# Topic 3.8 The Tangent Function (Daily Video 1)

AP Precalculus

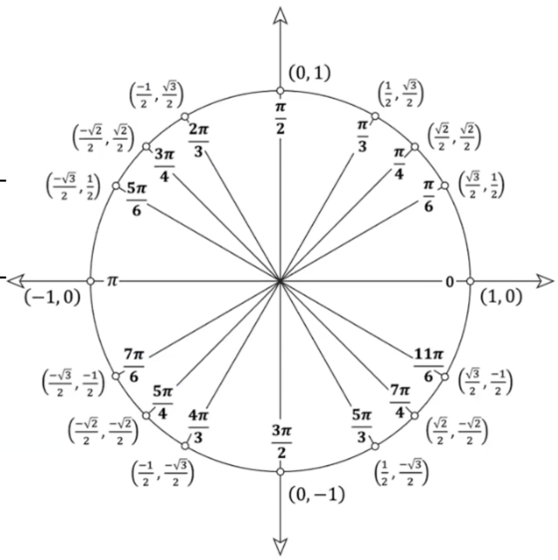
**In this video, we will introduce a third trigonometric function with a strong relationship to sine, cosine and the slope created by the terminal ray of an angle.**

## Let's Review!

Recall **sine** and **cosine** as defined with triangles and the unit circle.

If $r = 1$, then $\sin\theta =$ ____________

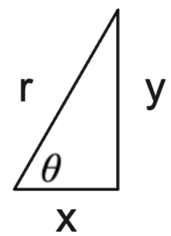
If $r = 1$, then $\cos\theta =$ ____________




Definition of tangent

$f(\theta) = \tan\theta = \frac{\quad}{\quad}$

$\tan\theta = \frac{y/r}{x/r} = \frac{\quad}{\quad}$




We can think of tangent in three significant ways:

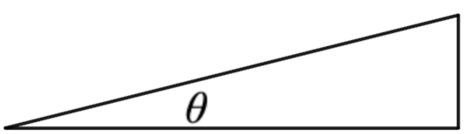
- As the __________ of the ray created by $\theta$
- As the ratio of the $y$ and $x$ coordinates from the _______ _______
- As the ratio of the _______ and __________ of $\theta$

## The Period of the Tangent Function

The tangent function repeats values every half-circle. It has a period of ____.

## Let's look at an EXAMPLE!

A plane is climbing and rises ½ mile for every 2 miles it travels parallel to the ground. What is the tangent of the angle of observation for an observer located on the ground? Label the diagram and show the calculation of the tangent ratio.




## What should we take away?

- $f(x) = tan\ x$ can be evaluated in several ways.
  - The tangent of the angle a ray makes with the horizontal is the ________ of the ray.
  - The tangent of an angle is the ratio of the ___ and ___ coordinates at that angle on the unit circle.
  - $f(x) = \tan x$ can be rewritten or evaluated using $f(x) = \frac{\quad}{\quad}$.
- The period of the tangent function is a ______________, or ____ units.

# Topic 3.8 The Tangent Function (Daily Video 2)

AP Precalculus

In this video, we will explore graphs of the tangent function with transformations and restrictions on their domains.

## Let's WARM UP!

Evaluate

$\tan 0$ $\qquad$ $\tan\frac{2\pi}{3}$

$\tan\frac{\pi}{6}$ $\qquad$ $\tan\frac{3\pi}{4}$

$\tan\frac{\pi}{4}$ $\qquad$ $\tan\frac{5\pi}{6}$

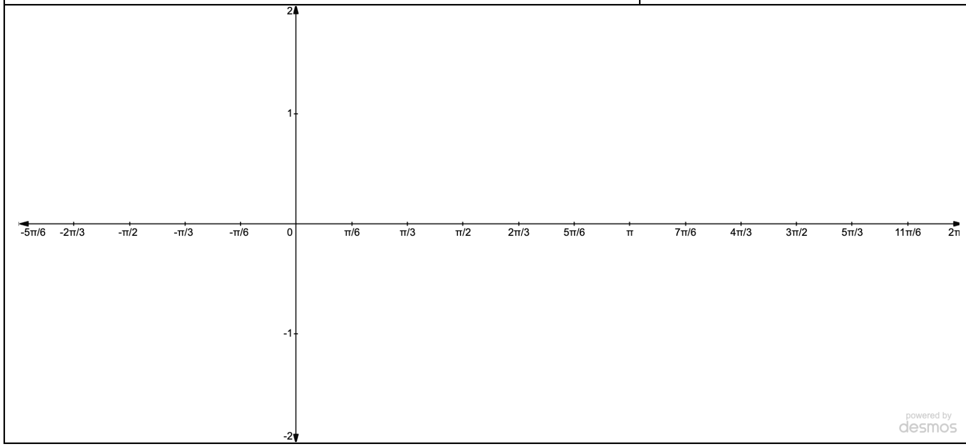
$\tan\frac{\pi}{3}$ $\qquad$ $\tan\pi$

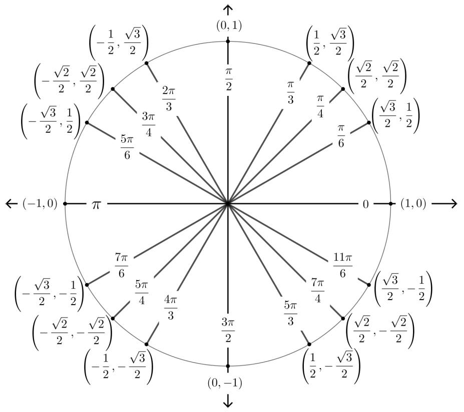
$\tan\frac{\pi}{2}$

Plot the ordered pairs $(\theta, \tan\theta)$ on the axes below. Recall that he period of $f(x) = \tan x$ is $\pi$ units.

