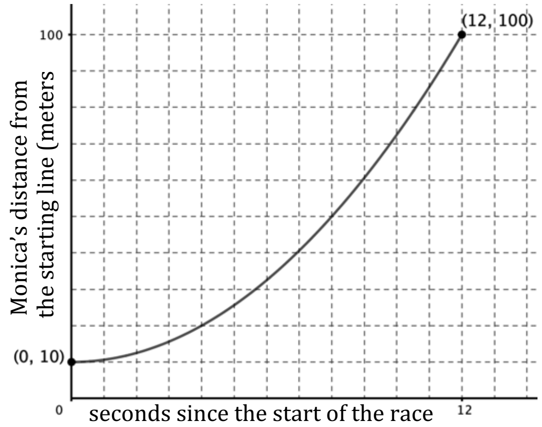
AP Precalculus

In this video, we will review the idea of average rate of change and explore what an average rate of change value conveys about how two quantities' values are related.

Let's WARM UP!

Monica is running a 100-meter race. Since she is younger than the other runners, the race official gave her a 10-meter head start. We are given a graph that represents Monica's distance from the start "in terms of" the number of seconds since the race began.

The points on the graph represent the corresponding distance-time pairs as Monica is running the race



The point (12,100) indicates that Monica is _________meters from the _______________________, ___________seconds after the race began.

Write an interpretation of what the *y*-intercept indicates in the context of Monica's 100-meter race?

____

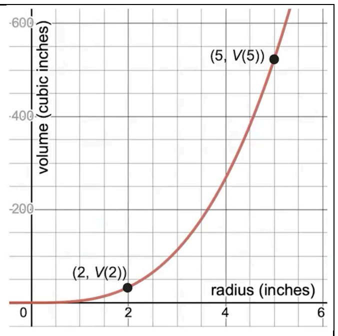
____

Calculate Monica's average rate of change. Show all your work. Draw and label a rate of change triangle on the graph above.	What constant speed is needed by Monica to run 90 meters in 12 seconds? Draw a graph on the grid above of Monica's distance from the start in terms of the number of seconds since the race began, if Monica ran at this constant speed.

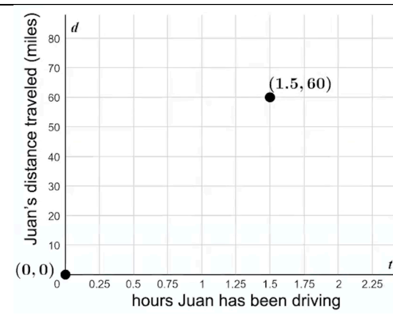
Example: Representing Average Rate of Change Using Function Notation

We are given that the volume of air in a spherical balloon varies with the balloon's radius, r, according to the formula $V(r) = \frac{4}{3}\pi r^3$.

Use **function notation** to represent the average rate of change of the balloon's volume, $V(r)$, in terms of its radius, r, as the balloon's radius increases from 2 to 5 inches. Include units in your answer. Show all work.



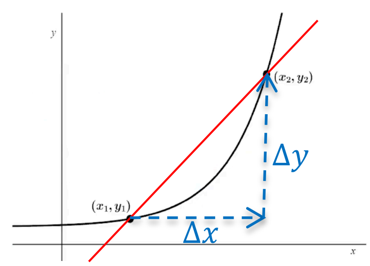
Example: Using Constant Rate of Change to Estimate Future Values

Juan is traveling on a curvy road to attend his friend's wedding. After driving for 90 minutes (3/2 hours) on the curvy road, Juan's odometer indicated he had traveled 60 miles. What was Juan's **average speed** over that 60-mile stretch of the road. Include units in your answer. Show all work. along the line $\Delta y = y_2 - y_1$,	Do you think Juan drove at a constant speed on this curvy road? If he could drive at a constant speed, what is the value of this constant speed, in miles per hour, so he went 60 miles in 90 minutes?
As Juan is driving, he notices that his friend's wedding begins in 15 minutes (1/4 hours). According to his navigation system, he has 7 more miles (of driving on the curvy road) to the wedding destination. Using the average rate of change 40 mph you already computed, will Juan make it to the wedding on time? Justify your answer.	

What should we take away?

The **average rate of change** of a function over some interval of its domain is the __________ **rate of change**, m , that produces the same ______ ___________ in the function's output quantity on the specified interval of the function's domain, as what was achieved by the function.

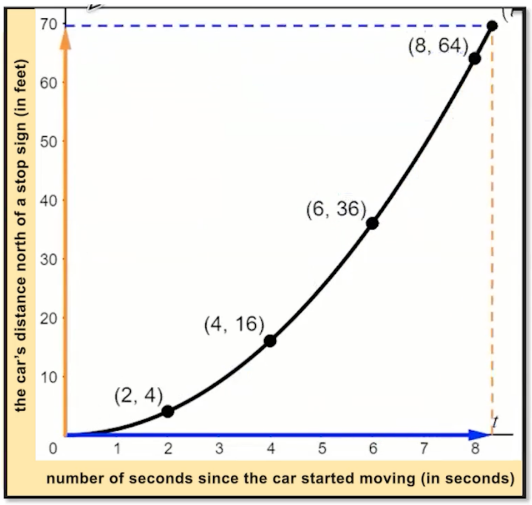
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



Topic 1.3 Rates of Change in Linear and Quadratic Functions (Daily Video 2)

AP Precalculus

In this video, we will explore how the average rate of change of the average rate of change varies over intervals of a function's domain.



