Key

A. The average monthly temperature in Grand Rapids can be modeled by the function $T(m) = 20.6\cos(0.5m + 2.3) + 51.8$ for $1 \le m \le 12$, where $T(m)$ is the temperature in °Fahrenheit and m is the month ($m = 1$ corresponds to January). Find the average rate of change in the average monthly temperature in Grand Rapids between February and May.

$$\frac{T(5) - T(2)}{5-2} \approx 7.382° \text{ per month}$$

B. Determine the number of real and imaginary zeros of $f(x) = x^5 - 3x^4 + 5x^3 + x^2 - 17x + 5$.

3 real, 2 imaginary

C. Carbon-14 dating is a method for determining the age of objects containing organic material. The half-life of Carbon-14 is generally accepted as 5730 years. How old is a fossilized tree sample found to have 35% of its original Carbon-14?

$0.35 = \left(\frac{1}{2}\right)^{t/5730}$ $\quad 5730 \log_{\frac{1}{2}}(0.35) = 8678.504$ years

D. Let $g(x) = 9\sin(2x) - 4$. Find all input values on the interval $[0, \pi]$ that yield an output of -2.

$x = 0.112$ and $x = 1.459$

E. An element is known to decay at a rate of 28% every 4 days. If after 7 days there are 65 grams remaining, how much of the element must there have been on day 0? Round to the nearest thousandth.

$A(t) = A_0(72)^{t/4}$ $\quad 65 = A_0(0.72)^{7/4}$ $\quad A_0 = \dfrac{65}{(0.72)^{7/4}} = \boxed{115.500 \text{ g}}$

F. Data about the population of a small town, in thousands, is given in the table.

Year	2000	2005	2010	2015	2020
Population (in thousands)	6.1	7.8	9.5	11.2	13.1

An exponential model for the data is given by $f(x) = 6.3114 \cdot 1.0385^x$ where $f(x)$ is the population, in thousands, and x is the number of years since 2000. For which years does the regression model provide an overestimate of the true population?

2000 and 2020 (residual is negative)

G. Let $f(x) = -4x^3 + 7x^2 - 2x + 1$. The graph of f has an inflection point at $x = 0.583$. For $(-\infty, 0.583)$ is the rate of change of f increasing or decreasing? Explain.

increasing; The graph of f is decreasing concave up, which means the rate of change is increasing.

H. The amount of money in an account earning continuously compounded interest can be modeled by $A(t) = A_0 e^{rt}$ where A_0 is the initial amount deposited, r is the interest rate, and t is the time in years. Suppose that $800 were deposited into such an account. After 10 years, the account held $1030. Assume that no additional deposits or withdrawals were made. Determine the interest rate of the account, to the nearest tenth of a percent.

$1030 = 800e^{10r}$ $\qquad \dfrac{\ln\left(\frac{1030}{800}\right)}{10} = r = 0.02527 \approx 2.5\%$

I. Let $f(x) = \sin(2.4x - 1)$ and $g(x) = 0.5 + f(x)$. On which subinterval(s) of $[-\pi, 0]$ is $g(x) > 0$?

$(-2.419, -0.674)$

J. Consider the graph of $h(x) = x^4 - 18x^2 - 32x - 15$.

a. At which x-value, if any, does h have a relative maximum? $x = -1$

b. Find the absolute maximum of h or explain why it does not exist.

Does not exist because h increases without bound as $x \to \pm\infty$

c. Find the absolute minimum of h or explain why it does not exist.

-198.285

K. A random sample of 8 families was selected. Each family was asked how many people are currently living in their household and how many gallons of milk the household drinks in a week. The results are given in the table.

Number of people in household, x	2	5	3	2	3	6	4	4
Gallons of milk consumed, y	0.5	4	2	3	1	7	6	2

$\hat{y} = -1.4 + 1.26577$

a. A linear regression is used to model these data. What is the expected increase in the number of gallons per milk consumed in a week by a household when an additional household member is added?

1.266 gallons per additional household member

b. What does the model predict will be the number of gallons of milk consumed by a household with 5 people in it, in one week?

4.929 gallons

c. Is the residual for $x = 5$ positive or negative? Explain what this means.

The residual is negative because actual − predicted $= 4 - 4.929 < 0$ so the linear regression gives an overestimate of the true value.

L. The average water temperature in Osaka, Japan experiences significant seasonal variation over the course of the year. The average water temperature in a given month m can be modeled by the function $T = 11\cos\left(\frac{\pi}{6}(m-8)\right) + 69$ for $1 \leq m \leq 12$. Let $m = 1$ correspond to the first month of the year. In which month(s) can a person living in Osaka expect the daily water temperature to be around 72 degrees?

(A) January and July
(B) March and November
(C) May and October
(D) February and August