Describe what is happening to the slope of the ray as $x$ approaches $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

How is this related to evaluations of $f(x) = \tan x$?




## Transformations

$$y = a\tan(b(x - h)) + k$$

- $h$ shifts the graph _________ and $k$ shifts the graph ______________. The new "center" of the graph is at the point ________.
- The graph will have its inflection at $y =$ _____ and vertical asymptotes will be at $h \pm$ **half of** the period length. *Note: video error: The length of one period is distance between two vertical asymptotes. "…vertical asymptotes will be at $h \pm$ **half of** the period length."
- $a$ is the _________ stretch or shrink.
- $b$ affects the ___________ of the function.

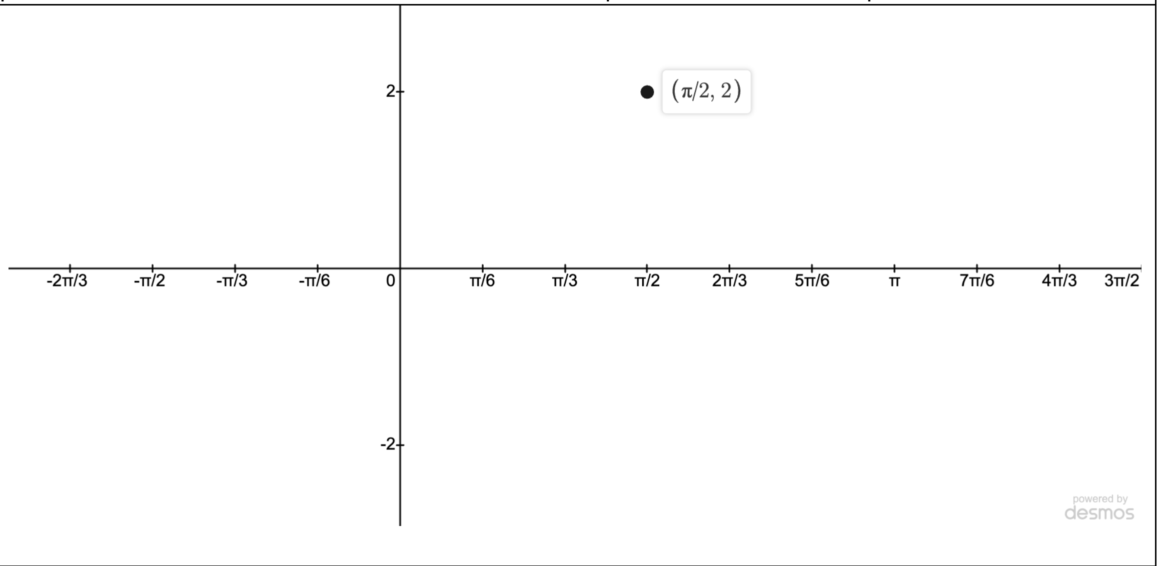
**Let's look at an EXAMPLE!**

Transformations: Shifts/Translations

What is the horizontal shift and vertical shift of $y = \tan\left(x - \frac{\pi}{2}\right) + 2$? Sketch the graph below, including the asymptotes. Note: The new "center" is already plotted on the graph.

Horizontal:

Vertical:



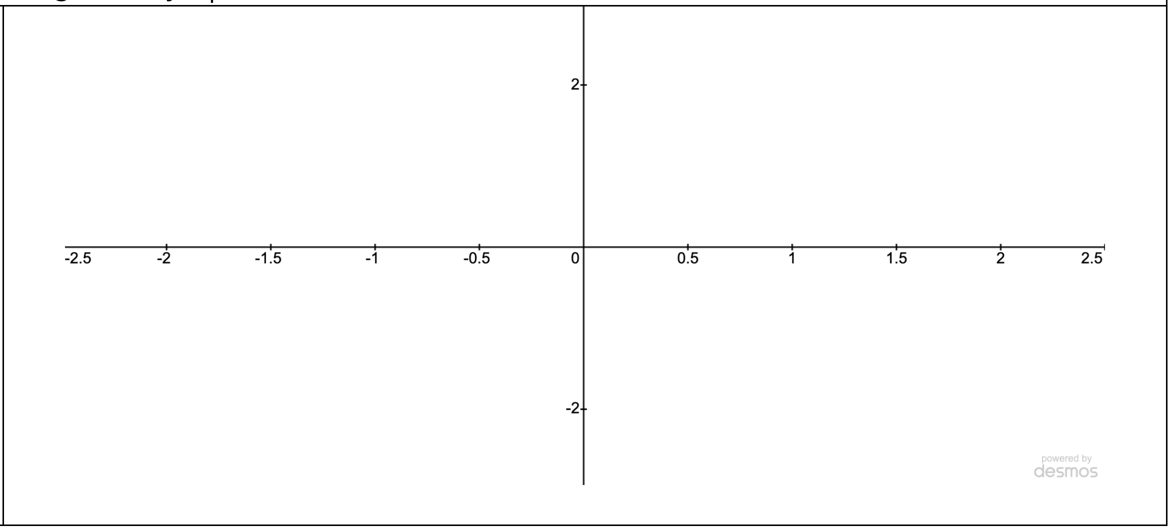

**Let's PRACTICE!**

Horizontal and Vertical Stretches

How does the graph of $y = \frac{1}{2}\tan(\pi x)$ compare to the graph of the parent function? Sketch the graph below, including the asymptotes.

$a = \frac{1}{2}$

$b = \pi$




**What should we take away?**

The old rules for transformations still apply.

- $h$ is the ________________ shift
- $k$ is the ____________ shift
- $a$ affects the ___________ heights of function values
- $b$ affects the period (new period = ——)

The new "center" is at $(h, k)$. Asymptotes are a ______________ to the left and right of $(h, k)$.

