Topic 1.4 Polynomial Functions and Rates of Change (Daily Video 1)

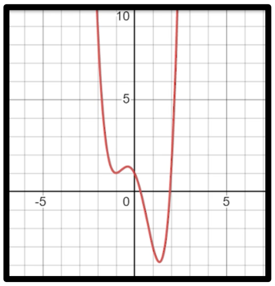
AP Precalculus

In this video, we will learn the vocabulary for polynomials and discuss maximums, minimums, global maximums, global minimums, and inflection points.

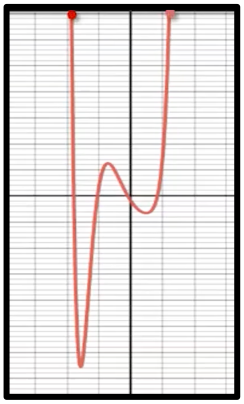
Definition of a Polynomial Function:	Example of a Polynomial Function
Let n be a nonnegative integer and let $a_n, a_{n-1}, a_{n-2}, \cdots, a_2, a_1, a_0$ be real numbers where $a_n \neq 0$. $$\boldsymbol{p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0}$$ then $\boldsymbol{p(x)}$ is a polynomial function in terms of $\boldsymbol{x}$ with degree $\boldsymbol{n}$.	$$\boldsymbol{f(x) = x^4 + 2x^3 - 4x^2 - x + 2}$$ The **leading term** is the term with the $\underline{\hspace{3cm}}$degree of the given variable and the **leading coefficient** is the coefficient of the leading term.

A polynomial function of degree n has at most $n-1$ turning points. ***Turning points*** are the relative (or local) maximums and the minimums on a graph. In other words, **turning points** are where a function switches from increasing to decreasing, or vice versa.

Example: Given $f(x) = x^4 - 3x^2 - 2x + 1$, how many turning points does the graph have? At most $\underline{\hspace{3cm}}$.



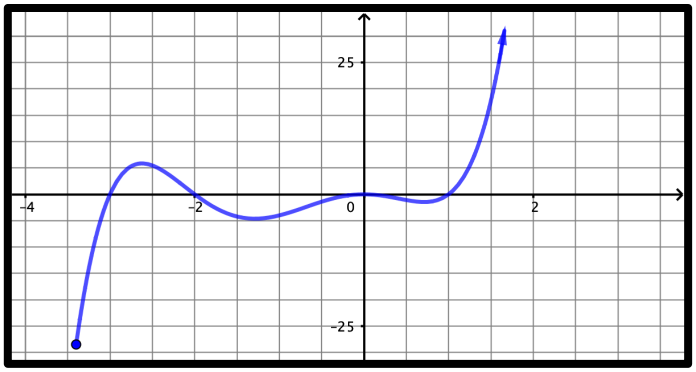
Relative Minimum	Relative Maximum
Where a graph switches from $\underbrace{\hspace{3cm}}_{\text{increasing/decreasing}}$ to $\underbrace{\hspace{3cm}}_{\text{increasing/decreasing}}$. This can also occur at an end point of a polynomial that has a restricted domain.	Where a graph switches from $\underbrace{\hspace{3cm}}_{\text{increasing/decreasing}}$ to $\underbrace{\hspace{3cm}}_{\text{increasing/decreasing}}$ This can also occur at an end point of a polynomial that has a restricted domain.
Global Minimum The lowest of all the minimums on a graph.	**Global Minimum** The highest of all the maximums on a graph.



Inflection Point: This occurs where the graph of a polynomial function changes from concave-up to concave-down or concave-down to concave-up. Occurs at input values where the rate of change of the function changes from increasing to $\underbrace{\hspace{3cm}}_{\text{increasing/decreasing}}$ or from decreasing to $\underbrace{\hspace{3cm}}_{\text{increasing/decreasing}}$

Let's Practice!

Problem #1: Given the graph to the right, which of the points are the following: Give approximate values.



1. Global maximum

2. The global minimum occurs at ...

3. The relative maximums occur at ...

4. The relative minimums occur at ...

5. Inflection points

Note: While the *x* values of the points indicated are where the extrema (minima; maxima) occur, the output (or *y*) value is considered the actual maximum or minimum.

Problem #2: Given the polynomial $f(x) = 2x^3 - 3x^2 - 8x + 1$

A. What is the degree of the polynomial? ______________

B. What is the leading coefficient? _________________

C. How many turning points does the graph have? _____________

What should we take away?

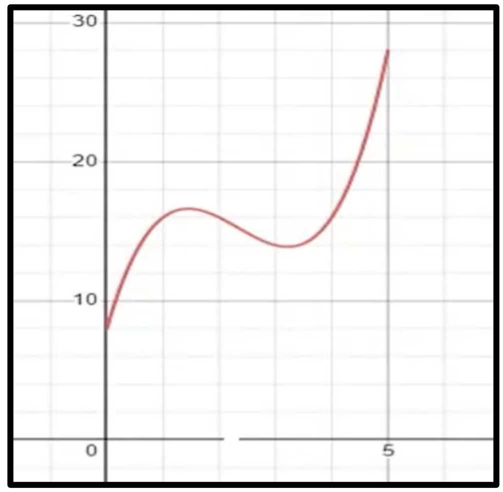
- The definition of a **polynomial**
- What the **leading term** is and what it tells us about the graph.
- The **key points on a polynomial:** maximums, minimums, and inflection points
- The difference between **relative** and **global**
- How to find the number of **turning points** on a graph

Topic 1.4 Polynomial Functions and Rates of Change (Daily Video 2)

AP Precalculus

In this video, we will explore how rates of change behave at different key points of a polynomial function.