Let's WARM UP!

Let's consider a graph that represents a car's distance (in feet) north of a stop sign in relation to the number of seconds since the car began to move.

Use the graph to fill in the blanks.

The car's distance north of the stop sign $\underbrace{\text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}}_{\text{increases/decreases}}$ as the time after the car started moving increases from t = ______ to t = ______ seconds.

Complete the table to the right.

Is the car speeding up, slowing down, or moving at a constant rate of change? (Circle one)

How do you know? Use the values in the table to explain your answer below.

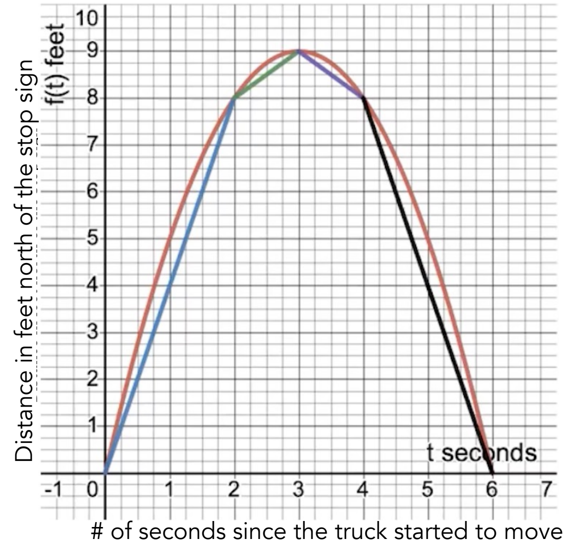
Δt	t	$d(t)$	$\Delta d(t)$	$\frac{\Delta d(t)}{\Delta t}$ ft/sec
	0	0		
	2	4		
	4	16		
	6	36		
	8	64		

What kind of pattern are the average rates of change following over $\underbrace{\text{successive}}_{\text{in order}}$ equally sized intervals of 2? Explain your reasoning below.

The pattern is ___________________________ because ____

Example: A truck's distance north of a stop sign

The **concave down** graph to the right represents a truck's distance north of a stop sign in relation to the number of seconds since it started moving.



Use the information in the graph to fill in the blanks below.

The truck stops heading north away from the stop sign, turns around, and heads south toward the stop sign at time $t =$ ____ seconds because the graph stops $\underbrace{\qquad\qquad\qquad}_{\text{increasing/decreasing}}$ and starts $\underbrace{\qquad\qquad\qquad}_{\text{increasing/decreasing}}$.

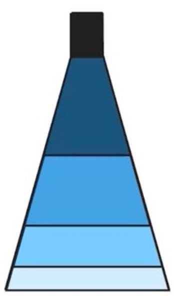
Fill in the values in the table to the right and use the values in the table to fill in the following blanks.

Δt	t	$d(t)$	$\Delta d(t)$	$\frac{\Delta d(t)}{\Delta t}$
	0	0		
2				
	2	8		
1				
	3	9		
1				
	4	8		
2				
	6	0		

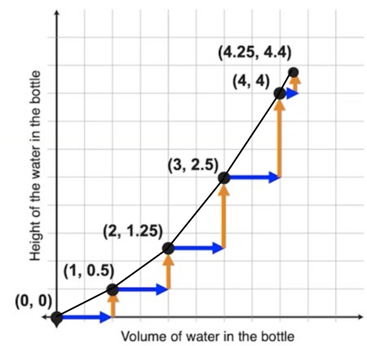
The truck's average rate of change in $\underbrace{\qquad\qquad}_{\text{units}}$ is $\underbrace{\qquad\qquad\qquad}_{\text{increasing/decreasing}}$ on the interval $0 < t < 6$ seconds because the graph of the truck's distance north of the stop sign is $\underbrace{\qquad\qquad\qquad}_{\text{concave up/concave down}}$.

Example: Water flowing in a bottle

The height of the water's average rate of change in inches per cup is $\underbrace{\qquad\qquad\qquad}_{\text{increasing/decreasing}}$ on the interval because the graph of the height of the water is $\underbrace{\qquad\qquad\qquad}_{\text{concave up/concave down}}$.



Volume of Water, v (cups)	Height of Water, h (inches)
0	0
1	0.5
2	1.25
3	2.5
4	4
4.25	4.4



What should we take away?

If a function's graph is **concave up** on an interval of the function's domain, the function's average rate of change ______________ on successive fixed intervals of this interval of the function's domain.

If a function's graph is ______________ on an interval of the function's domain, the function's average rate of change **decreases** on successive fixed intervals of this interval of the function's domain.