# Topic 3.9 Inverse Trigonometric Functions (Daily Video 1)

AP Precalculus

**In this video, we will use the concept of inverse functions to investigate the inverses of the sine, cosine, and tangent functions.**

### Let's REVIEW!

Suppose you are given $f(3) = 4, f(4) = 7,$ and $f(7) = 10$.

Predict: $f^{-1}(7) =$_______ Think: If the output value of $f$ is 7, what is the input value?

$f^{-1}(4) =$_______ Think: If the output value of $f$ is 4, what is the input value?

### Let's look at an EXAMPLE!

$\sin^{-1}(x) = \arcsin x$ is the inverse of $\sin(x)$

$\cos^{-1}(x) = \arccos x$ is the inverse of $\cos(x)$

$\tan^{-1}(x) = \arctan x$ is the inverse of $\tan(x)$

The inverse trigonometric function will find a specific __________ related to a given __________.

$\tan^{-1}(1) =$ ________ because $\tan$ _____ $= 1$.

### Let's PRACTICE!

$\cos^{-1}\left(\frac{1}{2}\right)$

$\arctan\left(\frac{1}{\sqrt{3}}\right)$

$\arcsin\left(\frac{\sqrt{2}}{2}\right)$

$\sin^{-1}(0.3)$

### What should we take away?

Trignometric functions find __________ that go with angles.

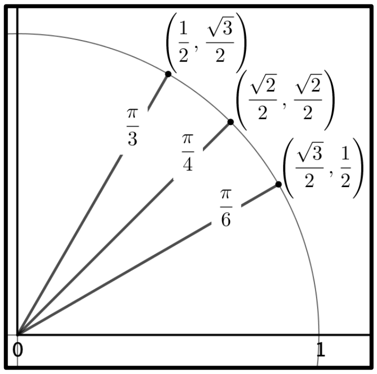
Inverse trignometric functions find _________ that go with ratios.

We really need to know our unit circle for special angles.

$\sin(\theta) = y - \text{coordinate}$

$\cos(\theta) = x - \text{coordinate}$

$\tan(\theta) = \frac{y}{x}$



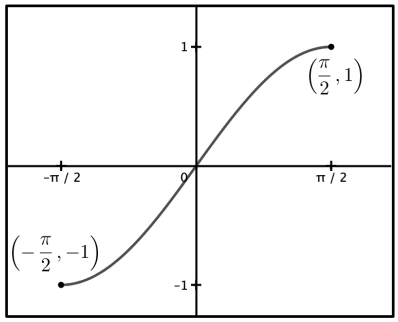
# Topic 3.9 Inverse Trigonometric Functions (Daily Video 2)

AP Precalculus

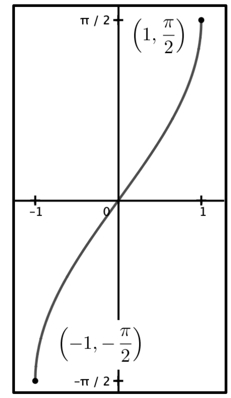
**In this video, we will further explore inverse trigonometric functions and consider restrictions on their domains.**

**Let's REVIEW!**

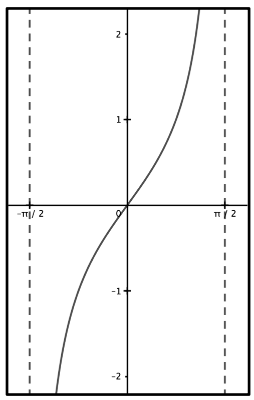
If we restrict $f(x) = \sin x$ to $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ , we get this graph:



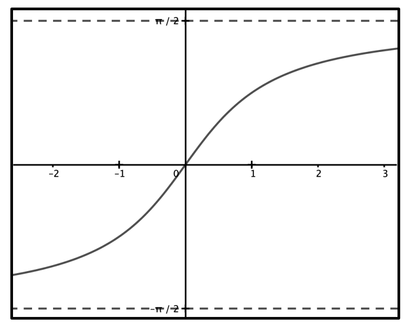

Then $y = \arcsin x$ is a function and looks like this:



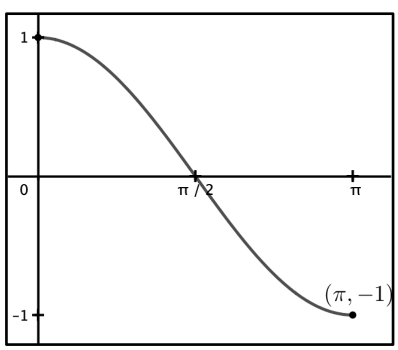

If we restrict $f(x) = \tan x$ to $-\frac{\pi}{2} < x < \frac{\pi}{2}$ , we get this graph:



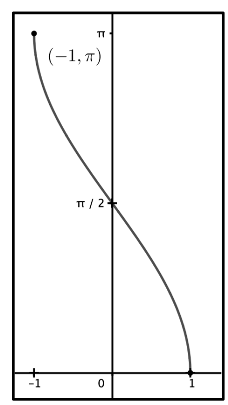

Then $y = \arctan x$ is a function and looks like this:




If we restrict $f(x) = \cos x$ to $0 \leq x \leq \pi$ , we get this graph:




Then $y = \arccos x$ is a function and looks like this:



Let's PRACTICE!