Roller Coaster Polynomial

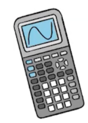


A portion of a roller coaster's path is modeled by the polynomial $h(t) = t^3 - 7t^2 + 14t + 8$, where t is time in seconds, and $h(t)$ is the height, in meters.

Where on the graph is the average rate of change of $h(t)$ **positive**? Choose any two points where the graph is **increasing**. Mark them on the graph and draw a line between them.

Where on the graph is the average rate of change of $h(t)$ **negative**? Choose any two points where the graph is **decreasing**. Mark them on the graph and draw a line between them.

What is the average rate of change between the relative maximum of the graph and the relative minimum of the graph? Use a graphing calculator to find the relative maximum and minimum points of $h(t) = t^3 - 7t^2 + 14t + 8$. You will need both coordinates of these points. What is the average rate of change between these two points? Show your work below.



What is the average rate of change between the absolute maximum of the graph and the absolute minimum of the graph? Show your work below.	One of the inflection points is at $(2.333, 15.259)$. What happens to the rate of change before that point and after that point? The rate of change approaching the inflection point is $\underbrace{\qquad\qquad\qquad}_{\text{increasing/decreasing}}$ ... Then, after inflection point, the rate of change begins to $\underbrace{\qquad\qquad\qquad}_{\text{increase/decrease}}$...

What should we take away?

- When a graph is ______________, its average rate of change is positive.
- When a graph is ______________, its average rate of change is negative.
- At an inflection point, the rate of change changes from _______________to _______________ or vice versa.

Topic 1.5 Polynomial Functions and Complex Zeros (Daily Video 1)

AP Precalculus

In this video, we will learn how to find the zeros (both real and complex) of a polynomial function.

Let's WARM UP!

Zeros of a Polynomial Function

If a is a complex number and $p(a) = 0$ then a is called a ____________ of p, or a ___________ of the polynomial function p.

If a is a real number, then $(x - a)$ is a ___________ factor of p if and only if a is a zero of p.

Example: $\boldsymbol{p(x) = x^3 - 4x}$

Factor:

$p(x) =$________________________________

Solve: $0 = x(x+2)(x-2)$

$x =$________________________

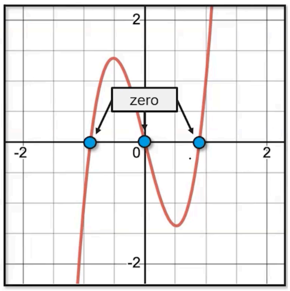
Check:

$p(0) =$________________________

$p(2) =$________________________

$p(-2) =$________________________

Looking at Zeros Graphically

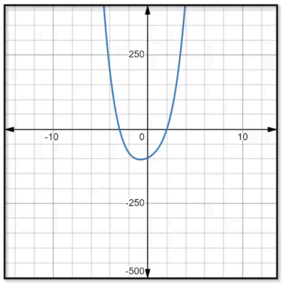


Remember that a real ____________of a function is a point at which the graph crosses the x-axis, which is called an ____________________.

Don't forget that there can be **nonreal zeros** that can't be seen on the graph. They come in **complex conjugate pairs**.

For example, if $x = 2 + 3i$ is a zero of the function, then its **complex conjugate** $x =$ ______________________ is also a zero.

Number of Complex Roots



The number of **complex roots** a polynomial has is equal to the __________________ of the polynomial.

Example: $\boldsymbol{f(x) = x^4 + x^3 + 10x^2 - 16x - 96}$

There should be _______ complex zeros. Using a graphing calculator, we find that zeros are

$x =$______ and $x =$______ , and two complex zeros $x =$_____ and $x =$_____.

Let's Find the Zeros of a Polynomial Function

Example 1: Find the zeros of

$$f(x) = x^3 + 2x^2 - 15x$$

Factor: $f(x) =$ _________________________

Set $f(x) = 0$: ________________________

Set the factors equal to zero and solve. Show your work in the space below.

Example 2: Find the zeros of

$$f(x) = x^4 + 2x^2 - 64$$

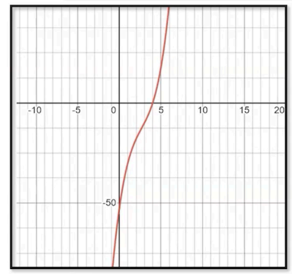
Factor: $f(x) =$ _________________________

Set $f(x) = 0$: ________________________

Set the factors equal to zero and solve. Show your work in the space below.

Example 3: Given the zero $x = 2 + 3i$, find the other zeros of the polynomial function

$f(x) = x^3 - 8x^2 + 29x - 52.$



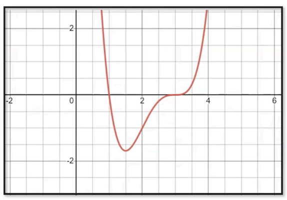
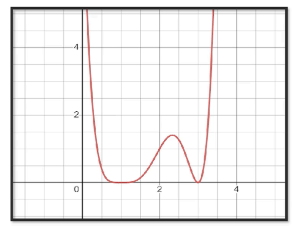
Find the second complex zero: $x =$________________

Graph the function to find the third zero: $x =$__________

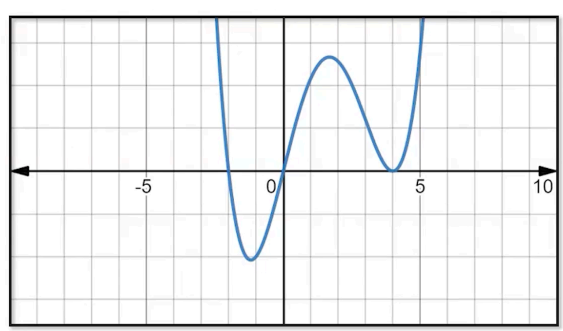
Write out all of the zeros: $x =$______; $x =$______; $x =$______

Multiplicty of a Zero

The multiplicty of a zero is how many times the zero's factor appears in a given polynomial.

