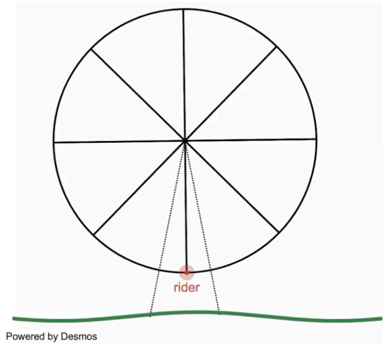
Topic 1.1 Change in Tandem (Daily Video 1)

AP Precalculus

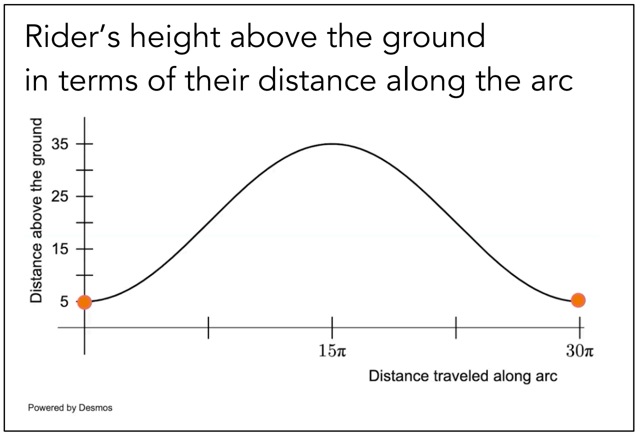
In this video, we will explore how a graph allows us to track how the values of two quantities change together.



Goal: Track one rider as they complete one full trip around the Ferris wheel with a diameter of 30 feet (Circumference 30π)

1. What quantities are we tracking? Sketch them on the diagram.

2. How are these two quantities changing?



Interpretation of the *y*-intercept (0,5): The rider's height above the ground is ______feet when they get on the Ferris wheel and have traveled _____feet along the arc.

Minimum height: What is the rider's minimum height above the ground? How many times is that height reached?
Express your answers to the above question as ordered pairs.

Maximum height: Locate a point on the graph where the rider is the maximum height above the ground. What are the coordinates of this point? _______ Write an interpretation of the coordinates of this point in the context of a Ferris wheel ride?

How is the height of the rider above the ground changing before reaching the maximum height? Circle one.

increasing   decreasing

How is the height of the rider above the ground changing after reaching the maximum height? Circle one.

increasing   decreasing

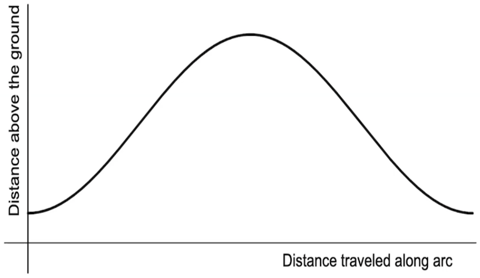
What should we take away?

- Graphs track how ____.
- **When we describe a graph, we should talk about:**
  - ____-intercept(s) and/or ____-intercept(s)
  - Intervals over which the function is _______________ or _______________
  - _______________ and/or _________________ points

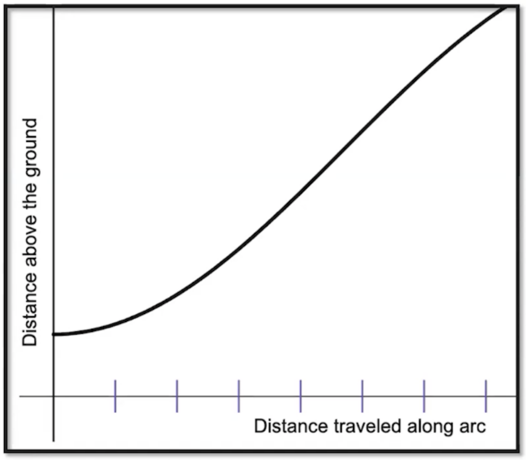
Topic 1.1 Change in Tandem (Daily Video 2)
AP Precalculus

In this video, we will investigate how two quantities change together, how to calculate that rate of change, and how to visualize this through the concavity of the graph.

Example 1: So, let's explore why the graph from Topic 1.1 was curved and what that curvature tells us about HOW these quantities change together.



Coordinating Amounts of Change: Let's systematically explore how the output changes for equal changes in the input.



As you watch the video use red and blue to fill in the picture.

1. Partition the horizontal axis into equal-sized chunks
2. Focus on one of these intervals
3. Identify corresponding points on the graph
4. Determine corresponding change in the output
5. Compare how the output changes for equal changes in the input.

What does the "red" segment represent? ____

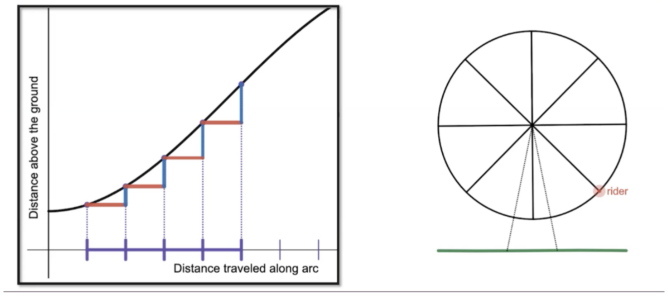
What does the "blue" segment represent? ____

This type of diagram, that shows both the change in the _______________ and the change in the _______________ is sometimes called? ____

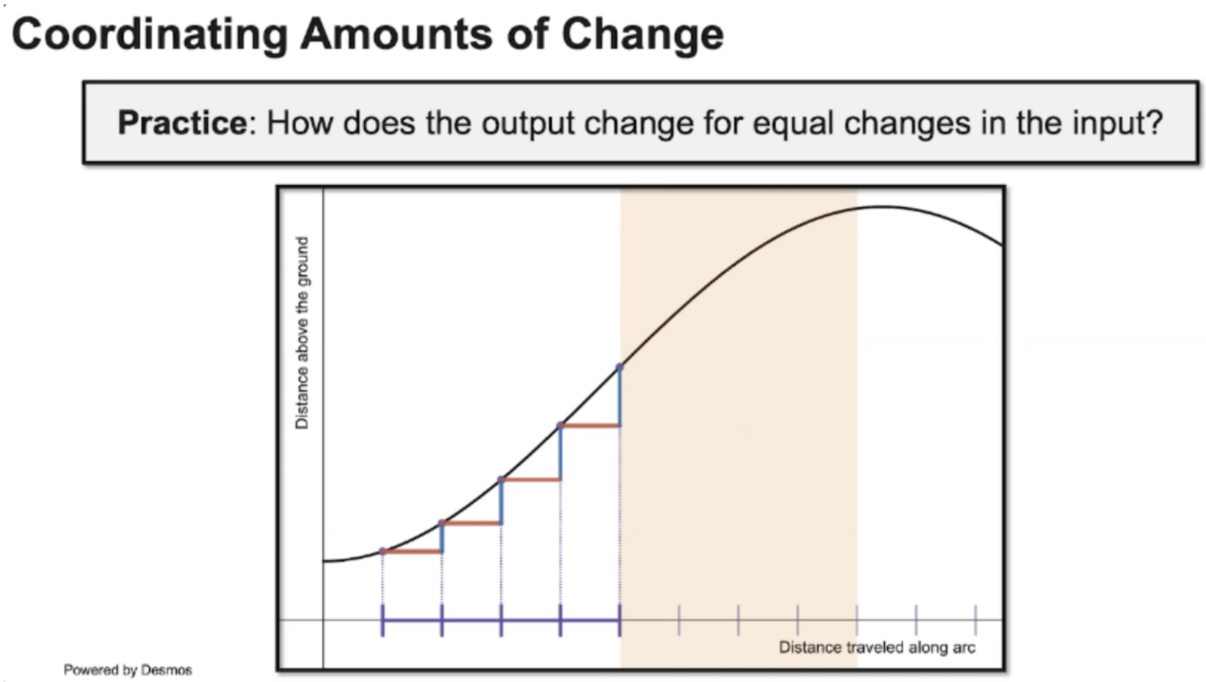
If, for equal changes in the __________, the corresponding change in the __________ is increasing, then the graph is _________________________.

So, how does the graph describe aspects of the Ferris Wheel phenomena? Complete the diagram in red and blue as you watch the video.

As the rider travels equal distances along the __________, the distance above the ground increases by ________________________.



Let's Practice:



If, for equal changes in the ___________, the corresponding change in the ____________ is decreasing, then the graph is ________________________.

What should we take away?

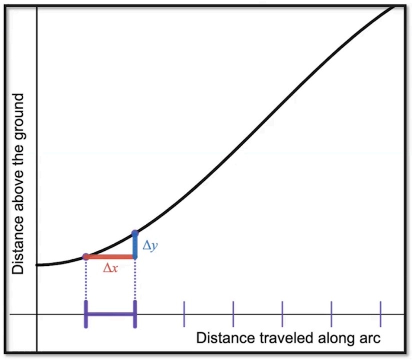
- ______________________________ describes how two quantities change together.
- A graph is _____________________ if, for equal changes in the ____________, the corresponding change in ___________ is _____________________________
- A graph is _____________________ if, for equal changes in the ____________, the corresponding change in the _____________ is ________________________.

Topic 1.2 Rates of Change (Daily Video 1)

AP Precalculus

In this video, we will define average rate of change and explore how to use average rate of change to solve problems.

Let's Review!



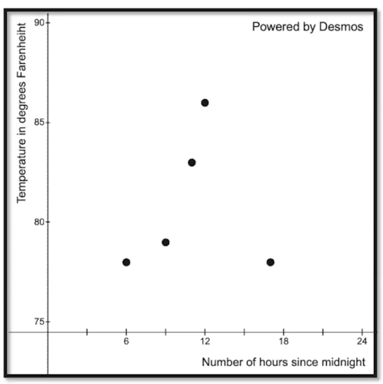
The **Rate of change** describes how the independent and dependent variables change together.

We can visualize how variables change together on the graph by looking at the corresponding change in x (Δx) and the change in y (Δy) on the graph.

Example: In What Way Does the Temperature Change?

The table below gives the temperature in Baltimore, MD, on July 5, 2022. The independent variable is the number of hours since midnight and the dependent variable is the temperature in degrees Fahrenheit.

x	$f(x)$
6	78
9	79
11	83
12	86
17	78



In what way does the temperature change between 6 a.m. and 9 a.m.?

