

Limits, Continuity, Differentiability

Evaluate $\lim_{x \rightarrow a} f(x)$



Limits, Continuity, Differentiability

$\lim_{x \rightarrow \infty} f(x)$ means



Limits, Continuity, Differentiability

$\lim_{x \rightarrow a} f(x)$ exists if



Limits, Continuity, Differentiability

$f(x)$ is continuous
at $x = a$ if



Limits, Continuity, Differentiability

$f(x)$ is differentiable
at $x = a$ if



Limits, Continuity, Differentiability

L'hôspital's Rule



Derivatives

Limit definition of $f'(a)$
(Derivative at the point
 $x = a$)



Derivatives

Limit definition of $f'(x)$
(Equation of derivative
function)



Derivatives

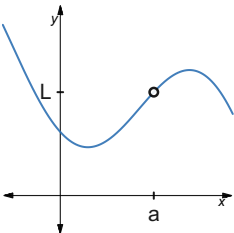
$\frac{d}{dx}(c)$



Derivatives

$\frac{d}{dx}(x^n)$



<p>Find end behavior. Use TFEPLC and rational function rules comparing the degree of the numerator and denominator.</p>	<p>Simplify. Let $x = a$. (Direct substitution)</p>
$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$ <p>and</p> $\lim_{x \rightarrow a} f(x) = f(a)$	$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$ 
<p>If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ is indeterminate $\left(\frac{0}{0} \text{ or } \frac{\infty}{\infty}\right)$ Then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$</p>	<p>$f(x)$ is continuous at $x = a$ and $\lim_{x \rightarrow a^-} f'(x) = \lim_{x \rightarrow a^+} f'(x)$</p>
$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$	$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \quad \text{or}$ $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$
nx^{n-1}	0

Derivatives

$$\frac{d}{dx}(c(f(x)))$$



Derivatives

Product Rule

$$\frac{d}{dx}(f(x) \cdot g(x))$$



Derivatives

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$$



Derivatives

Chain Rule - Derivative of a composite function

$$\frac{d}{dx}(f(g(x)))$$



Derivatives

$$\frac{d}{dx}(\sin x)$$



Derivatives

$$\frac{d}{dx}(\cos x)$$



Derivatives

$$\frac{d}{dx}(\tan x)$$



Derivatives

$$\frac{d}{dx}(\cot x)$$



Derivatives

$$\frac{d}{dx}(\sec x)$$



Derivatives

$$\frac{d}{dx}(\csc x)$$



$$f(x)g'(x) + g(x)f'(x)$$

$$c \cdot f'(x)$$

$$g'(x) \cdot f'(g(x))$$

$$\frac{g(x) \cdot f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$-\sin x$$

$$\cos x$$

$$-\csc^2 x$$

$$\sec^2 x$$

$$-\csc x \cot x$$

$$\sec x \tan x$$

Derivatives

$$\frac{d}{dx}(\ln x)$$



Derivatives

$$\frac{d}{dx}(e^x)$$



Derivatives

$$\frac{d}{dx}(a^x)$$



Derivatives

$$\frac{d}{dx}(\sin^{-1} x)$$



Derivatives

$$\frac{d}{dx}(\tan^{-1} x)$$



Derivatives

Very rare!

$$\frac{d}{dx}(\sec^{-1} x)$$



Derivatives

Derivatives of inverse
cofunctions
($\arccos x$, $\operatorname{arccot} x$, $\operatorname{arccsc} x$)



Derivatives

Derivatives of inverse
functions
Let $g(x) = f^{-1}(x)$
What is $g'(x)$?



Derivatives

$$\frac{d}{dx}(|x|)$$













Derivatives

Implicit
differentiation



e^x	$\frac{1}{x}$
$\frac{1}{\sqrt{1-x^2}}$	$a^x \cdot \ln a$
$\frac{1}{ x \sqrt{x^2-1}}$	$\frac{1}{1+x^2}$
$g'(x) = \frac{1}{f'(g(x))}$ <p>Slopes at inverse points are reciprocals.</p>	Opposite of the derivative of the corresponding function
Differentiate both sides with respect to x . Treat y as a function of x and apply chain rule.	Find the slope using the graph. There is no shortcut rule for this function!