<table>
<tr>
<td>If the zero's multiplicity is odd, then the graph will cross through the zero on the x-axis.

$f(x) = x^4 - 10x^3 + 36x^2 - 54x + 27$
Factor: $f(x) =$ ________________________
Set $f(x) = 0$: ________________________
Set the factors equal to zero and solve. Show your work in the space below.</td>
<td>If the zero's multiplicity is even, then the graph will be tangent to the x-axis at that point, because the signs of the output values are the same for input values near $x = a$.

$f(x) = (x-3)^2(x-1)^4$
Set $f(x) = 0$: ________________________
Set the factors equal to zero and solve. Show your work in the space below.</td>
</tr>
<tr>
<td>Multiplicty of a Zero Viewed Graphically

$f(x) = x(x-4)^2(x+2)$

Zeros: $x =$________ Multiplicity: ________
$x =$________ Multiplicity: ________
$x =$________ Multiplicity: ________</td>
<td>
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What should we take away?

- The highest power of a polynomial function's variable states how many zeros the function has.
- To find the zeros of a polynomial function, either ________ the polynomial (if possible) or use a graphing calculator.
- All complex zeros come in complex ____________ pairs.

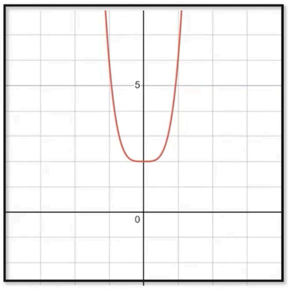
Topic 1.5 Polynomial Functions and Complex Zeros (Daily Video 2)

AP Precalculus

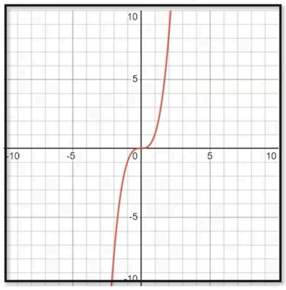
In this video, we will learn how to tell if a polynomial function is odd or even, practice finding zeros and matching graphs to their zeros.

What do *odd* and *even* mean in terms of a graph?

Even: The graph is symmetric over the *y*-axis or the line *x* = 0.



Odd: The graph is symmetric over the origin or the point $(0,0)$.



Even and Odd: Analytically

Even Symmetry	Odd Symmetry
$f(-x) = f(x)$	$f(-x) = -f(x)$
If substituting $(-x)$ for the variable and simplifying gives you the exact same signs as the original polynomial, then it has even symmetry.	If substituting $(-x)$ for the variable and simplifying gives you the exact opposite signs as the original polynomial, then it has odd symmetry.
Example: $f(x) = x^4 - 2x^2$	Example: $f(x) = 2x^3 - 2x$
Substituting: $f(-x) =$ ____________________	Substituting: $f(-x) =$ ____________________
Simplifying: $f(-x) =$ ____________________	Simplifying: $f(-x) =$ ____________________

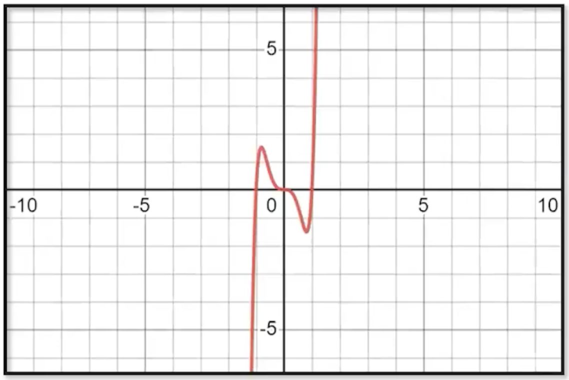
State Whether the Graph is Even or Odd

Is the function $f(x) = 5x^7 - 5x^3$ even or odd?
For it to be even, $f(-x) = f(x)$

Substituting: $f(-x) =$ ____________________

Simplifying: $f(-x) =$ ____________________

Is the function $f(x) = 5x^7 - 5x^3$ even or odd?
Circle the correct choice.



Practice Problem #1

Given $f(x) = (x-3)(x+2)^4(x+6)^3$, state the zeros, their multiplicity, and what the graph does at those points: **cross** through the *x*-axis or be **tangent** to the *x*-axis.

Zeros: $x =$________ Multiplicity: ________ Graph: ______________

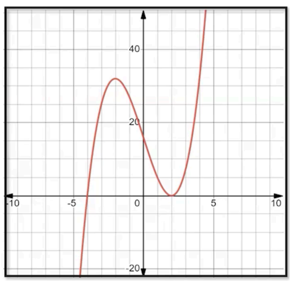
$x =$________ Multiplicity: ________ Graph: ______________

$x =$________ Multiplicity: ________ Graph: ______________

Practice Problem #2

Given the following graph of a polynomial equation, what is its equation?

Circle the correct choice.