What is the change in time? $\Delta x =$ ______

What is the change in temperature?
$\Delta f(x) =$ ______

Draw the slope triangle on the graph.

Average Rate of Change over the interval $[\boldsymbol{x_1}, \boldsymbol{x_2}]$

The ratio of the change in output values to the change in input values over the specified interval of the domain.

$$\frac{\Delta f(x)}{\Delta x} = \frac{\rule{4cm}{0.4pt}}{x_2 - x_1}$$

Average Rate of Change over the interval $[\boldsymbol{6}, \boldsymbol{9}]$

$$\frac{\Delta f(x)}{\Delta x} = \frac{f(9) - f(6)}{9 - 6} = \rule{2.5cm}{0.4pt} = \rule{1cm}{0.4pt}$$

Interpretation: If the temperature changed by the same amount each hour between 6 a.m. and 9 a.m., the temperature would have increased by _________ degrees Fahrenheit per hour.

Practice Computing Average Rate of Change

The table below gives the temperature in Baltimore, MD, on July 5, 2022. The independent variable is the number of hours since midnight and the dependent variable is the temperature in degrees Fahrenheit.

x	$f(x)$
6	78
9	79
11	83
12	86
17	78

Compute the average rate of change for the following intervals of the domain, then interpret that average rate of change. Show your work.

A. [9,11] $\dfrac{\Delta f(x)}{\Delta x} =$

If the temperature changed by the same amount each hour between _____ a.m. and ______ a.m., the temperature would have increased by _________ degrees Fahrenheit per hour.

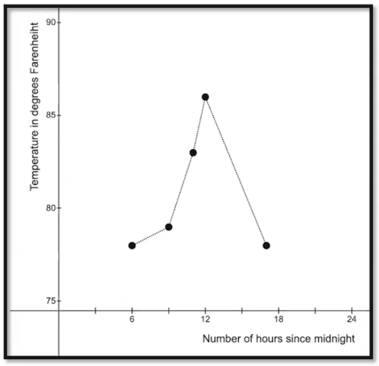
B. [11,12] $\dfrac{\Delta f(x)}{\Delta x} =$

If the temperature changed by the same amount each hour between _____ a.m. and ______ a.m., the temperature would have increased by _________ degrees Fahrenheit per hour.

C. [12,17] $\dfrac{\Delta f(x)}{\Delta x} =$

If the temperature changed by the same amount each hour between _____ a.m. and ______ a.m., the temperature would have decreased by _________ degrees Fahrenheit per hour.

Assumptions of Average Rate of Change



Average rate of change assumes constant rate of change— the same rate of change over the entire interval of the domain.

What should we take away?

- Average rate of change is the ratio of the change in _________ _________ to the change in input values over the specified interval of the domain.
- Average rate of change describes how two quantities would have changed together if the output consistently changed by the same amount over a specified interval of the domain.

Topic 1.2 Rates of Change (Daily Video 2)

AP Precalculus

In this video, we will attempt to improve our estimate of a function's rate of change by working with average rate of change over various intervals.

Example!

In 2008, Usain Bolt set a world-record time running the 100-meter sprint; he ran 100 meters in 9.69 seconds. What was Bolt's average speed over the entire race?

$$\frac{\Delta\text{distance}}{\Delta\text{time}} = \frac{100\text{ m}}{9.69\text{ s}} = \underline{\qquad\qquad\qquad}$$

Average speed is a type of average rate of change.

Interpreting Average Speed

Did Bolt run 10.32 meters every second?

$$\frac{\Delta\text{distance}}{\Delta\text{time}} = \frac{50-0}{5.50-0} = \underline{\qquad\qquad}$$

$$\frac{\Delta\text{distance}}{\Delta\text{time}} = \frac{100-50}{9.69-5.50} = \underline{\qquad\qquad}$$

Was 10.32 meters per second the fastest that Bolt ran?

The table below gives the time recorder every 50 meters during Bolt's 2008 race.

Time (seconds)	0	5.50	9.69
Distance (meters)	0	50	100
Average Speed (m/s)			

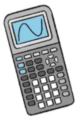
Is a speed of 10.32 meters per second a good approximation for Bolt's speed 4 seconds into the race?

The table below gives the time recorded every 10 meters during Bolt's 2008 race.

Time (seconds)	0	1.85	2.87	3.78	4.65	5.50	6.32	7.14	7.96	8.79	9.69
Distance (meters)	0	10	20	30	40	50	60	70	80	90	100
Average Speed (m/sec)											

Fill in the third row of the table by calculating the average speed over each of these 10 m intervals.

Workspace:



Was Bolt running the fastest as he crossed the finish line? ____________________

What was Bolt's fastest speed? __________________

What should we take away?

- Computing average speed over a long period of time gives an estimate of bolt's actual speed. Looking at the average speed over shorter intervals of time gives a ____________ approximation of Bolt's actual speed throughout the race.
- In general, we can better describe _______ _____ __________ by determining the average rate of change over smaller and smaller intervals of the domain.

Topic 1.3 Rates of Change in Linear and Quadratic Functions (Daily Video 1)

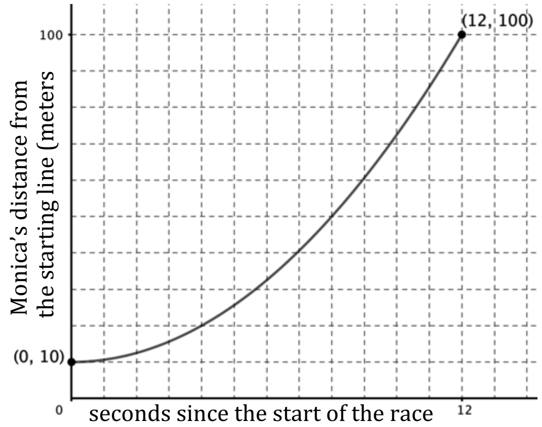
AP Precalculus

In this video, we will review the idea of average rate of change and explore what an average rate of change value conveys about how two quantities' values are related.

Let's WARM UP!

Monica is running a 100-meter race. Since she is younger than the other runners, the race official gave her a 10-meter head start. We are given a graph that represents Monica's distance from the start "in terms of" the number of seconds since the race began.

The points on the graph represent the corresponding distance-time pairs as Monica is running the race



The point (12,100) indicates that Monica is _________meters from the _______________________, ___________seconds after the race began.

Write an interpretation of what the *y*-intercept indicates in the context of Monica's 100-meter race?

____

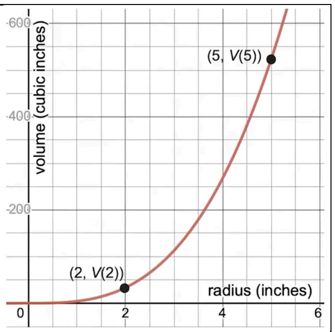
____

Calculate Monica's average rate of change. Show all your work. Draw and label a rate of change triangle on the graph above.	What constant speed is needed by Monica to run 90 meters in 12 seconds? Draw a graph on the grid above of Monica's distance from the start in terms of the number of seconds since the race began, if Monica ran at this constant speed.

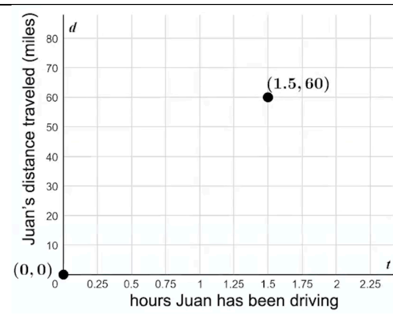
Example: Representing Average Rate of Change Using Function Notation

We are given that the volume of air in a spherical balloon varies with the balloon's radius, r, according to the formula $V(r) = \frac{4}{3}\pi r^3$.

Use **function notation** to represent the average rate of change of the balloon's volume, $V(r)$, in terms of its radius, r, as the balloon's radius increases from 2 to 5 inches. Include units in your answer. Show all work.



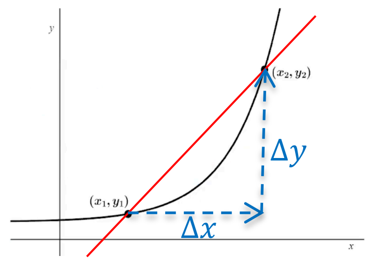
Example: Using Constant Rate of Change to Estimate Future Values

Juan is traveling on a curvy road to attend his friend's wedding. After driving for 90 minutes (3/2 hours) on the curvy road, Juan's odometer indicated he had traveled 60 miles. What was Juan's **average speed** over that 60-mile stretch of the road. Include units in your answer. Show all work. along the line $\Delta y = y_2 - y_1$,	Do you think Juan drove at a constant speed on this curvy road? If he could drive at a constant speed, what is the value of this constant speed, in miles per hour, so he went 60 miles in 90 minutes?
As Juan is driving, he notices that his friend's wedding begins in 15 minutes (1/4 hours). According to his navigation system, he has 7 more miles (of driving on the curvy road) to the wedding destination. Using the average rate of change 40 mph you already computed, will Juan make it to the wedding on time? Justify your answer.	

What should we take away?

The **average rate of change** of a function over some interval of its domain is the __________ **rate of change**, *m* , that produces the same ______ __________ in the function's output quantity on the specified interval of the function's domain, as what was achieved by the function.

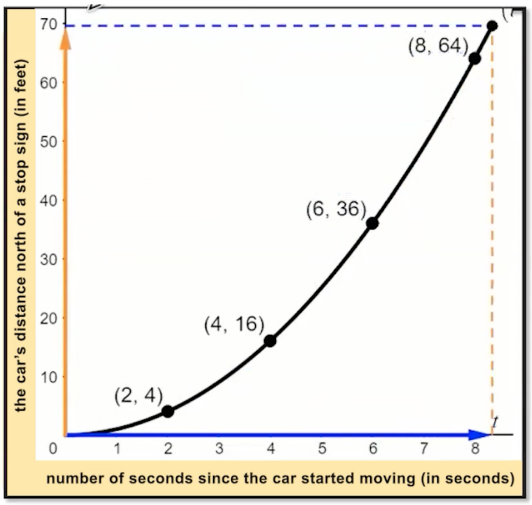
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



Topic 1.3 Rates of Change in Linear and Quadratic Functions (Daily Video 2)

AP Precalculus

In this video, we will explore how the average rate of change of the average rate of change varies over intervals of a function's domain.



Let's WARM UP!

Let's consider a graph that represents a car's distance (in feet) north of a stop sign in relation to the number of seconds since the car began to move.