1. $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

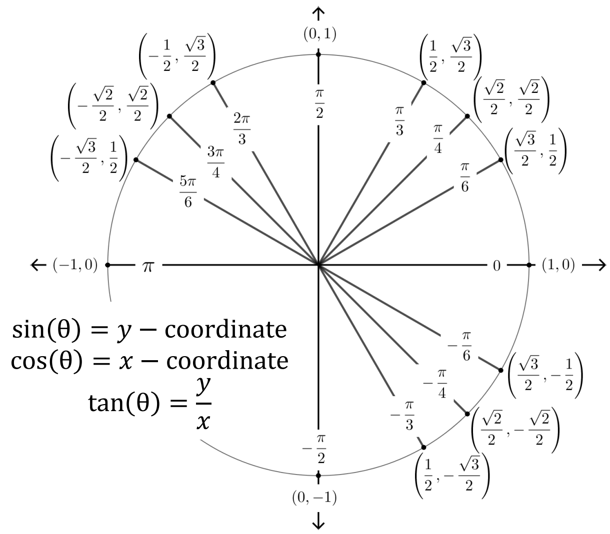
4. $\arccos\left(-\frac{1}{2}\right)$

2. $\tan^{-1}(-1)$

5. $\sin^{-1}(1)$

3. $\cos^{-1}(-1)$

6. $\arctan\left(-\frac{1}{\sqrt{3}}\right)$




## What should we take away?

- For inverses to be functions, the original function must be _________________________.
- To make trigonometric functions one-to-one, the domain needs to be restricted:
  $\sin(x)$ is restricted to _______________

  $\tan(x)$ is restricted to _______________

  $\cos(x)$ is restricted to _______________

- We can use our knowledge of common ratios and the unit circle to determine the value of inverse trigonometric functions.

CALC MEDIC

# Topic 3.10 Trigonometric Equations and Inequalities (Daily Video 1)

AP Precalculus

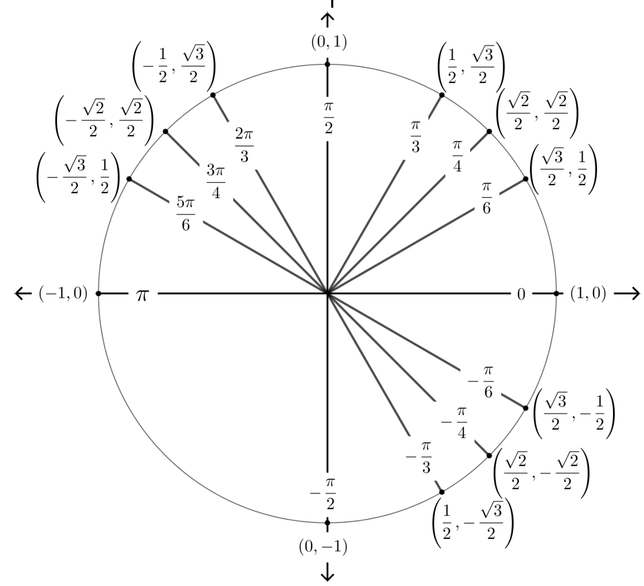
In this video, we will solve trigonometric equations and inequalities using inverse functions.

**Let's REVIEW!**

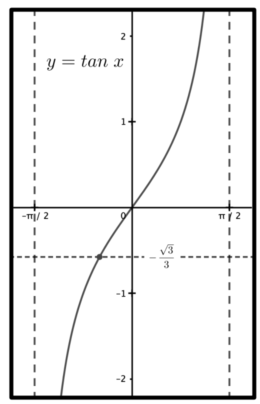
What is $\sin^{-1}\left(\frac{1}{2}\right)$ ?

What is $\cos\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$ ?

Unit Circle from Topic 3.9 video 2




**Let's look at an EXAMPLE!**

| | |
|---|---|
| Find all solutions for $\theta$ using an inverse. Note: Presenter only found solutions on the interval $0 < \theta < \pi$.<br>$2\sin(2\theta) = \sqrt{3}$<br>$\sin(2\theta) =$ ____________<br>$\sin^{-1}(\sin(2\theta)) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$<br>________ $= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$<br>$2\theta =$        $2\theta =$<br>$\theta =$        $\theta =$<br> | Find the solution for $\theta$ where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.<br>$\tan(3\theta) > -\frac{\sqrt{3}}{3}$<br>One period of $\tan x$ is shown to the right.<br><br>$3\theta > \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$<br>$\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) =$ _____<br>$-\frac{\pi}{6} < 3\theta < \frac{\pi}{6}$<br>_______ $< \theta <$ _______ or $\left(\text{—},\text{—}\right)$<br>One period of $\tan(3\theta)$ is ______ so all solutions would include<br>$-\frac{\pi}{3}$     $+\frac{\pi}{3}$<br>$\left(\text{—},\text{—}\right) \cup \left(-\frac{\pi}{18},\frac{\pi}{6}\right) \cup \left(\text{—},\text{—}\right)$<br>Rewrite the intervals using decimals.<br>(     ,     ) ∪ (     ,     ) ∪ (     ,     ) |

**Let's PRACTICE!** Let $3\cos(4\theta) = \frac{3}{2}$. Solve for $\theta$ where $0 \le \theta \le 2\pi$.

A) $\theta = \frac{\pi}{24}$ B) $\theta = \frac{\pi}{12}$ C) $\theta = -\frac{\pi}{24}$ D) $\theta = -\frac{\pi}{12}$

**What should we take away?**

Applying an inverse can be used to solve trigonometric equations and inequalities.