A. $f(x) = x(x-4)(x+2)$

B. $f(x) = (x+4)(x-2)$

C. $f(x) = (x+4)(x-2)^2$

D. $f(x) = (x-2)(x+4)^2$

Practice Problem #3

Given the following table, what is the lowest-degree polynomial function it could represent?

A. Degree: 2

B. Degree: 3

C. Degree: 4

D. Degree: 1

x	y
–5	–24
–4	0
–3	6
–2	0
–1	–12
0	–24
1	–30
2	–24
3	0
4	48

Open-Response Practice Problem

Given the polynomial function: $f(x) = x^3 - 2x^2 - 4x - 16$. You are allowed to use your graphing calculator to graph $f(x)$.

A. How many complex zeros could the function have? Explain how you know below.

B. What is (are) the *x*-intercept(s) of the graph? ______________________

C. Are there any non-real zeros? Explain how you know below.

D. Does the graph of the polynomial have even or odd symmetry? _____________

Substituting: $f(-x) =$____

Simplifying: $f(-x) =$____

What should we take away?

The graph of a function has _________ symmetry when $f(-x) = f(x)$ and _________ symmetry when $f(-x) = -f(x)$.

Topic 1.6 Polynomial Functions and End Behavior (Daily Video 1)

AP Precalculus

In this video, we will learn explore how to know the end behavior of polynomial functions based on their equations.

Let's WARM UP!

Leading Term Practice

Find the leading term of the following polynomials.

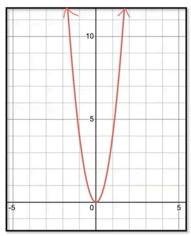
1. $f(x) = 3x^6 - 2x^3 - 4x + 1$ __________ 2. $f(x) = -2x^4 + x^3 - 4x^2 + 1x - 8$ __________

3. $f(x) = -5x^4 + x^2 + 4x^9 + 12x^8 + 2$ __________

Definition of End Behavior

The **end behavior** of a graph is what the graph is doing as the input values move to the: **right without bound** (to positive infinity) and the **left without bound** (to negative infinity).

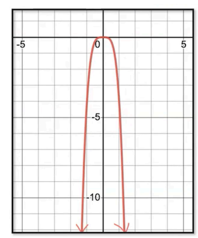
Graph the function $f(x) = 4x^2$



As **input values** of the nonconstant polynomial function increase without bound the ouput values ________________ without bound.

As **input values** of the nonconstant polynomial function decrease without bound the ouput values ________________ without bound.

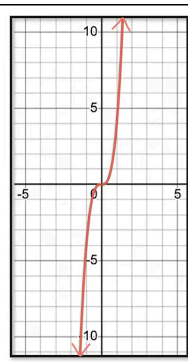
Graph the function $f(x) = -4x^2$



As **input values** of the nonconstant polynomial function increase without bound the ouput values ________________ without bound.

As **input values** of the nonconstant polynomial function decrease without bound the ouput values ________________ without bound.

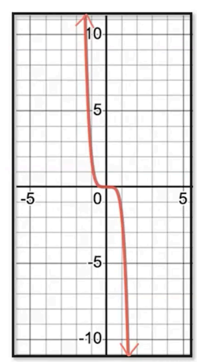
Graph the function $f(x) = 4x^3$



As **input values** of the nonconstant polynomial function increase without bound the ouput values ________________ without bound.

As **input values** of the nonconstant polynomial function decrease without bound the ouput values ________________ without bound.

Graph the function $f(x) = -2x^5$



As **input values** of the nonconstant polynomial function increase without bound the ouput values ________________ without bound.

As **input values** of the nonconstant polynomial function decrease without bound the ouput values ________________ without bound.

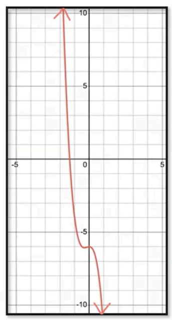
End Behavior Summary

Even-Powered Leading Term	Odd-Powered Leading Term
Even and positive: $x \to \infty$ $y \to \infty$ $x \to -\infty$ $y \to \infty$	**Odd and positive:** $x \to \infty$ $y \to \infty$ $x \to -\infty$ $y \to -\infty$
Even and negative: $x \to \infty$ $y \to -\infty$ $x \to -\infty$ $y \to -\infty$	**Odd and negative:** $x \to \infty$ $y \to -\infty$ $x \to -\infty$ $y \to \infty$

Let's look at an EXAMPLE!

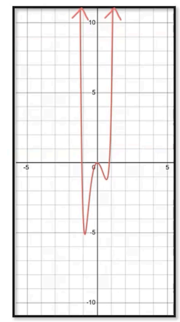
Given $f(x) = -4x^3 - 2x^2 - 6$ describe the end behavior of the graph of the polynomial.