Use the graph to fill in the blanks.

The car's distance north of the stop sign $\underbrace{\qquad\qquad\qquad}_{\text{increases/decreases}}$ as the time after the car started moving increases from t = ______ to t = ______ seconds.

Complete the table to the right.

Is the car speeding up, slowing down, or moving at a constant rate of change? (Circle one)

How do you know? Use the values in the table to explain your answer below.

Δt	t	$d(t)$	$\Delta d(t)$	$\frac{\Delta d(t)}{\Delta t}$ ft/sec
	0	0		
	2	4		
	4	16		
	6	36		
	8	64		

What kind of pattern are the average rates of change following over $\underbrace{\text{successive}}_{\text{in order}}$ equally sized intervals of 2? Explain your reasoning below.

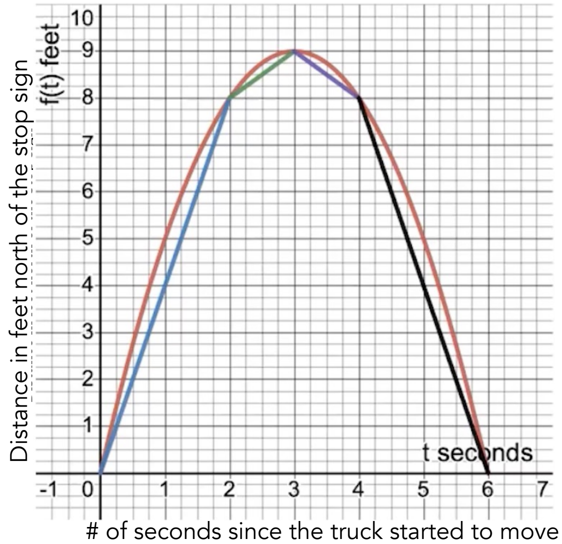
The pattern is ____________________________ because ____

____

____

Example: A truck's distance north of a stop sign

The **concave down** graph to the right represents a truck's distance north of a stop sign in relation to the number of seconds since it started moving.



Use the information in the graph to fill in the blanks below.

The truck stops heading north away from the stop sign, turns around, and heads south toward the stop sign at time $t =$ ____ seconds because the graph stops $\underbrace{\qquad\qquad\qquad\qquad}_{\text{increasing/decreasing}}$ and starts $\underbrace{\qquad\qquad\qquad\qquad}_{\text{increasing/decreasing}}$.

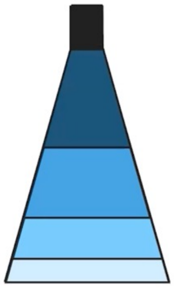
Fill in the values in the table to the right and use the values in the table to fill in the following blanks.

Δt	t	$d(t)$	$\Delta d(t)$	$\frac{\Delta d(t)}{\Delta t}$
	0	0		
2				
	2	8		
1				
	3	9		
1				
	4	8		
2				
	6	0		

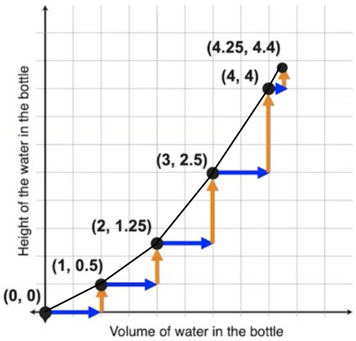
The truck's average rate of change in $\underbrace{\qquad\qquad\qquad}_{\text{units}}$ is $\underbrace{\qquad\qquad\qquad\qquad}_{\text{increasing/decreasing}}$ on the interval $0 < t < 6$ seconds because the graph of the truck's distance north of the stop sign is $\underbrace{\qquad\qquad\qquad\qquad}_{\text{concave up/concave down}}$.

Example: Water flowing in a bottle

The height of the water's average rate of change in inches per cup is $\underbrace{\qquad\qquad\qquad\qquad}_{\text{increasing/decreasing}}$ on the interval because the graph of the height of the water is $\underbrace{\qquad\qquad\qquad\qquad}_{\text{concave up/concave down}}$.



Volume of Water, *v* (cups)	Height of Water, *h* (inches)
0	0
1	0.5
2	1.25
3	2.5
4	4
4.25	4.4



What should we take away?

If a function's graph is **concave up** on an interval of the function's domain, the function's average rate of change ______________ on successive fixed intervals of this interval of the function's domain.

If a function's graph is ______________ on an interval of the function's domain, the function's average rate of change **decreases** on successive fixed intervals of this interval of the function's domain.

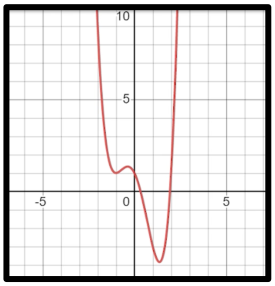
Topic 1.4 Polynomial Functions and Rates of Change (Daily Video 1)

AP Precalculus

In this video, we will learn the vocabulary for polynomials and discuss maximums, minimums, global maximums, global minimums, and inflection points.

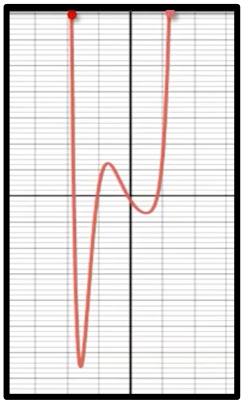
Definition of a Polynomial Function:	Example of a Polynomial Function
Let *n* be a nonnegative integer and let $a_n, a_{n-1}, a_{n-2}, \cdots, a_2, a_1, a_0$ be real numbers where $a_n \neq 0$. $\boldsymbol{p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0}$ then $\boldsymbol{p(x)}$ is a polynomial function in terms of $\boldsymbol{x}$ with degree $\boldsymbol{n}$.	$\boldsymbol{f(x) = x^4 + 2x^3 - 4x^2 - x + 2}$ The **leading term** is the term with the ___________degree of the given variable and the **leading coefficient** is the coefficient of the leading term.

A polynomial function of degree n has at most $n-1$ turning points. ***Turning points*** are the relative (or local) maximums and the minimums on a graph. In other words, **turning points** are where a function switches from increasing to decreasing, or vice versa.



Example: Given $f(x) = x^4 - 3x^2 - 2x + 1$, how many turning points does the graph have? At most ___________.

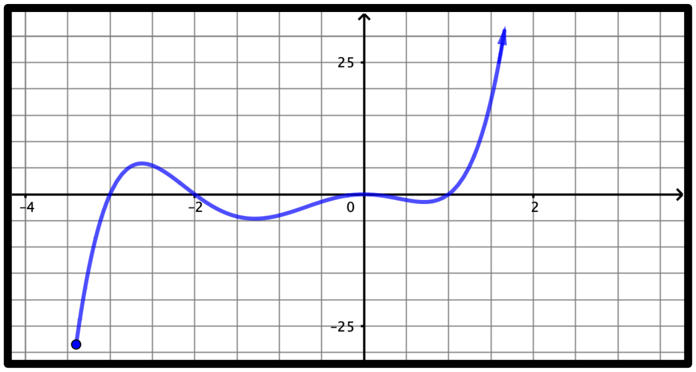
Relative Minimum	Relative Maximum
Where a graph switches from $\underbrace{\hspace{3cm}}_{\text{increasing/decreasing}}$ to $\underbrace{\hspace{3cm}}_{\text{increasing/decreasing}}$. This can also occur at an end point of a polynomial that has a restricted domain.	Where a graph switches from $\underbrace{\hspace{3cm}}_{\text{increasing/decreasing}}$ to $\underbrace{\hspace{3cm}}_{\text{increasing/decreasing}}$ This can also occur at an end point of a polynomial that has a restricted domain.
Global Minimum The lowest of all the minimums on a graph.	**Global Minimum** The highest of all the maximums on a graph.



Inflection Point: This occurs where the graph of a polynomial function changes from concave-up to concave-down or concave-down to concave-up. Occurs at input values where the rate of change of the function changes from increasing to $\underbrace{\hspace{3cm}}_{\text{increasing/decreasing}}$ or from decreasing to $\underbrace{\hspace{3cm}}_{\text{increasing/decreasing}}$

Let's Practice!

Problem #1: Given the graph to the right, which of the points are the following: Give approximate values.



1. Global maximum

2. The global minimum occurs at ...

3. The relative maximums occur at ...

4. The relative minimums occur at ...

5. Inflection points

Note: While the *x* values of the points indicated are where the extrema (minima; maxima) occur, the output (or *y*) value is considered the actual maximum or minimum.

Problem #2: Given the polynomial $f(x) = 2x^3 - 3x^2 - 8x + 1$

A. What is the degree of the polynomial? ______________

B. What is the leading coefficient? _________________

C. How many turning points does the graph have? _____________

What should we take away?

- The definition of a **polynomial**
- What the **leading term** is and what it tells us about the graph.
- The **key points on a polynomial:** maximums, minimums, and inflection points
- The difference between **relative** and **global**
- How to find the number of **turning points** on a graph

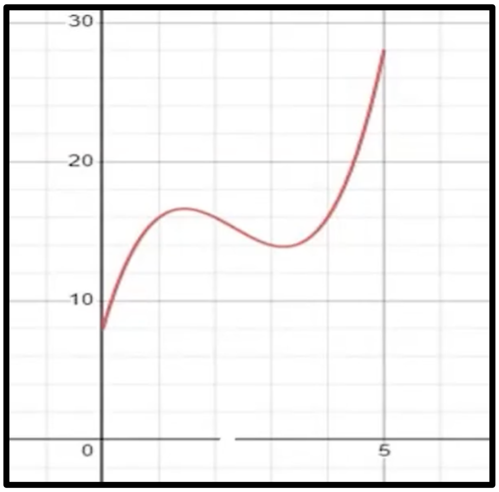
Topic 1.4 Polynomial Functions and Rates of Change (Daily Video 2)

AP Precalculus

In this video, we will explore how rates of change behave at different key points of a polynomial function.

Roller Coaster Polynomial

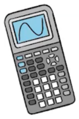
A portion of a roller coaster's path is modeled by the polynomial $h(t) = t^3 - 7t^2 + 14t + 8$, where t is time in seconds, and $h(t)$ is the height, in meters.



Where on the graph is the average rate of change of $h(t)$ **positive**? Choose any two points where the graph is **increasing**. Mark them on the graph and draw a line between them.

Where on the graph is the average rate of change of $h(t)$ **negative**? Choose any two points where the graph is **decreasing**. Mark them on the graph and draw a line between them.