# Topic 3.10 Trigonometric Equations and Inequalities (Daily Video 2)

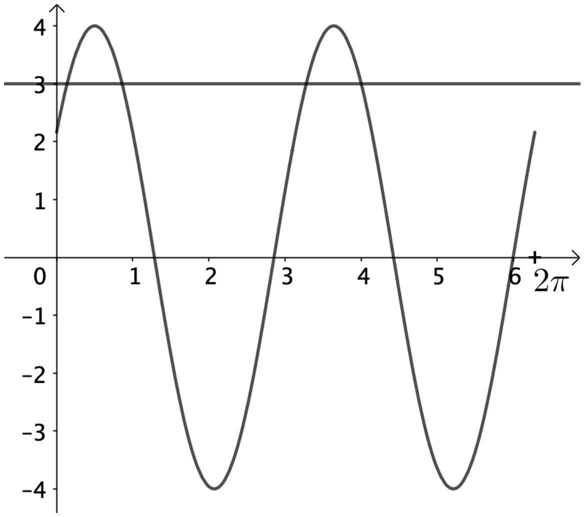
**AP Precalculus**

**In this video, we will solve trigonometric equations and inequalities using inverse functions, with and without technology, to answer a question.**

**Let's look at an EXAMPLE!**

Reminders: Make sure your calculator is in RADIAN mode and be sure to write your solutions to three decimal places.

Where is $4\cos(2x-1)=3$? Restrict your answers to $0 \le x < 2\pi$.




Where is $4\cos(2x-1)>3$? Restrict your answers to $0 \le x < 2\pi$.

Let $2=\sin^2 x-\sin x$. Solve for $x$, where $0 \le x < 2\pi$, without using a calculator. Show how you arrived at your answer.

**What should we take away?**

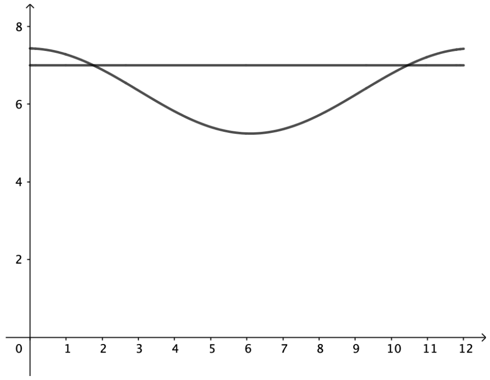
- Trigonometric equations and inequalities can be solved analytically or graphically.
- Solutions to trignometric inequalites must be written in interval notation.

# Topic 3.10 Trigonometric Equations and Inequalities (Daily Video 3)

## AP Precalculus

**In this video, we will continue solving trigonometric equations and inequalities and explore models in context with reasonable solutions.**

### Modeling the Sunrise

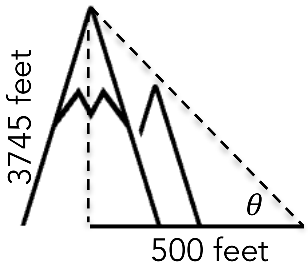



In a certain town in the United States, the time of sunrise can be modeled by the equation

$$y = 1.096\sin(0.510x + 1.600) + 6.339$$

where $x$ represents the month (January = 1, December = 12), and $y$ represents the time of the sunrise. If you woke up everyday at sunrise, during what months would you have to wake up before 7:00 AM? Show how you arrived at your answer.

### Angle of Elevation




The peak of a mountain is 3,745 feet high. If you are standing 500 feet from the mountain's base, what is the measure, in radians, of the angle of elevation of the mountain's peak? Show how you arrived at your answer.

### Let's PRACTICE!

What is $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta$ ?

A) $-\frac{\pi}{3}$ B) $-\frac{2\pi}{3}$ C) $\frac{\pi}{3}$ D) $\frac{2\pi}{3}$

Solve the inequality: $\cos(2x) > -\frac{1}{2}$ for $0 \leq x \leq \pi$.

A) $\left(0, \frac{\pi}{3}\right)$ B) $\left[0, \frac{\pi}{3}\right)$

C) $\left(\frac{2\pi}{3}, \pi\right]$ D) $\left[0, \frac{\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \pi\right]$

Let $y = 2x\cos(0.3x - 9) + 5$ represent the height of a kite bobbing in the wind, where $x$ represents the number of minutes the kite has been in the air and $y$ is how high the kite is, in feet. After how many minutes will the kite first reach 20 feet?

A) $-18.409$ min B) $7.941$ min C) $25.682$ min D) $34.318$ min

### What should we take away?

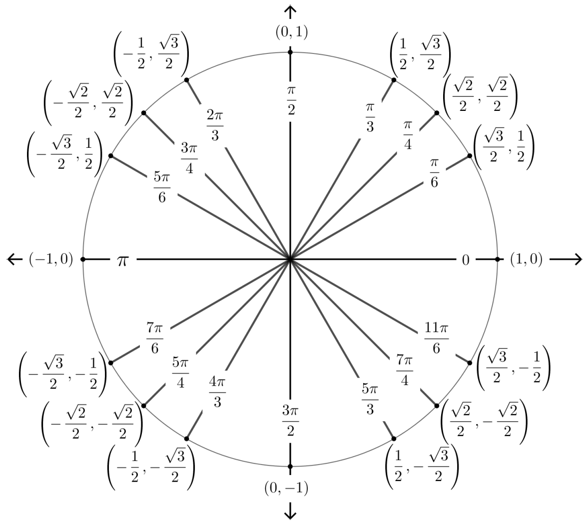
Trigonometric equations and inequalities can be solved analytically or graphically.

# Topic 3.11 The Secant, Cosecant, and Cotangent Functions (Daily Video 1)

AP Precalculus

In this video, we will introduce the reciprocal functions of the sine, cosine, and tangent functions.

### Let's look at an EXAMPLE!




The cosecant function:

$$f(\theta) = \csc\theta = \frac{1}{\sin\theta} = \frac{r}{y}$$

$$\csc\frac{\pi}{6} =$$

$$\csc\frac{3\pi}{4} =$$

The secant function:

$$f(\theta) = \sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$$

$$\sec\frac{\pi}{6} =$$

$$\sec\frac{3\pi}{4} =$$

The cotangent function:

$$f(\theta) = \cot\theta = \frac{1}{\tan\theta} = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$$

$$\cot\frac{\pi}{6} =$$

$$\cot\frac{3\pi}{4} =$$

### Let's PRACTICE!

| | |
|---|---|
| $\csc\frac{\pi}{3} =$ | $\cot\frac{11\pi}{6} =$ |
| $\sec\frac{3\pi}{4} =$ | $\sec 1.2 =$ |

### What should we take away?

- Cosecant, secant and cotangent are reciprocals of sine, cosine and tangent, respectively.