- Leading Coefficient: _______
- Power: _______
- Sign: _______
- Answer: $x \to \infty$ $y \to$ ______
  $x \to -\infty$ $y \to$ ______

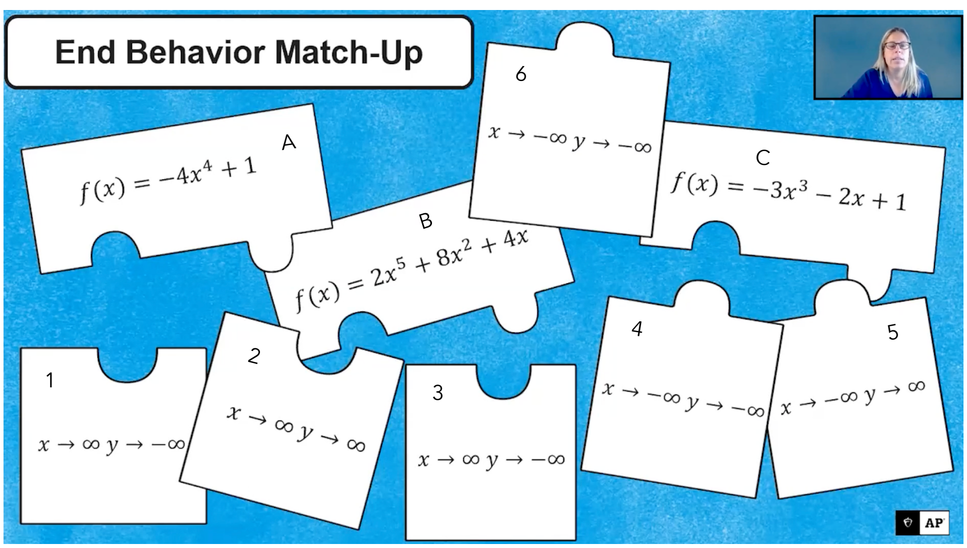


Given
$f(x) = -x^6 + 4x^3 - 6x^2 + 5x^8$
describe the end behavior of the graph of the polynomial.

- Leading Coefficient: _______
- Power: _______
- Sign: _______
- Answer: $x \to \infty$ $y \to$ ______
  $x \to -\infty$ $y \to$ ______



Let's PRACTICE!



What should we take away?

- If the leading term is even powered, then both ends of the graph go in the ___________. If the sign is ____________, both ends go up. If the sign is __________, both ends go down.
- If the leading term is odd powered, then the ends of the graph go in the _______________ directions. If the sign is _________, the left side goes down and the right side goes up. If the sign is __________, the left side goes up and the right side goes down.

Topic 1.7 Rational Functions and End Behavior (Daily Video 1)

AP Precalculus

In this video, we will learn explore how rational functions are expressed and what effect changes in the degrees of the numerator and denominator have on the function's end behavior.

Let's WARM UP!

Determine the degree of each of the following polynomials.

Example 1: $f(x) = 2x^4 - 7x^3 + 1$ degree = __________

Example 2: $f(x) = 2 + 3x$ degree = __________

Example 3: $f(x) = 5$ degree = __________

What is a rational function? A rational function is represented as a quotient of two polynomials. Let $f(x)$ and $g(x)$ represent polynomial functions. Then the rational function $r(x)$ is given by

$$r(x) = \frac{f(x)}{g(x)}, \text{where } g(x) \neq 0.$$

Your turn!

Determine which of the following represents a rational function. Circle the rational function(s).

$f(x) = 3x^{\frac{2}{3}} - 4x - 1$ $\qquad h(x) = \frac{2}{3x-1}$ $\qquad g(x) = \frac{3x^2 - 4x - 1}{3}$

End Behavior of Rational Functions

After examining the degree of the numerator and the degree of the denominator of a rational function, the following ideas can be used to determine the end behavior of the function.	• Numerator degree > Denominator degree The end behavior will mirror the polynomial of the resulting quotient of __________ __________. • Numerator degree = Denominator degree The end behavior approach the __________ asymptote determined by the ratio of the leading terms. • Numerator degree < Denominator degree The end behavior approach the horizontal asymptote ________.

Let's look at an EXAMPLE!

$$f(x) = \frac{x^3 + 1}{4x^2 + 5x + 1}$$

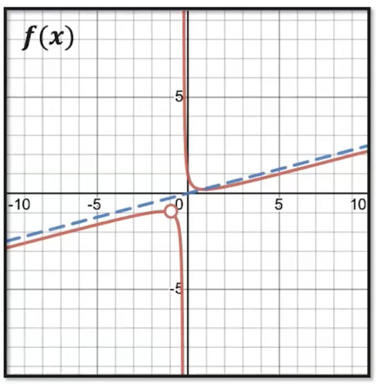
Degree of the numerator: ________

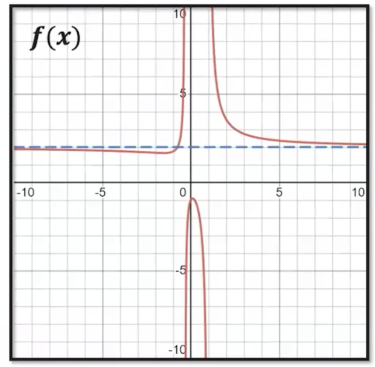
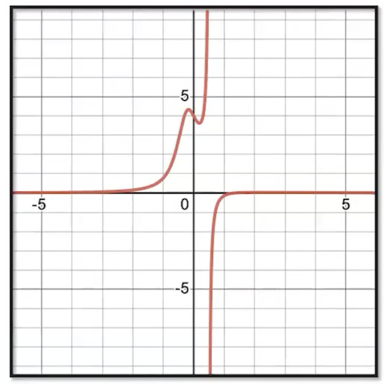
Degree of the denominator: ________

Quotient of Leading Terms: $\frac{x^3}{4x^2} =$ __________

End Behavior: $\lim_{x \to -\infty} f(x) =$ _________

$\lim_{x \to \infty} f(x) =$ _________



$f(x) = \dfrac{6x^2+1}{3x^2-2x-1}$ Degree of the numerator: ________ Degree of the denominator: ________ Ratio of Leading coefficients: $\dfrac{6}{3} =$ ________ End Behavior: $\lim_{x \to -\infty} f(x) =$ ________ $\lim_{x \to \infty} f(x) =$ ________	
$f(x) = \dfrac{3x+1}{8x^3-1}$ Degree of the numerator: ________ Degree of the denominator: ________ End Behavior: $\lim_{x \to -\infty} f(x) =$ ________ $\lim_{x \to \infty} f(x) =$ ________	

What should we take away?