What is the average rate of change between the relative maximum of the graph and the relative minimum of the graph? Use a graphing calculator to find the relative maximum and minimum points of $h(t) = t^3 - 7t^2 + 14t + 8$. You will need both coordinates of these points. What is the average rate of change between these two points? Show your work below.



What is the average rate of change between the absolute maximum of the graph and the absolute minimum of the graph? Show your work below.	One of the inflection points is at $(2.333, 15.259)$. What happens to the rate of change before that point and after that point? The rate of change approaching the inflection point is $\underbrace{\qquad\qquad\qquad}_{\text{increasing/decreasing}}$ ... Then, after inflection point, the rate of change begins to $\underbrace{\qquad\qquad\qquad}_{\text{increase/decrease}}$...

What should we take away?

- When a graph is ______________, its average rate of change is positive.
- When a graph is ______________, its average rate of change is negative.
- At an inflection point, the rate of change changes from _______________to _______________ or vice versa.

Topic 1.5 Polynomial Functions and Complex Zeros (Daily Video 1)

AP Precalculus

In this video, we will learn how to find the zeros (both real and complex) of a polynomial function.

Let's WARM UP!

Zeros of a Polynomial Function

If a is a complex number and $p(a) = 0$ then a is called a ____________ of p, or a ___________ of the polynomial function p.

If a is a real number, then $(x - a)$ is a ___________ factor of p if and only if a is a zero of p.

Example: $\boldsymbol{p(x) = x^3 - 4x}$

Factor:

$p(x) =$________________________________

Solve: $0 = x(x+2)(x-2)$

$x =$________________________

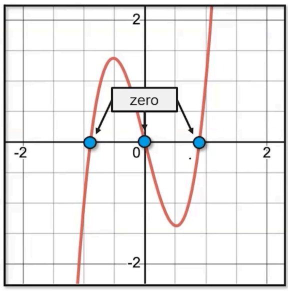
Check:

$p(0) =$________________________

$p(2) =$________________________

$p(-2) =$________________________

Looking at Zeros Graphically

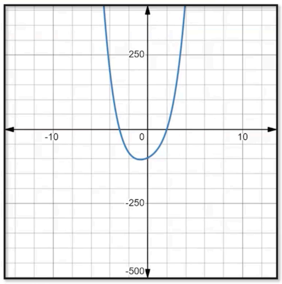


Remember that a real ____________of a function is a point at which the graph crosses the x-axis, which is called an ____________________.

Don't forget that there can be **nonreal zeros** that can't be seen on the graph. They come in **complex conjugate pairs**.

For example, if $x = 2 + 3i$ is a zero of the function, then its **complex conjugate** $x =$ ______________________ is also a zero.

Number of Complex Roots



The number of **complex roots** a polynomial has is equal to the ________________ of the polynomial.

Example: $\boldsymbol{f(x) = x^4 + x^3 + 10x^2 - 16x - 96}$

There should be _______ complex zeros. Using a graphing calculator, we find that zeros are

$x =$______ and $x =$______ , and two complex zeros $x =$_____ and $x =$_____.

Let's Find the Zeros of a Polynomial Function

Example 1: Find the zeros of

$$f(x) = x^3 + 2x^2 - 15x$$

Factor: $f(x) =$ _________________________

Set $f(x) = 0$: _________________________

Set the factors equal to zero and solve. Show your work in the space below.

Example 2: Find the zeros of

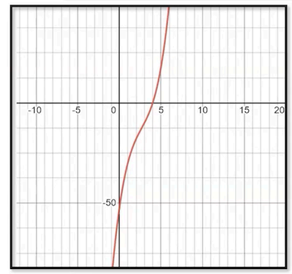
$$f(x) = x^4 + 2x^2 - 64$$

Factor: $f(x) =$ _________________________

Set $f(x) = 0$: _________________________

Set the factors equal to zero and solve. Show your work in the space below.

Example 3: Given the zero $x = 2 + 3i$, find the other zeros of the polynomial function $f(x) = x^3 - 8x^2 + 29x - 52$.



Find the second complex zero: $x =$________________

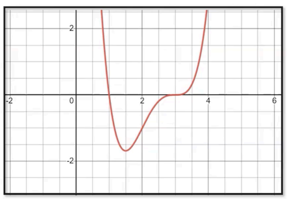
Graph the function to find the third zero: $x =$__________

Write out all of the zeros: $x =$______; $x =$______; $x =$______

Multiplicty of a Zero

The multiplicty of a zero is how many times the zero's factor appears in a given polynomial.

If the zero's multiplicity is **odd**, then the graph will cross through the zero on the x-axis.



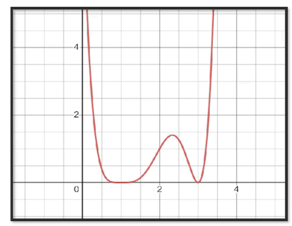
$f(x) = x^4 - 10x^3 + 36x^2 - 54x + 27$

Factor: $f(x) =$ ________________________

Set $f(x) = 0$: ________________________

Set the factors equal to zero and solve. Show your work in the space below.

If the zero's multiplicity is **even**, then the graph will be tangent to the x-axis at that point, because the signs of the output values are the same for input values near $x = a$.



$f(x) = (x-3)^2(x-1)^4$

Set $f(x) = 0$: ________________________

Set the factors equal to zero and solve. Show your work in the space below.

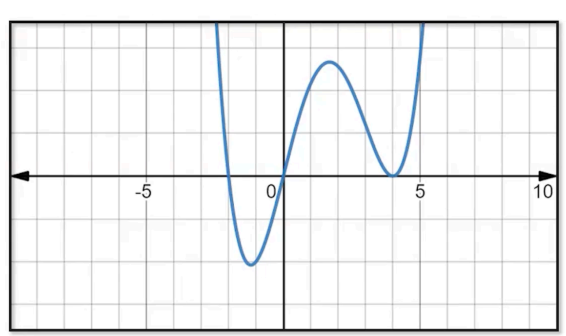
Multiplicty of a Zero Viewed Graphically

$f(x) = x(x-4)^2(x+2)$

Zeros: $x =$________ Multiplicity: ________

$x =$________ Multiplicity: ________

$x =$________ Multiplicity: ________



What should we take away?

- The highest power of a polynomial function's variable states how many zeros the function has.
- To find the zeros of a polynomial function, either ________ the polynomial (if possible) or use a graphing calculator.
- All complex zeros come in complex ____________ pairs.

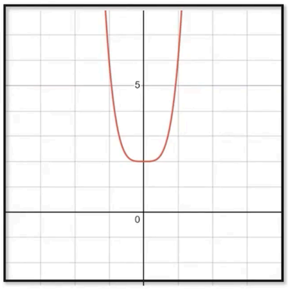
Topic 1.5 Polynomial Functions and Complex Zeros (Daily Video 2)

AP Precalculus

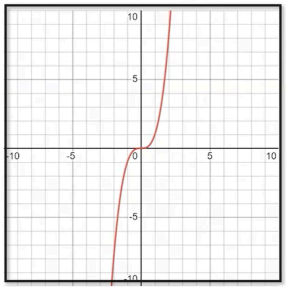
In this video, we will learn how to tell if a polynomial function is odd or even, practice finding zeros and matching graphs to their zeros.

What do *odd* and *even* mean in terms of a graph?

Even: The graph is symmetric over the *y*-axis or the line *x* = 0.



Odd: The graph is symmetric over the origin or the point $(0,0)$.



Even and Odd: Analytically

Even Symmetry $\boldsymbol{f(-x) = f(x)}$	Odd Symmetry $\boldsymbol{f(-x) = -f(x)}$
If substituting $(-x)$ for the variable and simplifying gives you the exact same signs as the original polynomial, then it has even symmetry.	If substituting $(-x)$ for the variable and simplifying gives you the exact opposite signs as the original polynomial, then it has odd symmetry.
Example: $\boldsymbol{f(x) = x^4 - 2x^2}$	Example: $\boldsymbol{f(x) = 2x^3 - 2x}$
Substituting: $\boldsymbol{f(-x)}$ =____________________	Substituting: $\boldsymbol{f(-x)}$ =____________________
Simplifying: $\boldsymbol{f(-x)}$ =____________________	Simplifying: $\boldsymbol{f(-x)}$ =____________________

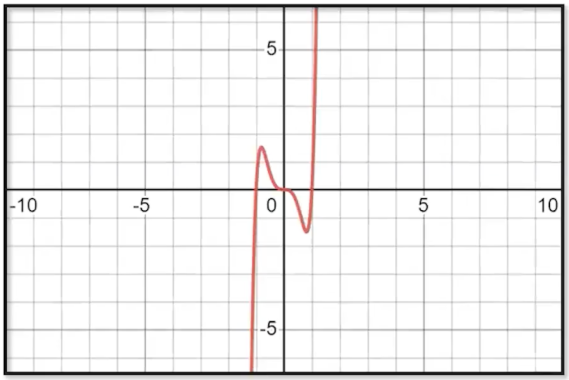
State Whether the Graph is Even or Odd

Is the function $f(x) = 5x^7 - 5x^3$ even or odd?
For it to be even, $f(-x) = f(x)$

Substituting: $f(-x)$ =____________________

Simplifying: $f(-x)$ =____________________

Is the function $f(x) = 5x^7 - 5x^3$ even or odd?
Circle the correct choice.



Practice Problem #1

Given $f(x) = (x-3)(x+2)^4(x+6)^3$, state the zeros, their multiplicity, and what the graph does at those points: **cross** through the *x*-axis or be **tangent** to the *x*-axis.

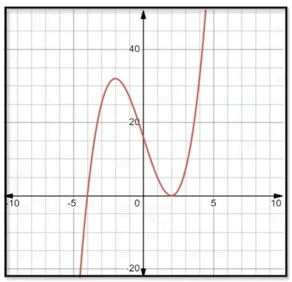
Zeros: $x =$________ Multiplicity: ________ Graph: ______________

$x =$________ Multiplicity: ________ Graph: ______________

$x =$________ Multiplicity: ________ Graph: ______________

Practice Problem #2

Given the following graph of a polynomial equation, what is its equation?
Circle the correct choice.



A. $f(x) = x(x-4)(x+2)$

B. $f(x) = (x+4)(x-2)$

C. $f(x) = (x+4)(x-2)^2$

D. $f(x) = (x-2)(x+4)^2$

Practice Problem #3

Given the following table, what is the lowest-degree polynomial function it could represent?