$f(\theta) = \csc\theta = \frac{1}{\_\_\_\_} = \frac{r}{\_\_\_\_}$ $\quad f(\theta) = \sec\theta = \frac{1}{\_\_\_\_} = \frac{r}{\_\_\_\_}$ $\quad f(\theta) = \cot\theta = \frac{1}{\_\_\_\_} = \_\_\_\_ = \frac{\theta}{\theta}$

# Topic 3.11 The Secant, Cosecant, and Cotangent Functions (Daily Video 2)

AP Precalculus

**In this video, we will further explore the secant, cosecant, and cotangent functions by analyzing their graphs, domains, and ranges.**

### Let's WARM UP!

Solve for $-\pi < x \leq 2\pi$

$\sin x = 0$ $\qquad\qquad$ $\cos x = 0$ $\qquad\qquad$ $\tan x = 0$

### Let's REVIEW!

Vertical Asymptotes usually occur when the denominator of a function that is a ratio is zero. Where can we predict asymptotes for secant, cosecant, and cotangent on the interval $[-\pi, 2\pi]$?

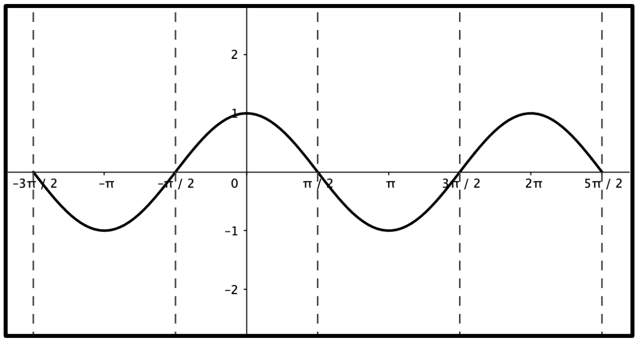
$f(x) = \sec x = \dfrac{1}{\_\_\_\_\_\_}$ $\qquad$ vertical asymptotes at $x =$

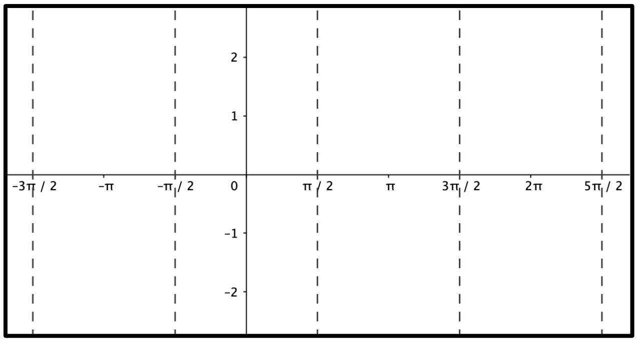
$f(x) = \csc x = \dfrac{1}{\_\_\_\_\_\_}$ $\qquad$ vertical asymptotes at $x =$

$f(x) = \cot x = \dfrac{1}{\_\_\_\_\_\_} = \dfrac{\_\_\_\_\_\_}{\_\_\_\_\_\_}$ $\qquad$ vertical asymptotes at $x =$

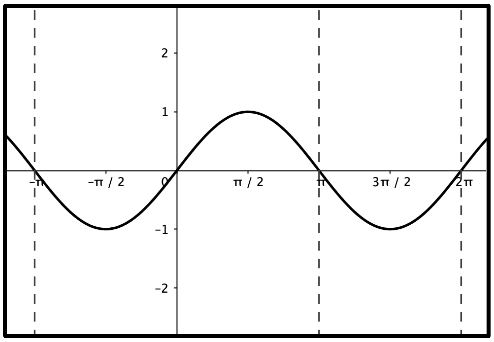
### Let's look at an EXAMPLE!

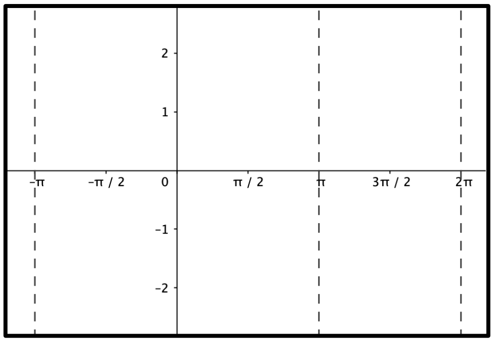
Sketch a graph of $f(x) = \sec x$ from the graph of $\cos x$.




Sketch a graph of $f(x) = \csc x$ from the graph of $\sin x$.