- Numerator degree > Denominator degree
  The end behavior will mirror the polynomial of the resulting quotient of leading terms.
- Numerator degree = Denominator degree
  The end behavior approach the horizontal asymptote determined by the ratio of the leading terms.
- Numerator degree < Denominator degree
  The end behavior approach the horizontal asymptote $y = 0$.

Topic 1.7 Rational Functions and End Behavior (Daily Video 2)

AP Precalculus

In this video, we will learn explore about the connected relationship between a function's end behavior and the asymptotes of rational functions.

Limits at Infinity-Horizontal Asymptotes

Example 1: $F(x) = \dfrac{x-4}{6x^2-1}$

For $F(x)$, as the input values increase without bounds, what happens to the output values?

$$\lim_{x\to\pm\infty} \frac{x-4}{6x^2-1} = \lim_{x\to\pm\infty}\left(\frac{\frac{1x}{x^2}-\frac{4}{x^2}}{\frac{6x^2}{x^2}-\frac{1}{x^2}}\right) = \underbrace{\lim_{x\to\pm\infty}\left(\frac{\frac{1}{\square}-\frac{4}{x^2}}{\square-\frac{1}{x^2}}\right)}_{\text{Simplify,fill in the boxes}} = \frac{\square-\square}{\square-\square} = \underline{\qquad\qquad}$$

The horizontal asymptote is at $y = 0$.

Example 2: $F(x) = \dfrac{8x^2+3x+4}{2x^2-3x-1}$

For $F(x)$, as the input values increase without bounds, what happens to the output values?

$$\lim_{x\to\pm\infty} \frac{8x^2+3x+4}{2x^2-3x-1} = \lim_{x\to\pm\infty}\left(\frac{\frac{8x^2}{x^2}+\frac{3x}{x^2}+\frac{4}{x^2}}{\frac{2x^2}{x^2}-\frac{3x}{x^2}-\frac{1}{x^2}}\right) = \underbrace{\lim_{x\to\pm\infty}\left(\frac{8+\frac{3}{\square}+\frac{4}{\square}}{2-\frac{3}{\square}-\frac{1}{\square}}\right)}_{\text{Simplify,fill in the boxes}} = \frac{\square-\square-\square}{\square-\square-\square} = \underline{\qquad}$$

The horizontal asymptote is at $y =$_______.

Let's Practice! Determine the limits of each of the following functions, then determine the equation of the horizontal asymptote(s), if any.

Example 1:	Example 2:
$F(x) = \dfrac{x^3-4x^2+3x-5}{3x^3+x^2+3x-4}$	$F(x) = \dfrac{x+4}{5x^2-6x-4}$
$\lim_{x\to\pm\infty} F(x) =$ ______	$\lim_{x\to\pm\infty} F(x) =$ ______
Horizontal Asymptote: $y =$________	Horizontal Asymptote: $y =$________

What should we take away?

Determining the end behavior of a rational function by finding limits at infinity can lead to determining the ______________ asymptote(s) of the given function.

Topic 1.8 Rational Functions and Zeros (Daily Video 1)

AP Precalculus

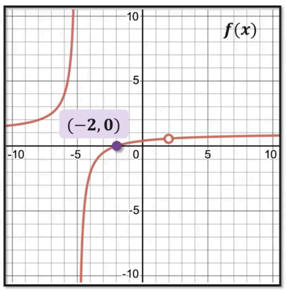
In this video, we will learn how to find the zeros of a rational function and determine the intervals of positive, negative, or undefined output values.

Let's look at an EXAMPLE!

Example 1: Find the zeros of the rational function $f(x)$. First factor the rational function and then simplify.

$$f(x) = \frac{x^2 - 4}{x^2 + 3x - 10}$$

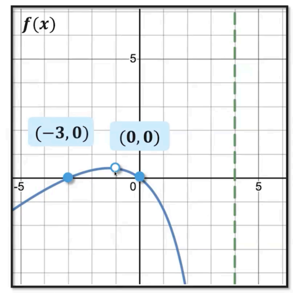
Find the zero(s) of $f(x)$.



Example 2: Find the zeros of the rational function $f(x)$. First factor the rational function and then simplify.

$$f(x) = \frac{x^3 + 4x^2 + 3x}{x^2 - 3x - 4}$$

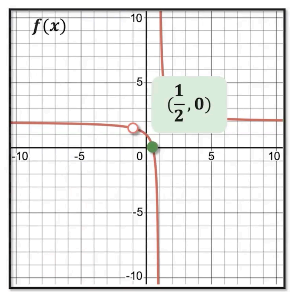
Find the zero(s) of $f(x)$.



Let's PRACTICE!

Find the zero(s) of $f(x)$.

$$f(x) = \frac{2x^2 + x - 1}{x^2 - 1}$$

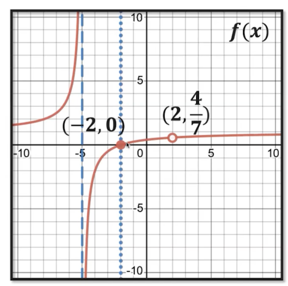


Let's look at an EXAMPLE!