A. Degree: 2

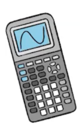
B. Degree: 3

C. Degree: 4

D. Degree: 1

x	y
–5	–24
–4	0
–3	6
–2	0
–1	–12
0	–24
1	–30
2	–24
3	0
4	48

Open-Response Practice Problem



Given the polynomial function: $f(x) = x^3 - 2x^2 - 4x - 16$. You are allowed to use your graphing calculator to graph $f(x)$.

A. How many complex zeros could the function have? Explain how you know below.

B. What is (are) the *x*-intercept(s) of the graph? ____________________

C. Are there any non-real zeros? Explain how you know below.

D. Does the graph of the polynomial have even or odd symmetry? ____________

Substituting: $f(-x) =$____

Simplifying: $f(-x) =$____

What should we take away?

The graph of a function has ________ symmetry when $f(-x) = f(x)$ and ________ symmetry when $f(-x) = -f(x)$.

Topic 1.6 Polynomial Functions and End Behavior (Daily Video 1)

AP Precalculus

In this video, we will learn explore how to know the end behavior of polynomial functions based on their equations.

Let's WARM UP!

Leading Term Practice

Find the leading term of the following polynomials.

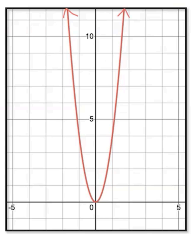
1. $f(x) = 3x^6 - 2x^3 - 4x + 1$ ____________　　2. $f(x) = -2x^4 + x^3 - 4x^2 + 1x - 8$ ____________

3. $f(x) = -5x^4 + x^2 + 4x^9 + 12x^8 + 2$ ______________

Definition of End Behavior

The **end behavior** of a graph is what the graph is doing as the input values move to the: **right without bound** (to positive infinity) and the **left without bound** (to negative infinity).

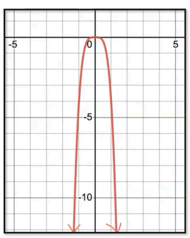
Graph the function $f(x) = 4x^2$



As **input values** of the nonconstant polynomial function increase without bound the ouput values _________________ without bound.

As **input values** of the nonconstant polynomial function decrease without bound the ouput values _________________ without bound.

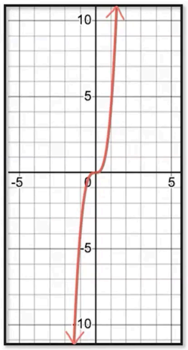
Graph the function $f(x) = -4x^2$



As **input values** of the nonconstant polynomial function increase without bound the ouput values _________________ without bound.

As **input values** of the nonconstant polynomial function decrease without bound the ouput values _________________ without bound.

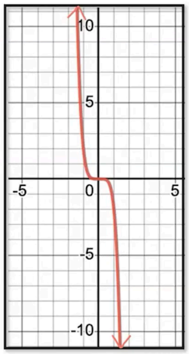
Graph the function $f(x) = 4x^3$



As **input values** of the nonconstant polynomial function increase without bound the ouput values _________________ without bound.

As **input values** of the nonconstant polynomial function decrease without bound the ouput values _________________ without bound.

Graph the function $f(x) = -2x^5$



As **input values** of the nonconstant polynomial function increase without bound the ouput values _________________ without bound.

As **input values** of the nonconstant polynomial function decrease without bound the ouput values _________________ without bound.

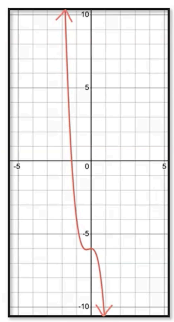
End Behavior Summary

Even-Powered Leading Term	Odd-Powered Leading Term
Even and positive: $x \to \infty\ y \to \infty$ $x \to -\infty\ y \to \infty$	**Odd and positive:** $x \to \infty\ y \to \infty$ $x \to -\infty\ y \to -\infty$
Even and negative: $x \to \infty\ y \to -\infty$ $x \to -\infty\ y \to -\infty$	**Odd and negative:** $x \to \infty\ y \to -\infty$ $x \to -\infty\ y \to \infty$

Let's look at an EXAMPLE!

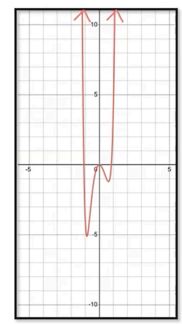
Given $f(x) = -4x^3 - 2x^2 - 6$ describe the end behavior of the graph of the polynomial.

- Leading Coefficient: _______
- Power: _______
- Sign: _______
- Answer: $x \to \infty\ \ y \to$ _______
  $x \to -\infty\ \ y \to$ _______

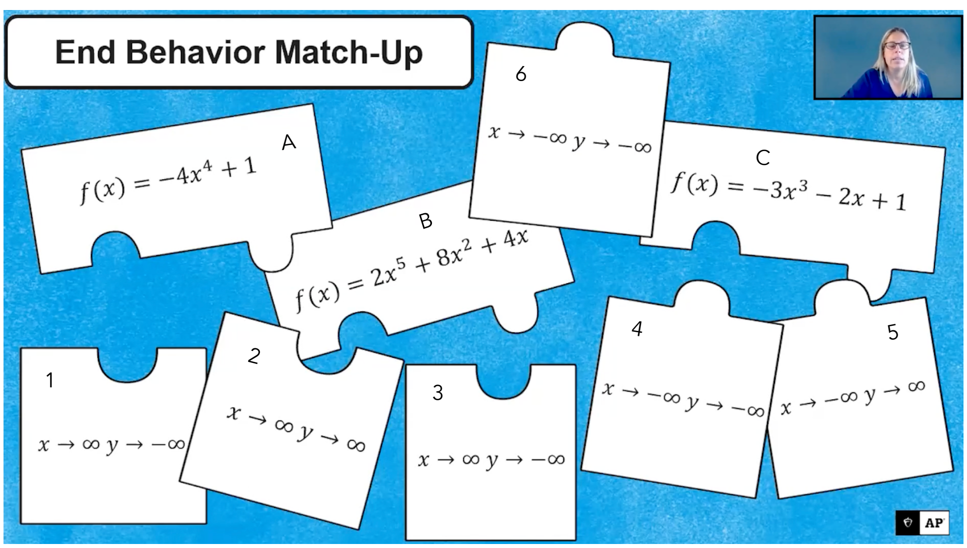


Given
$f(x) = -x^6 + 4x^3 - 6x^2 + 5x^8$
describe the end behavior of the graph of the polynomial.

- Leading Coefficient: _______
- Power: _______
- Sign: _______
- Answer: $x \to \infty\ \ y \to$ _______
  $x \to -\infty\ \ y \to$ _______



Let's PRACTICE!



What should we take away?

- If the leading term is even powered, then both ends of the graph go in the ___________. If the sign is ____________, both ends go up. If the sign is __________, both ends go down.
- If the leading term is odd powered, then the ends of the graph go in the ______________ directions. If the sign is _________, the left side goes down and the right side goes up. If the sign is __________, the left side goes up and the right side goes down.

Topic 1.7 Rational Functions and End Behavior (Daily Video 1)

AP Precalculus

In this video, we will learn explore how rational functions are expressed and what effect changes in the degrees of the numerator and denominator have on the function's end behavior.

Let's WARM UP!

Determine the degree of each of the following polynomials.

Example 1: $f(x) = 2x^4 - 7x^3 + 1$ degree = __________

Example 2: $f(x) = 2 + 3x$ degree = __________

Example 3: $f(x) = 5$ degree = __________

What is a rational function? A rational function is represented as a quotient of two polynomials. Let $f(x)$ and $g(x)$ represent polynomial functions. Then the rational function $r(x)$ is given by

$$r(x) = \frac{f(x)}{g(x)}\text{, where } g(x) \neq 0.$$

Your turn!

Determine which of the following represents a rational function. Circle the rational function(s).

$f(x) = 3x^{\frac{2}{3}} - 4x - 1$ $\quad h(x) = \frac{2}{3x - 1}$ $\quad g(x) = \frac{3x^2 - 4x - 1}{3}$

End Behavior of Rational Functions

After examining the degree of the numerator and the degree of the denominator of a rational function, the following ideas can be used to determine the end behavior of the function.	• Numerator degree > Denominator degree The end behavior will mirror the polynomial of the resulting quotient of ____________ ____________. • Numerator degree = Denominator degree The end behavior approach the ______________ asymptote determined by the ratio of the leading terms. • Numerator degree < Denominator degree The end behavior approach the horizontal asymptote ________.

Let's look at an EXAMPLE!

$$f(x) = \frac{x^3 + 1}{4x^2 + 5x + 1}$$

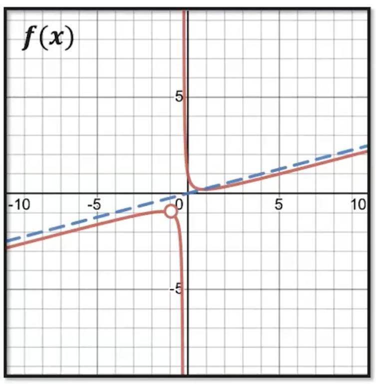
Degree of the numerator: ________

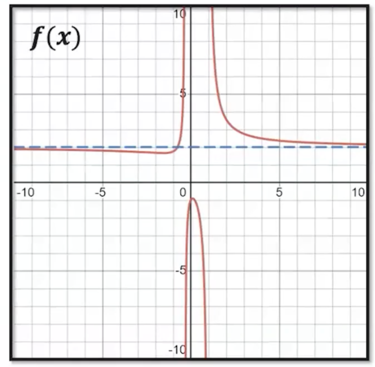
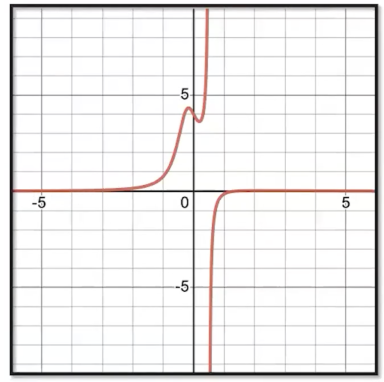
Degree of the denominator: ________

Quotient of Leading Terms: $\frac{x^3}{4x^2} =$ __________

End Behavior: $\lim\limits_{x \to -\infty} f(x) =$ __________

$\lim\limits_{x \to \infty} f(x) =$ __________



$f(x) = \dfrac{6x^2+1}{3x^2-2x-1}$ Degree of the numerator: ________ Degree of the denominator: ________ Ratio of Leading coefficients: $\dfrac{6}{3} =$ ___________ End Behavior: $\lim\limits_{x\to-\infty} f(x) =$ __________ $\lim\limits_{x\to\infty} f(x) =$ __________	
$f(x) = \dfrac{3x+1}{8x^3-1}$ Degree of the numerator: ________ Degree of the denominator: ________ End Behavior: $\lim\limits_{x\to-\infty} f(x) =$ __________ $\lim\limits_{x\to\infty} f(x) =$ __________	

What should we take away?