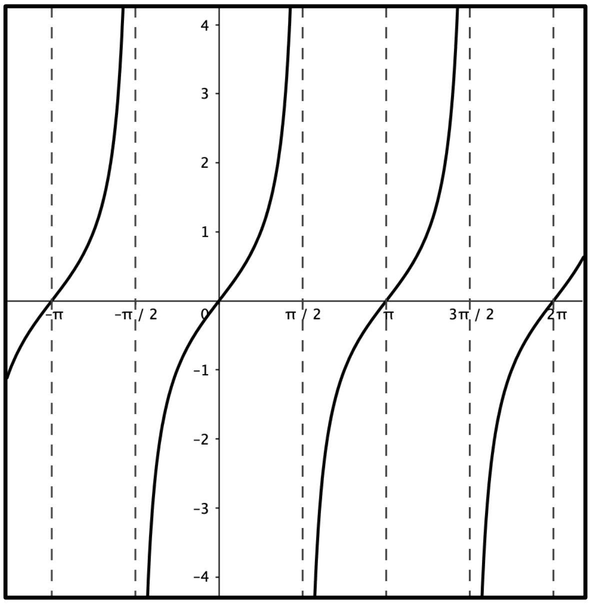


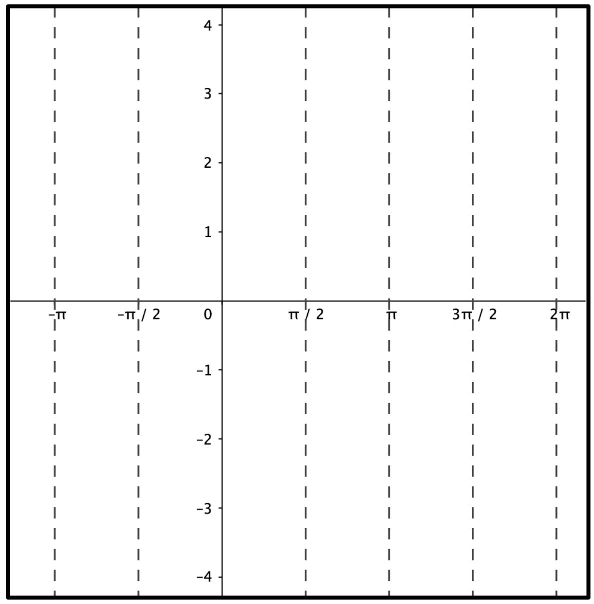




Sine and cosine have ranges of _________

Secant and cosecant have ranges of ____________________

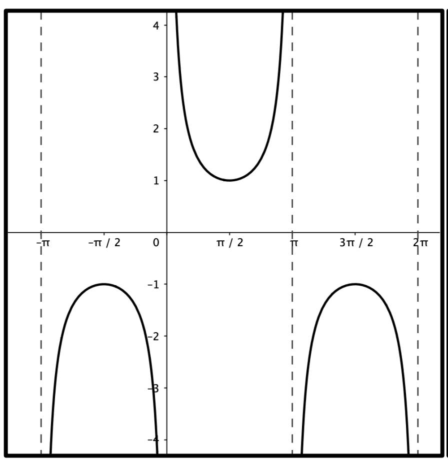
Sketch a graph of $f(x) = \cot x$ from the graph of $\tan x$.



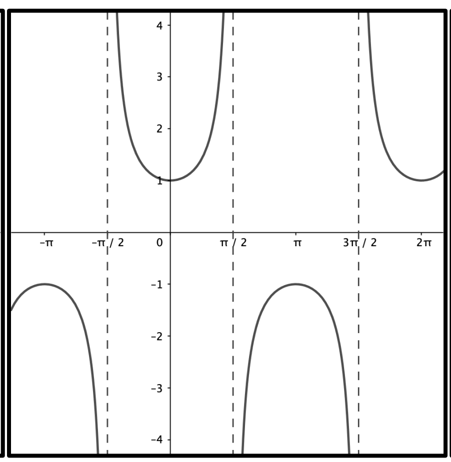




**What should we take away?**

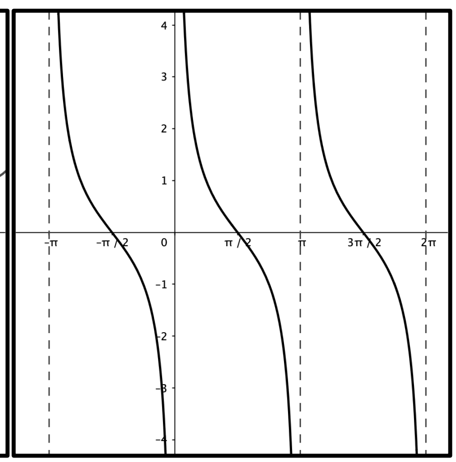
secant



cosecant



cotangent

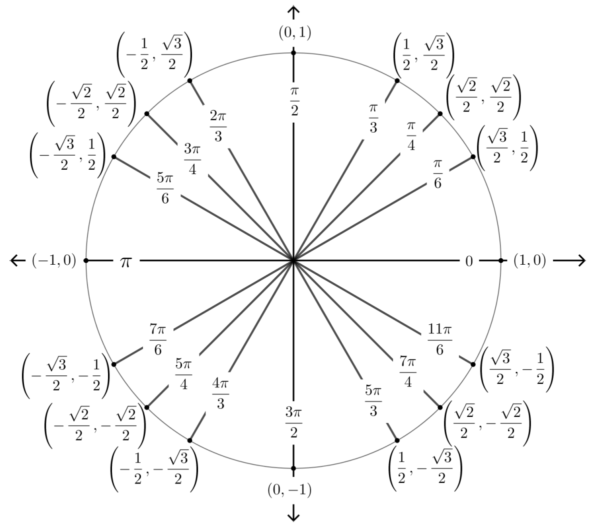


- We can use the zeros of cosine, sine, and tangent to predict the ______________ asymptotes of secant, cosecant, and cotangent.
- The tangent graph is always increasing between consecutive asymptotes so the cotangent is always _______________ between consecutuve asymptotes.

# Topic 3.12 Equivalent Representations of Trigonometric Functions (Daily Video 1)

AP Precalculus

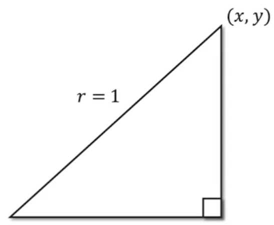
**In this video, we will explore the Pythagorean identities and use them to write equivalent trigonometric expressions.**




### Let's REVIEW!

On the unit circle $(x, y) = ($_______, _______$)$

The Pythagorean theorem can be applied to a right triangle at a point on the unit circle.