<p>Other</p> <p>Average rate of change of f on $[a, b]$</p> 	<p>Other</p> <p>How to find critical points</p> 
<p>Other</p> <p>How to find the absolute maxima and minima of f</p> 	<p>Other</p> <p>How to find inflection points</p> 
<p>Other</p> <p>How to find vertical asymptotes</p> 	<p>Other</p> <p>How to find horizontal asymptotes</p> 
<p>Other</p> <p>How to solve related rates problems</p> 	<p>Other</p> 
<p>Integration</p> <p>$\int dx$</p> 	<p>Integration</p> <p>$\int a \, dx$</p> 

<p>A critical point is any interior point where $f'(x) = 0$ or $f'(x)$ is undefined.</p>	<p>Find the slope of the secant line.</p> $\frac{f(b) - f(a)}{b - a}$
<p>Inflection points will exist where f'' changes signs.</p>	<p>Candidate's test: Compare function outputs at endpoints and points where f' changes sign. The y-value is the max/min!</p>
<p>Use end behavior. Find $\lim_{x \rightarrow -\infty} f(x)$ and $\lim_{x \rightarrow \infty} f(x)$</p>	<p>Set the denominator equal to 0 and solve for x.</p>
	<p>Write an equation that models the situation. Implicitly differentiate all variables that are changing with respect to time.</p>
$ax + C$	$x + C$

Integration

$$\int x^n dx$$



Integration

$$\int x^{-1} dx \text{ OR } \int \frac{1}{x} dx \text{ OR } \int \frac{dx}{x}$$



Integration

$$\int \cos x dx$$



Integration

$$\int \sin x dx$$



Integration

$$\int \sec^2 x dx$$



Integration

$$\int \csc^2 x dx$$



Integration

$$\int \sec x \tan x dx$$



Integration

$$\int \csc x \cot x dx$$



Integration

$$\int e^x dx$$













Integration

$$\int a^x dx$$












$\ln x + C$	$\frac{x^{n+1}}{n+1} + C$
$-\cos x + C$	$\sin x + C$
$-\cot x + C$	$\tan x + C$
$-\csc x + C$	$\sec x + C$
$\frac{1}{\ln a} \cdot a^x + C$	$e^x + C$

<p>Integration</p> $\int \frac{1}{\sqrt{1-x^2}} dx$ 	<p>Integration</p> $\int \frac{1}{1+x^2} dx$ 
<p>Integration <i>Very rare!</i></p> $\int \frac{1}{ x \sqrt{x^2-1}} dx$ 	<p>Integration</p> $\int_a^b \sqrt{r^2-x^2} dx$ 
<p>Interpretations and Justifications</p> <p>Intermediate Value Theorem (IVT)</p> 	<p>Interpretations and Justifications</p> <p>Mean Value Theorem</p> 
<p>Interpretations and Justifications</p> <p>A particle is speeding up if</p> 	<p>Interpretations and Justifications</p> <p>A particle is slowing down if</p> 
<p>Interpretations and Justifications</p> <p>Interpreting a derivative at $x = a$ in context</p> 	<p>Interpretations and Justifications</p> <p>Interpreting an integral over $[a, b]$ in context</p> 











$\tan^{-1} x + C$	$\sin^{-1} x + C$
<p>Use geometry!</p> <p>This is a semicircle of radius r!</p>	$\sec^{-1} x + C$
<p>If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) then</p> $\frac{f(b) - f(a)}{b - a} = f'(c)$ <p>For some value c in (a, b).</p> <p>Slope of the secant line = slope of the tangent line. Average rate of change = instantaneous rate of change.</p>	<p>If $f(x)$ is continuous on $[a, b]$ then $f(x)$ takes on every value between $f(a)$ and $f(b)$.</p>
<p>$v(t)$ and $a(t)$ have opposite signs</p>	<p>$v(t)$ and $a(t)$ have the same sign</p>
<p>From $x = \underline{\hspace{1cm}}$ to $x = \underline{\hspace{1cm}}$ the net change in $\underline{\hspace{1cm}}$ is $\underline{\hspace{1cm}}$ (units).</p> <ol style="list-style-type: none"> Interval Context Net change or accumulation Units 	<p>At $x = \underline{\hspace{1cm}}$, the $\underline{\hspace{1cm}}$ is changing at a rate of $\underline{\hspace{1cm}}$ (units).</p> <ol style="list-style-type: none"> Instant Context Rate of change Units

<div>Interpretations and Justifications</div> <div>f is increasing when</div> <div>+</div>	<div>Interpretations and Justifications</div> <div>f is decreasing when</div> <div>+</div>
<div>Interpretations and Justifications</div> <div>f is concave up when</div> <div>+</div>	<div>Interpretations and Justifications</div> <div>f is concave down when</div> <div>+</div>
<div>Interpretations and Justifications</div> <div>f has a relative max when</div> <div>+</div>	<div>Interpretations and Justifications</div> <div>f has a relative min when</div> <div>+</div>
<div>Interpretations and Justifications</div> <div>When a question asks you to “interpret the meaning”</div> <div>+</div>	<div>Interpretations and Justifications</div> <div>When a question asks you to “justify”</div> <div>+</div>
<div>Interpretations and Justifications</div> <div>+</div>	<div>Interpretations and Justifications</div> <div>+</div>