- Numerator degree > Denominator degree
  The end behavior will mirror the polynomial of the resulting quotient of leading terms.
- Numerator degree = Denominator degree
  The end behavior approach the horizontal asymptote determined by the ratio of the leading terms.
- Numerator degree < Denominator degree
  The end behavior approach the horizontal asymptote $y = 0$.

Topic 1.7 Rational Functions and End Behavior (Daily Video 2)

AP Precalculus

In this video, we will learn explore about the connected relationship between a function's end behavior and the asymptotes of rational functions.

Limits at Infinity-Horizontal Asymptotes

Example 1: $F(x) = \dfrac{x-4}{6x^2-1}$

For $F(x)$, as the input values increase without bounds, what happens to the output values?

$$\lim_{x\to\pm\infty} \frac{x-4}{6x^2-1} = \lim_{x\to\pm\infty}\left(\frac{\frac{1x}{x^2}-\frac{4}{x^2}}{\frac{6x^2}{x^2}-\frac{1}{x^2}}\right) = \underbrace{\lim_{x\to\pm\infty}\left(\frac{\frac{1}{\square}-\frac{4}{x^2}}{\square-\frac{1}{x^2}}\right)}_{\text{Simplify,fill in the boxes}} = \frac{\square-\square}{\square-\square} = \_\_\_\_\_\_\_\_$$

The horizontal asymptote is at $y = 0$.

Example 2: $F(x) = \dfrac{8x^2+3x+4}{2x^2-3x-1}$

For $F(x)$, as the input values increase without bounds, what happens to the output values?

$$\lim_{x\to\pm\infty} \frac{8x^2+3x+4}{2x^2-3x-1} = \lim_{x\to\pm\infty}\left(\frac{\frac{8x^2}{x^2}+\frac{3x}{x^2}+\frac{4}{x^2}}{\frac{2x^2}{x^2}-\frac{3x}{x^2}-\frac{1}{x^2}}\right) = \underbrace{\lim_{x\to\pm\infty}\left(\frac{8+\frac{3}{\square}+\frac{4}{\square}}{2-\frac{3}{\square}-\frac{1}{\square}}\right)}_{\text{Simplify,fill in the boxes}} = \frac{\square-\square-\square}{\square-\square-\square} = \_\_\_\_$$

The horizontal asymptote is at $y =$________.

Let's Practice! Determine the limits of each of the following functions, then determine the equation of the horizontal asymptote(s), if any.

Example 1:	Example 2:
$F(x) = \dfrac{x^3-4x^2+3x-5}{3x^3+x^2+3x-4}$ $\lim_{x\to\pm\infty} F(x) =$ ______ Horizontal Asymptote: $y =$________	$F(x) = \dfrac{x+4}{5x^2-6x-4}$ $\lim_{x\to\pm\infty} F(x) =$ ______ Horizontal Asymptote: $y =$________

What should we take away?

Determining the end behavior of a rational function by finding limits at infinity can lead to determining the ______________ asymptote(s) of the given function.

Topic 1.8 Rational Functions and Zeros (Daily Video 1)

AP Precalculus

In this video, we will learn how to find the zeros of a rational function and determine the intervals of positive, negative, or undefined output values.

Let's look at an EXAMPLE!

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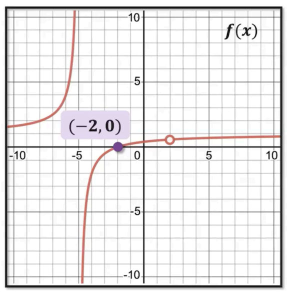
Example 1: Find the zeros of the rational function $f(x)$. First factor the rational function and then simplify.

$$f(x) = \frac{x^2 - 4}{x^2 + 3x - 10}$$

Find the zero(s) of $f(x)$.



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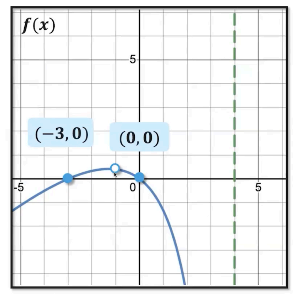
Example 2: Find the zeros of the rational function $f(x)$. First factor the rational function and then simplify.

$$f(x) = \frac{x^3 + 4x^2 + 3x}{x^2 - 3x - 4}$$

Find the zero(s) of $f(x)$.



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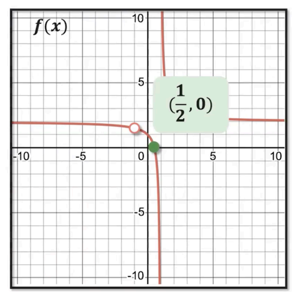
</table>

Let's PRACTICE!

Find the zero(s) of $f(x)$.

$$f(x) = \frac{2x^2 + x - 1}{x^2 - 1}$$

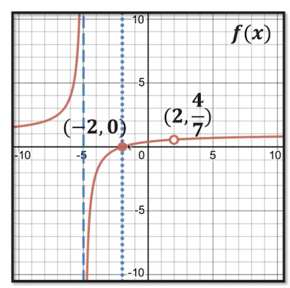


Let's look at an EXAMPLE!

The zeros of both the numerator and denominator of a rational function, $f(x)$, create intervals that satisfy the inequalities $f(x) \geq 0$ or $f(x) \leq 0$.

$$f(x) = \frac{x^2 - 4}{x^2 + 3x - 10} = \frac{(x-2)(x+2)}{(x-2)(x+5)}$$

To determine the **intervals** where $f(x)$ is positive or negative, analyze the sign of each factor at an *x*-value in the interval to determine the sign of the final output. The first one has been done as an example.



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Over the interval $x < -5$, $f(x) > 0$ because using

$$x = -6 \Rightarrow \frac{(-6-2)(-6+2)}{(-6-2)(-6+5)} \Rightarrow \frac{(-)(-)}{(-)(-)} \Rightarrow +$$

Over the interval $-5 < x < -2$, $f(x) < 0$ because using $x =$ _____ $\Rightarrow$

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Over the interval $-2 \leq x < 2$, $f(x) \geq 0$ because using $x =$ _____ $\Rightarrow$

Over the interval $x > 2$, $f(x) > 0$ because using $x =$ _____ $\Rightarrow$

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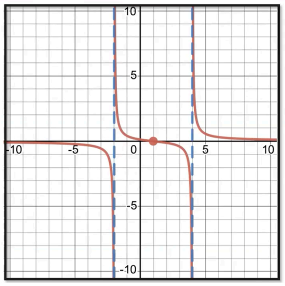
Let's PRACTICE!

Find the zero(s) of both the numerator and denominator of $f(x)$. Then determine the **intervals** where $f(x)$ is positive or negative.



$$f(x) = \frac{x-1}{x^2-2x-8}$$

Over the interval $x < -2,\ f(x) < 0$ because using $x =$ _____ $\Rightarrow$	Over the interval $1 \leq x < 4,\ f(x) \leq 0$ because using $x =$ _____ $\Rightarrow$
Over the interval $-2 < x \leq 1,\ f(x) \geq 0$ because using $x =$ _____ $\Rightarrow$	Over the interval $x > 4,\ f(x) > 0$ because using $x =$ _____ $\Rightarrow$

What should we take away?

- Finding zeros of a rational function requires simplifying rational functions, then finding the zeros of the resulting polynomial in the numerator.
- Zeros of the numerator and the denominator of rational functions can identify endpoints and/or asymptotes of intervals of positive and negative function values for the rational function.

Topic 1.9 Rational Functions and Vertical Asymptotes (Daily Video 1)

AP Precalculus

In this video, we will investigate how to determine the vertical asymptote(s) of rational functions.

Let's Warm up!

Find the real zeros of the following rational functions:

$f(x) = \dfrac{x^2 + 2x}{x^2 - 4x - 5}$	$f(x) = \dfrac{x^2 - x - 6}{x + 2}$

Reminder: The real zeros of a rational function correspond to the real zeros of the numerator for the values in the function's domain.

Let's look at an EXAMPLE!

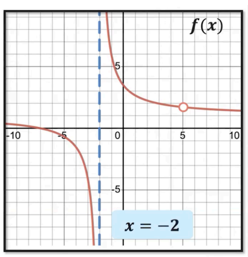
Find the vertical asymptote(s) of the given rational function.

Example 1:

$$f(x) = \frac{x^2 + 2x - 35}{x^2 - 3x - 10} =$$

Real zero of denominator: __________

Vertical asymptote at ___________



Example 3:

Determine $\lim_{x \to -2^-} f(x)$ and $\lim_{x \to -2^+} f(x)$.

$\lim_{x \to -2^-} f(x) =$_______ $\lim_{x \to -2^+} f(x) =$_______

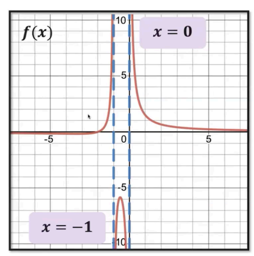
Example 2:

$$f(x) = \frac{x^2 + 3x + 2}{x^3 + 2x^2 + x} =$$

Real zeros of numerator: _______________

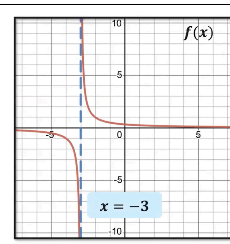
Real zeros of denominator: _______________

Vertical asymptote at ___________



Let's PRACTICE!

Find the vertical asymptote(s) of the given rational function $f(x)$. Then determine $\lim_{x \to -3^-} f(x)$ and $\lim_{x \to -3^+} f(x)$.



$$f(x) = \frac{x - 8}{x^2 - 5x - 24} =$$

Vertical asymptote at ___________ $\lim_{x \to -3^-} f(x) =$ ________ $\lim_{x \to -3^+} f(x) =$ ___________

What should we take away?

Finding vertical asymptotes of a rational function requires examining the real zeros unique to the __________________ and the behavior of the output values of a rational function near a vertical asymptote either increase or decrease ______________________.