The zeros of both the numerator and denominator of a rational function, $f(x)$, create intervals that satisfy the inequalities $f(x) \geq 0$ or $f(x) \leq 0$.

$$f(x) = \frac{x^2 - 4}{x^2 + 3x - 10} = \frac{(x-2)(x+2)}{(x-2)(x+5)}$$

To determine the **intervals** where $f(x)$ is positive or negative, analyze the sign of each factor at an *x*-value in the interval to determine the sign of the final output. The first one has been done as an example.



Over the interval $x < -5$, $f(x) > 0$ because using

$$x = -6 \Rightarrow \frac{(-6-2)(-6+2)}{(-6-2)(-6+5)} \Rightarrow \frac{(-)(-)}{(-)(-)} \Rightarrow +$$

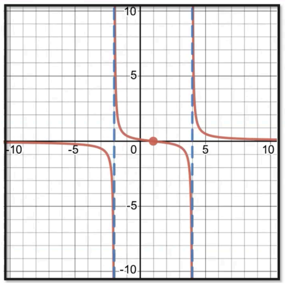
Over the interval $-5 < x < -2$, $f(x) < 0$ because using $x =$ _____ $\Rightarrow$

Over the interval $-2 \leq x < 2$, $f(x) \geq 0$ because using $x =$ _____ $\Rightarrow$

Over the interval $x > 2$, $f(x) > 0$ because using $x =$ _____ $\Rightarrow$

Let's PRACTICE!

Find the zero(s) of both the numerator and denominator of $f(x)$. Then determine the **intervals** where $f(x)$ is positive or negative.



$$f(x) = \frac{x-1}{x^2-2x-8}$$

Over the interval $x < -2,\ f(x) < 0$ because using $x =$ _____ $\Rightarrow$	Over the interval $1 \le x < 4,\ f(x) \le 0$ because using $x =$ _____ $\Rightarrow$
Over the interval $-2 < x \le 1,\ f(x) \ge 0$ because using $x =$ _____ $\Rightarrow$	Over the interval $x > 4,\ f(x) > 0$ because using $x =$ _____ $\Rightarrow$

What should we take away?

- Finding zeros of a rational function requires simplifying rational functions, then finding the zeros of the resulting polynomial in the numerator.
- Zeros of the numerator and the denominator of rational functions can identify endpoints and/or asymptotes of intervals of positive and negative function values for the rational function.

Topic 1.9 Rational Functions and Vertical Asymptotes (Daily Video 1)

AP Precalculus

In this video, we will investigate how to determine the vertical asymptote(s) of rational functions.

Let's Warm up!

Find the real zeros of the following rational functions:

$f(x) = \dfrac{x^2 + 2x}{x^2 - 4x - 5}$	$f(x) = \dfrac{x^2 - x - 6}{x + 2}$

Reminder: The real zeros of a rational function correspond to the real zeros of the numerator for the values in the function's domain.

Let's look at an EXAMPLE!

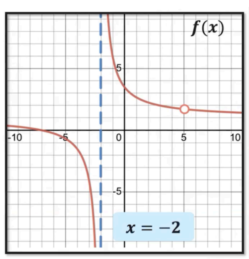
Find the vertical asymptote(s) of the given rational function.

Example 1:

$f(x) = \dfrac{x^2 + 2x - 35}{x^2 - 3x - 10} =$

Real zero of denominator: ___________

Vertical asymptote at ____________



Example 3:

Determine $\lim_{x \to -2^-} f(x)$ and $\lim_{x \to -2^+} f(x)$.

$\lim_{x \to -2^-} f(x) =$________ $\lim_{x \to -2^+} f(x) =$________

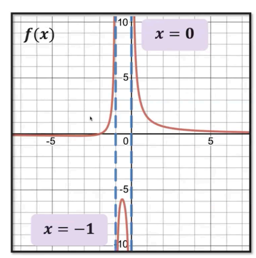
Example 2:

$f(x) = \dfrac{x^2 + 3x + 2}{x^3 + 2x^2 + x} =$

Real zeros of numerator: ________________

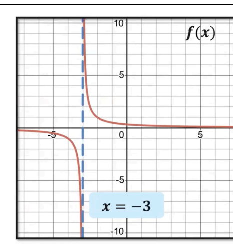
Real zeros of denominator: ________________

Vertical asymptote at ____________



Let's PRACTICE!

Find the vertical asymptote(s) of the given rational function $f(x)$. Then determine $\lim_{x \to -3^-} f(x)$ and $\lim_{x \to -3^+} f(x)$.



$f(x) = \dfrac{x - 8}{x^2 - 5x - 24} =$

Vertical asymptote at ____________ $\lim_{x \to -3^-} f(x) =$ _________ $\lim_{x \to -3^+} f(x) =$ ____________

What should we take away?

Finding vertical asymptotes of a rational function requires examining the real zeros unique to the ____________________ and the behavior of the output values of a rational function near a vertical asymptote either increase or decrease _________________________.

Topic 1.10 Rational Functions and Holes (Daily Video 1)

AP Precalculus

In this video, we will compare the multiplicity of zeros in the numerator and denominator of a rational function in order to identify and determine holes in the graph of the function.

Let's Warm up!

Determine the zeros and their multiplicity of the following polynomial.

$$F(x) = (x+3)(x-2)^2(x+1)^3$$

Zero: _______ Multiplicity _____ Zero: _______ Multiplicity ______ Zero: ______ Multiplicity ______

Let's look at an EXAMPLE and PRACTICE!