$f' < 0$	$f' > 0$
$f'' < 0$	$f'' > 0$
$f'(x) = 0$ or undefined and f' changes from - to + OR $f'(x) = 0$ and $f''(x) > 0$	$f'(x) = 0$ or undefined and f' changes from + to - OR $f'(x) = 0$ and $f''(x) < 0$
The response must include a definition or theorem. Be sure to verify the conditions of the theorem!	The response must include: 1. Meaning in context 2. Units 3. Time

<p>Area / Volume</p> <p>Area by Riemann sums (rectangles)</p> 	<p>Area / Volume</p> <p>Left Riemann sums are an overestimate if / underestimate if</p> <p>Right Riemann sums are an overestimate if / underestimate if</p> 
<p>Area / Volume</p> <p>Area by trapezoids</p> 	<p>Area / Volume</p> <p>Area using trapezoids will be an overestimate if... underestimate if...</p> 
<p>Area / Volume</p> <p>Total area =</p> 	<p>Area / Volume</p> <p>How to find the area between curves</p> 
<p>Area / Volume</p> <p>Volume: Disks</p> 	<p>Area / Volume</p> <p>Volume: Washers</p> 
<p>Area / Volume</p> <p>Volume: Cross- sections</p> 	<p>Area / Volume</p> 

<table><tr><td>Riemann Sum</td><td>Over estimate when f is</td><td>Under estimate when f is</td></tr><tr><td>Left</td><td>decreasing</td><td>increasing</td></tr><tr><td>Right</td><td>increasing</td><td>decreasing</td></tr></table>	Riemann Sum	Over estimate when f is	Under estimate when f is	Left	decreasing	increasing	Right	increasing	decreasing	<div>$\sum_{k=1}^n f(x_k) \cdot \Delta x$<div><div>↑</div><div>↑</div><div>height width</div></div><p>The y-value/height could be at the left, right, or midpoint of each interval.</p></div>
Riemann Sum	Over estimate when f is	Under estimate when f is								
Left	decreasing	increasing								
Right	increasing	decreasing								
<div>f is concave up f is concave down</div>	$A = \frac{1}{2} \Delta x (y_0 + y_1) + \frac{1}{2} \Delta x (y_1 + y_2) + \dots$									
<div>$\int_a^b (\text{higher} - \text{lower}) \, dx = \int_a^b [f(x) - g(x)] \, dx$<p>OR</p>$\int_c^d (\text{right} - \text{left}) \, dy = \int_c^d [f(y) - g(y)] \, dy$</div>	$\int_a^b f(x) \, dx$									
<div>$\int_a^b \pi(R^2 - r^2) \, dx$<p>OR</p>$\int_c^d \pi(R^2 - r^2) \, dy$<p>$R$ = larger measurement from axis of revolution</p></div>	<div>$\int_a^b \pi(\text{radius})^2 \, dx$<p>OR</p>$\int_c^d \pi(\text{radius})^2 \, dy$</div>									
	<div>$\int_a^b (\text{area of one cross section}) \, dx$<p>OR</p>$\int_c^d (\text{area of one cross section}) \, dy$</div>									

<p>Other</p> $\int_a^b f'(x) dx$ 	<p>Other</p> <p>Final amount / Final position</p> $f(b) =$ 
<p>Other</p> <p>Total distance traveled</p> 	<p>Other</p> <p>Position Velocity Acceleration</p> 
<p>Other</p> <p>Speed</p> 	<p>Other</p> <p>If $F(x) = \int_a^{v(x)} f(t) dt$ then $F'(x) =$</p> 
<p>Other</p> <p>Average value of a function f on $[a, b]$</p> 	<p>Other</p> <p>Solving a differential equation</p> 
<p>Other</p> <p>Solve $\frac{dy}{dx} = ky$</p> 	<p>Other</p> <p>Displacement</p> 

$f(a) + \int_a^b f'(x) dx$ <p>Starting amount + accumulation</p>	$f(x)]_a^b = f(b) - f(a)$ <p>This gives the net change in the amount or position.</p>
$f(t), x(t), \text{ or } y(t)$ $f'(t), x'(t), \text{ or } y'(t)$ $f''(t), x''(t), \text{ or } y''(t)$	$\int_a^b v(t) dt$
$f(v(x)) \cdot v'(x)$ <p>Plug in the upper limit of integration. Multiply by the derivative of the upper limit.</p>	$ v(t) $
<p>Separate Integrate Solve for C Isolate y Select the proper solution</p>	$\frac{1}{b-a} \int_a^b f(x) dx$
$\int_{t_1}^{t_2} v(t) dt$	<p>Exponential growth!</p> $y = y_0 e^{kt}$