$x^2 + y^2 =$ _______

Or in terms of sine and cosine we get the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$.

### Let's look at an EXAMPLE!

| Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to write the identity for $\tan^2\theta$. | Simplify<br>$\sin\theta + \cos\theta \cdot \cot\theta$ | Simplify<br>$\cot^2 x - \cot^2 x \cdot \cos^2 x$ |
|---|---|---|

### Let's PRACTICE!

| Simplify $\dfrac{1}{\tan^2\theta + 1}$<br><br>A) $\cot^2\theta$<br>B) $\sin^2\theta$<br>C) $\cos^2\theta$<br>D) $\csc^2\theta$ | Simplify $\cos\theta(\sec\theta - \cos\theta)$<br><br>A) $\cot^2\theta$<br>B) $\sin^2\theta$<br>C) $\cos^2\theta$<br>D) $\csc^2\theta$ |
|---|---|

### What should we take away?

There are two Pytagorean identities $\sin^2\theta +$ _______ $= 1$ and $\tan^2\theta =$ _______ $- 1$ that can used to simplify trigonometric expressions.

# Topic 3.12 Equivalent Representations of Trigonometric Functions (Daily Video 2)

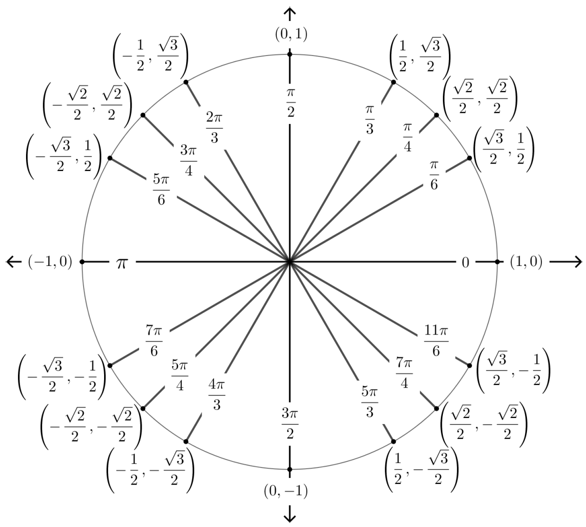
AP Precalculus

In this video, we will explore sum, difference, and double-angle identities of sine and cosine.

### Let's REVIEW!

$\tan\theta =$ $\qquad\qquad$ $\csc\theta =$

$\sin^2\theta + \cos^2\theta =$ $\qquad\qquad$ $\sec\theta =$




Verify $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ using $\alpha = \frac{\pi}{6}$ and $\beta = \frac{\pi}{3}$.

Verify $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ using $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{2}$.

### Let's Look at an EXAMPLE! / Let's PRACTICE!

| Sine Double-Angle Identities | Cosine Double-Angle Identities | Verify |
|---|---|---|
| $\sin(2\alpha) = \sin(\alpha+\alpha)$ | $\cos(2\alpha) = \cos(\alpha+\alpha)$ | $\csc^2\alpha = \dfrac{\csc\alpha}{\sin\alpha}$ |

### What should we take away?

$\sin(\alpha+\beta) =$ ____________________ $\qquad$ $\cos(\alpha+\beta) =$ ____________________

$\sin(\alpha-\beta) =$ ____________________ $\qquad$ $\cos(\alpha-\beta) =$ ____________________

$\sin(2\alpha) =$ ____________________ $\qquad$ $\cos(2\alpha) =$ ____________________

*Note: Difference identities were not verified in the video. Hint: Write the difference as the sum of the opposite, $(\alpha-\beta) = (\alpha+(-\beta))$. Then use the fact that sine is an odd function, which means $\sin(-\beta) = -\sin\beta$, and cosine is an even function, which means $\cos(-\beta) = \cos\beta$.

# Topic 3.12 Equivalent Representations of Trigonometric Functions (Daily Video 3)

## AP Precalculus

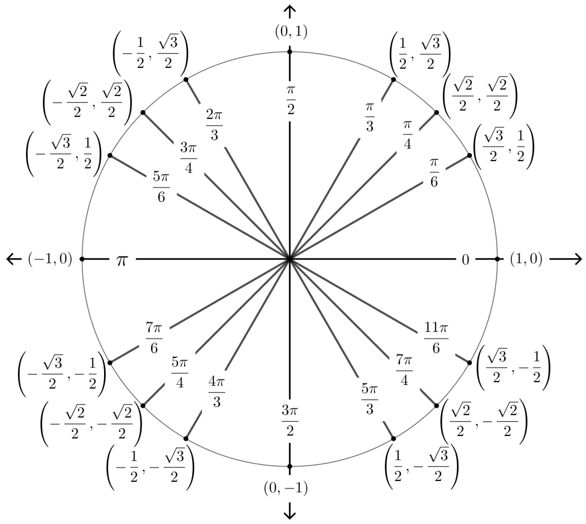
**In this video, we will use trigonometric identities to solve trigonometric equations and inequalities.**

### Let's look an EXAMPLE!

( the interval on the video is written incorrectly. The interval is corrected below)

Solve for $\theta$ when $0 \leq \theta < 2\pi$. Show your work.

$\sin(2\theta) - \sin\theta = 0$




Solve for $\theta$ when $0 \leq \theta < 2\pi$. Show your work.

$\sin(2\theta) - \sin\theta > 0$

* The note on the video has a typo. It should read: 0 is not included in the solutions since $\sin(2 \cdot 0) - \sin(0) = 0$.

### Let's PRACTICE!

Using a sum identity, find all values of $\theta$ on $[0,2\pi)$ where $\sin\left(\theta + \frac{\pi}{3}\right) = 0$.

### What should we take away?

We can solve trigonometric equations and inequalities using appropriate trigonometric identities.