Topic 1.10 Rational Functions and Holes (Daily Video 1)

AP Precalculus

In this video, we will compare the multiplicity of zeros in the numerator and denominator of a rational function in order to identify and determine holes in the graph of the function.

Let's Warm up!

Determine the zeros and their multiplicity of the following polynomial.

$$F(x) = (x+3)(x-2)^2(x+1)^3$$

Zero: _______ Multiplicity _____ Zero: _______ Multiplicity ______ Zero: ______ Multiplicity ______

Let's look at an EXAMPLE and PRACTICE!

Determine where the function $f(x)$ has a hole in its graph.

Example: Determine the y-coordinate of the hole in $f(x)$.

$$f(x) = \frac{x^2+2x-35}{x^2-3x-10} =$$

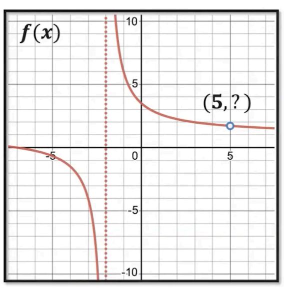
Real zeros of numerator: __________

Real zeros of denominator: __________

$$\lim_{x\to 5}\frac{x^2+2x-35}{x^2-3x-10}$$

$$= \lim_{x\to 5}\frac{x+7}{x+2} = \_\_\_\_\_\_\_\_$$

Coordinates of the hole:___________



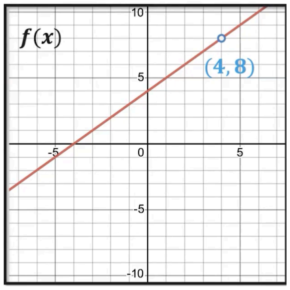
PRACTICE: Determine where the function $f(x)$ has a hole and then determine the y-coordinate of the hole.

$$f(x) = \frac{x^2-16}{x-4} =$$

Hole at __________

$$\lim_{x\to 4}\frac{x^2-16}{x-4} = \lim_{x\to 4}(x+4) = \_\_\_\_\_\_\_\_$$

Coordinates of the hole:___________



What should we take away?

- Finding the location of holes in the graph of a rational function requires examining the common zeros of the polynomials in both the ______________ and ________________.
- The y-coordinate of a hole can be determined by examining the limiting behavior of a function's output values arbitrarily close to the ________________ of the hole.

Topic 1.11 Equivalent Representations of Polynomial and Rational Expressions

(Daily Video 2) *Note: Video 2 should actually be Video 1. Videos 1 and 2 are incorrectly labeled on AP Classroom.

AP Precalculus

In this video, we will compare dividing a polynomial by a linear factor with dividing an integer by a smaller integer.

Long Division Warm up! Divide 425 by 12 and show your work.

$425 \div 12$	Steps
$12\overline{)\,4\,2\,5}$	• Put 12 on the outside and 425 on the inside. • Move from left to right. • Use the fewest digits possible at a time to divide, then subtract. • Bring down the next digit. • Repeat the process. • Once you can't go any further, place the remainder over the divisor.

Let's PRACTICE!

Polynomial Division

$(3x^2 + 7x + 55) \div (x + 2)$	Steps
$x + 2\overline{)\,3x^2 + 7x + 55}$	• Put $(x + 2)$ on the outside and $(3x^2 + 7x + 55)$ on the inside. • Move from left to right. • Use the fewest terms possible at a time by using the same number of terms. • Make the first terms match. • Subtract. • Bring down the next term, and repeat. • Place remainder over divisor.

What should we take away?

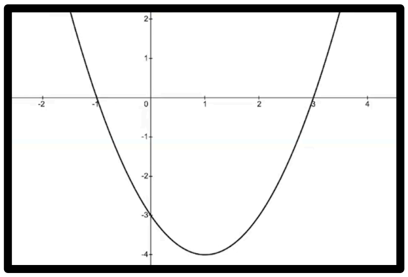
Dividing a polynomial by a linear factor is like dividing an integer by a smaller integer.

Topic 1.11 Equivalent Expressions of Polynomials and Rational Functions

(Daily Video 1) *Note: Video 1 should actually be Video 2. Videos 1 and 2 are incorrectly labeled on AP Classroom.

AP Precalculus

In this video, we will review how to convert polynomial and rational functions from standard form to factored form and from factored form to standard form.



Let's Warm up!

Basic Characteristics of a Polynomial Function

y-intercept: ______________ x-intercept: _____________________

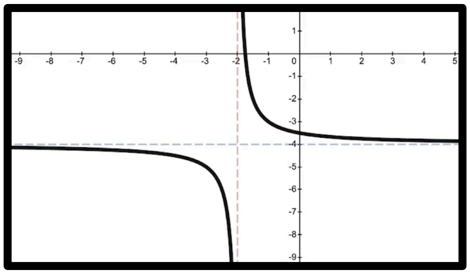
Zeros: __________________ Axis of symmetry: _______________

Domain: ________________ Range: ________________

$f(x)$ in factored form: ________________________ $f(x)$ in standard form: ________________________

Example: Write $f(x) = x^3 - x^2 - 2x$ in factored form.

Basic Characteristics of a Rational Function



y-intercept: __________ horizontal asymptote: ___________

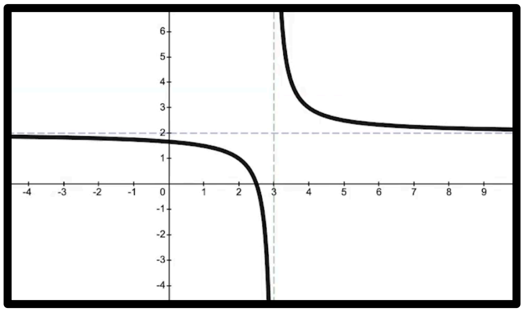
Zeros: ______________ vertical asymptote: _____________

Domain: ____________________ Range: __________________

$f(x)$ in standard form: Change to factored form.	$f(x)$ in factored form: Change to standard form.

Let's PRACTICE!

What is the equation of this function in factored form?



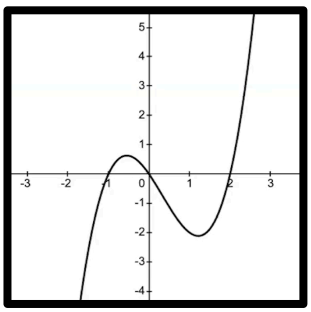
A) $f(x) = \dfrac{1}{x-3} + 2$

B) $f(x) = \dfrac{1}{x-2} + 3$

C) $f(x) = \dfrac{3x-5}{x-2}$

D) $f(x) = \dfrac{2x-5}{x-3}$

What is a possible equation of this function standard form?



Write the following function in standard form.

$$f(x) = \frac{2x-9}{x-3}$$

What should we take away?

We should be able to change __________________ functions and __________________ functions from _______________ form to _______________ form and vice versa.

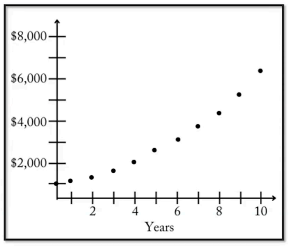
Topic 1.12 Transformations of Functions (Daily Video 1)

AP Precalculus

In this video, we will explore how and why an additive transformation impacts the graph of a function.

Let's WARMUP!

Invest $1,000 and Earn 20% Return per Year!



What is the equation of this function? ______________________

The equation of $f(x)$ is changed to $f(x) + 5$. Describe how the graph of $f(x)$ is changed.

The equation of $f(x)$ is changed to $f(x-1)$. Describe how the graph of $f(x)$ is changed.

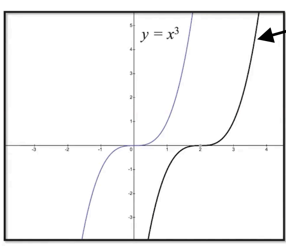
Let's REVIEW!

Match the Direction with the Transformation

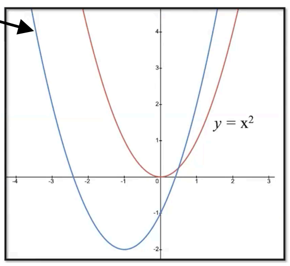
Direction of Translation		Function: $y = x^2$ is the Parent	
→	↑	$y = (x-5)^2$	$y = x^2 + 5$
↓	←	$y = (x+5)^2$	$y = x^2 - 5$

Let's PRACTICE!

What is the equation of the graph to the right?



What is the equation of the graph to the left?



What should we take away?

We should be able to recognize, based on graphs and/or equations, when an additive transformation has occurred. $f(x) + k$ is a ____________ shift and $f(x+k)$ is a _____________ shift of the graph of $f(x)$.

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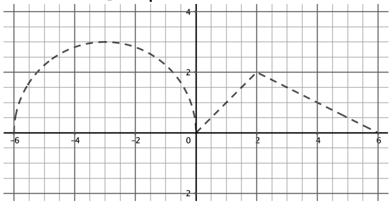
Topic 1.12 Transformations of Functions (Daily Video 2)

AP Precalculus

In this video, we will explore how and why a multiplicative transformation impacts the graph of a function.

The graph of $y = f(x)$

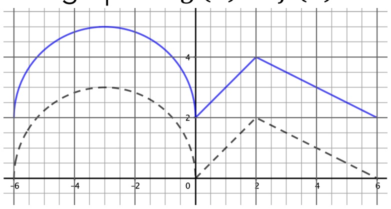


Let's Review!

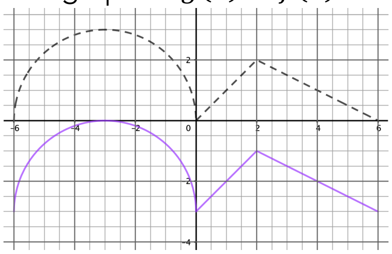
$f(x)$ is a piecewise defined function with a semicircle and 2 linear pieces.

Additive transformations

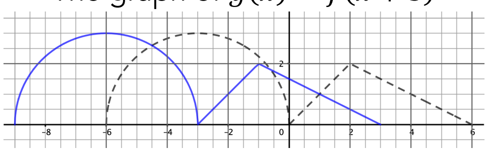
The graph of $g(x) = f(x) + 2$



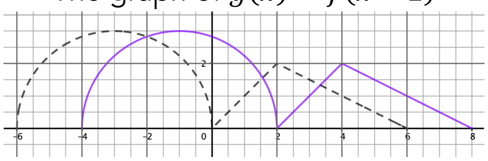
The graph of $g(x) = f(x) - 3$



The graph of $g(x) = f(x + 3)$



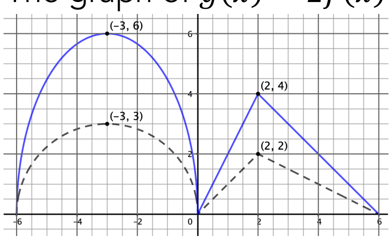
The graph of $g(x) = f(x - 2)$

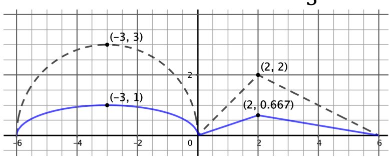


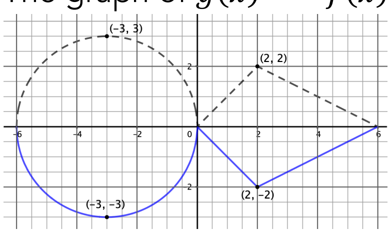
Let's look at an EXAMPLE!

$g(x) = af(x)$ is a multiplicative transformation of the function f. The result is a ______________ dilation of the graph of f by a factor of _______.	$g(x) = f(bx)$ is a multiplicative transformation of the function f. The result is a ______________ dilation of the graph of f by a factor of _______.

Effect of Multiplying a Function by a Constant $g(x) = af(x)$

<table>
<tr>
<td>The graph of $g(x) = 2f(x)$


How were the y −values changed?</td>
<td>The graph of $g(x) = \frac{1}{3}f(x)$


How were the y −values changed?</td>
<td>The graph of $g(x) = -f(x)$


This example was not included in the video. How were the y −values changed?</td>
</tr>
<tr>
<td colspan="2">

- If $|a| > 1$, the function is vertically ______________ by a factor of a.
- If $0 < |a| < 1$, the function is vertically ______________ by a factor of a.

</td>
<td>

- If $a < 0$, the function has a vertical dilation by a factor $|a|$ **and** is ______________ over the x −axis.

</td>
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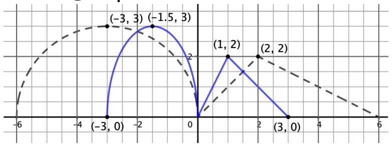
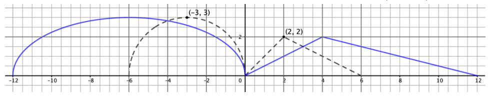
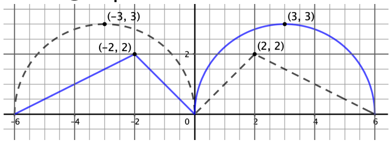
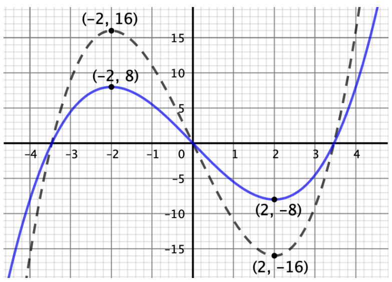
</table>

Effect of Multiplying x by a Constant $\boldsymbol{g(x) = f(bx)}$

<table>
<tr>
<td>The graph of $g(x) = f(2x)$

How were the x −values changed?</td>
<td>The graph of $g(x) = f\left(\frac{1}{2}x\right)$

How were the x −values changed?</td>
<td>The graph of $g(x) = f(-x)$

This example was not included in the video. How were the x −values changed?</td>
</tr>
<tr>
<td colspan="2">• If $|b| > 1$, the function is horizontally ____________ by a factor of $\frac{1}{b}$.
• If $0 < |b| < 1$, the function is horizontally ____________ by a factor of $\frac{1}{b}$.</td>
<td>• If $b < 0$, the function has a horizontal dilation by a factor $\left|\frac{1}{b}\right|$ and is ____________ over the y −axis.</td>
</tr>
</table>

Let's PRACTICE!



Identify the Transformation

The black (dashed) graph's equation is $f(x) = x^3 - 12x$.

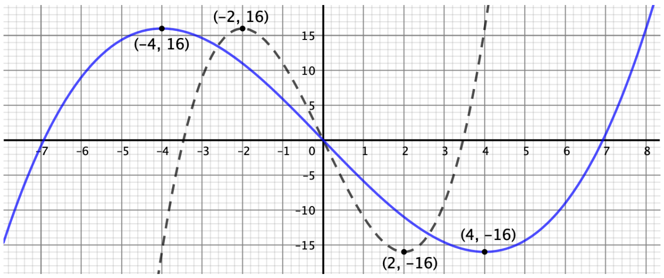
How is the blue (solid) graph different?

The blue graph is a ______________ dilation.

The equation of for the blue graph has a form of $g(x) =$ ___________

What is the value of a? Explain your reasoning.

The equation of the transformed function is $g(x) =$ ___________



How is the blue (solid) graph different?

The blue graph is a ______________ dilation.

The equation of for the blue graph has a form of $g(x) =$ ___________

What is the value of b? Explain your reasoning.

The equation of the transformed function is $g(x) =$ ___________

What should we take away?

1. Given a function, produce the graph of a new function with multiplicative transformations.

2. Create an equation for a function given its parent function and its horizontal and vertical dilations.

Topic 1.13 Function Model Selection and Assumption Articulation (Daily Video 1)

AP Precalculus

In this video, we will explore the most appropriate functions to use to model given data sets based on our knowledge of rates of change.

Let's REVIEW!

Linear data sets have a ________________rate of change.

Is Data Set A linear? Justify your answer.

Data Set A

x	$y = f(x)$
0	3
1	7
2	11
3	15
4	19

Quadratic data sets have constant _______________ differences for equal increments of input.

Is Data Set B linear or quadratic? Justify your answer.

Data Set B

x	$y = g(x)$
0	-12
1	-3
2	0
3	-3
4	-12

Let's PRACTICE!

Is the data, to the right, linear or quadratic? Justify your answer.

x	-2	2	4	6	12
$f(x)$	5	-1	-4	-7	-16

Is the data, to the right, linear or quadratic? Justify your answer.

Linear or Quadratic?	
x	$f(x)$
-2	-8
-1	-2
0	0
1	-2
2	-8

What should we take away?

Linear models always have a _______________ rate of change.

Quadratic models have a constant _______________ difference for equal increments of input.

Topic 1.13 Function Model Selection and Assumption Articulation (Daily Video 2)

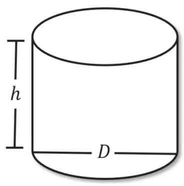
AP Precalculus

In this video, we will use quadratic and cubic functions to model given scenarios and discuss physical constraints on a function's domain and range.

Let's look at an EXAMPLE!

Volume of a Cylinder

Part 1: Suppose the volume of a right cylinder has a height, h, that is twice the length of its diameter, D. Identify, from the choices below, the function, $V(r)$, that represents the volume of the cylinder in terms of the radius. Recall: $V = \pi r^2 h$.



A. $V(r) = \pi r^2 h$ B. $V(r) = 2\pi r^2 D$ C. $V(r) = 2\pi r^3$ D. $V(r) = 4\pi r^3$

Part 2: Using the formula found in part 1, $V(r) =$_________, what is a reasonable domain and range for this problem and why?

Part 3: Using the formula found in part 1, $V(r) =$_________, what is a reasonable domain and range for this problem if the diameter can never be larger than 20 cm?

Domain: ____________________ Range: __________________________

What should we take away?

When we a dealing with "real-world" problems, we must always consider what restrictions the _________________ ___________________ of the scenario might put on the _________ values and _________ values.

Topic 1.14 Function Model Construction and Application (Daily Video 1)

AP Precalculus

In this video, we will explore quantities that are inversely proportional and build appropriate models.

Let's look at an EXAMPLE!

Suppose the output of a function, $f(x)$, is **inversely proportional** to the square of its input.

Write a generic equation for the function.

$f(x) =$ ______________

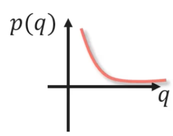
If we know that one data point of the function $f(x) = \frac{k}{x^2}$ is $(10, 30)$ then what is the value of the constant k? Show how you arrived at your answer.

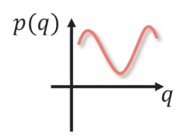
What is the output for an input of 50 units?

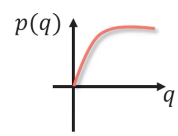
$f(50) =$ __________

Let's PRACTICE!

Suppose the price per pound, $p(q)$, of a particular whole grain is inversely proportional to the quantity, q, demanded. Which of the following graphs could represent this relationship? Circle a choice and explain your reasoning.







Write an equation for $p(q)$ if we know that the price per pound of whole grain is \$4 when 2,000 pounds are being demanded. Show how you arrived at your answer.

$p(q) =$ _______________

What should we take away?

If the output values and input values are inversely proportional, then as input values increase, output values ________________ and as input values ___________________, output values increase.

Topic 1.14 Function Model Construction and Application (Daily Video 2)

AP Precalculus

In this video, we will compute average rates of change and compare the changes in those average rates of change to draw conclusions about a given model.

Let's REVIEW! Rational Function Review: Topics 1.7 – 1.9

$$g(t) = \frac{3t+1}{t+2}$$

Asymptotes for $y = g(t)$: horizontal asymptote: ______________ vertical asymptote: _______________

List the domain and range: domain: ________________________ range: __________________________

Let's look at an EXAMPLE!

Suppose that the previous function, $g(t)$, can be used to model the population of a species since 1951 ($t \geq 0$, measured in years) and $g(t)$ is the population (in thousands).

Calculate the value of $g(0)$ and explain the meaning of $g(0)$ in the context of this problem.	State the range of $g(x)$ in the context of the problem. Explain your reasoning.
Find the average rate of change between $t = 1$ and $t = 2$ and the average rate of change between $t = 8$ and $t = 10$. Be sure to use proper units. Show your work.	Compare the average rate of change between $t = 1$ and $t = 2$ with the average rate of change between $t = 8$ and $t = 10$. Be sure to use the context of the problem in your discussion.
Both average rates of change are positive over these intervals so the graph of $g(t)$ is _________________.	The average rates of change are decreasing over these intervals so the graph of $g(t)$ is _____________________.

What should we take away?

When the rate of change over an interval is ____________, the function is increasing and when the rate of change over an interval is ____________, the function is decreasing.

When the rates of change over an interval are increasing, the function is _________________ and when the rates of change over an interval are decreasing, the function is _____________________.