Determine where the function $f(x)$ has a hole in its graph.

Example: Determine the y-coordinate of the hole in $f(x)$.

$$f(x) = \frac{x^2+2x-35}{x^2-3x-10} =$$

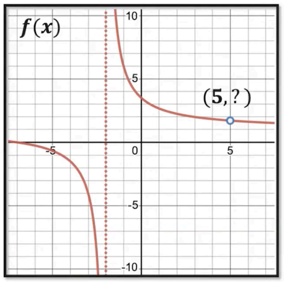
Real zeros of numerator: __________

Real zeros of denominator: __________

$$\lim_{x\to 5}\frac{x^2+2x-35}{x^2-3x-10}$$

$$= \lim_{x\to 5}\frac{x+7}{x+2} = \_\_\_\_\_\_\_\_$$

Coordinates of the hole:___________



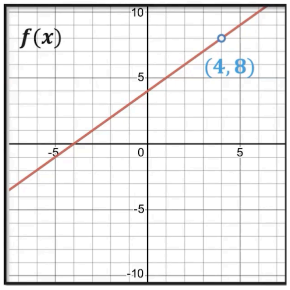
PRACTICE: Determine where the function $f(x)$ has a hole and then determine the y-coordinate of the hole.

$$f(x) = \frac{x^2-16}{x-4} =$$

Hole at __________

$$\lim_{x\to 4}\frac{x^2-16}{x-4} = \lim_{x\to 4}(x+4) = \_\_\_\_\_\_\_\_$$

Coordinates of the hole:___________



What should we take away?

- Finding the location of holes in the graph of a rational function requires examining the common zeros of the polynomials in both the ______________ and ________________.
- The y-coordinate of a hole can be determined by examining the limiting behavior of a function's output values arbitrarily close to the ________________ of the hole.

Topic 1.11 Equivalent Representations of Polynomial and Rational Expressions (Daily Video 2)

*Note: Video 2 should actually be Video 1. Videos 1 and 2 are incorrectly labeled on AP Classroom.

AP Precalculus

In this video, we will compare dividing a polynomial by a linear factor with dividing an integer by a smaller integer.

Long Division Warm up! Divide 425 by 12 and show your work.

425 ÷ 12	Steps
$12\overline{)4\,2\,5}$	• Put 12 on the outside and 425 on the inside. • Move from left to right. • Use the fewest digits possible at a time to divide, then subtract. • Bring down the next digit. • Repeat the process. • Once you can't go any further, place the remainder over the divisor.

Let's PRACTICE!

Polynomial Division

$(3x^2 + 7x + 55) \div (x + 2)$	Steps
$x + 2\overline{)3x^2 + 7x + 55}$	• Put $(x + 2)$ on the outside and $(3x^2 + 7x + 55)$ on the inside. • Move from left to right. • Use the fewest terms possible at a time by using the same number of terms. • Make the first terms match. • Subtract. • Bring down the next term, and repeat. • Place remainder over divisor.

What should we take away?

Dividing a polynomial by a linear factor is like dividing an integer by a smaller integer.

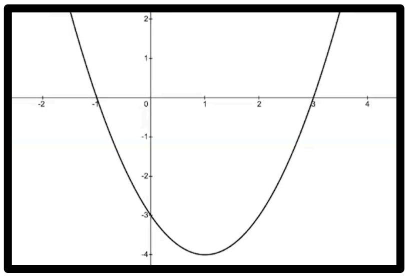
Topic 1.11 Equivalent Expressions of Polynomials and Rational Functions

(Daily Video 1) *Note: Video 1 should actually be Video 2. Videos 1 and 2 are incorrectly labeled on AP Classroom.

AP Precalculus

In this video, we will review how to convert polynomial and rational functions from standard form to factored form and from factored form to standard form.

Let's Warm up!



Basic Characteristics of a Polynomial Function

y-intercept: ______________ *x*-intercept: ____________________

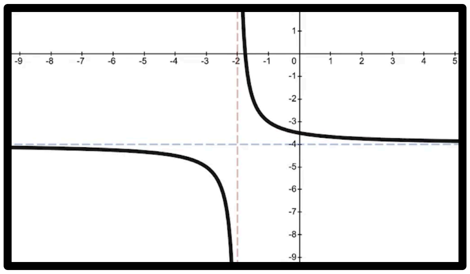
Zeros: __________________ Axis of symmetry: ______________

Domain: ________________ Range: ________________

$f(x)$ in factored form: ________________________ $f(x)$ in standard form: ________________________

Example: Write $f(x) = x^3 - x^2 - 2x$ in factored form.

Basic Characteristics of a Rational Function



y-intercept: __________ horizontal asymptote: ___________

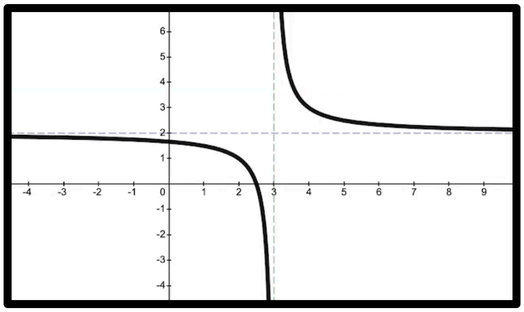
Zeros: ______________ vertical asymptote: _____________

Domain: ____________________ Range: __________________

$f(x)$ in standard form: Change to factored form.	$f(x)$ in factored form: Change to standard form.

Let's PRACTICE!

What is the equation of this function in factored form?



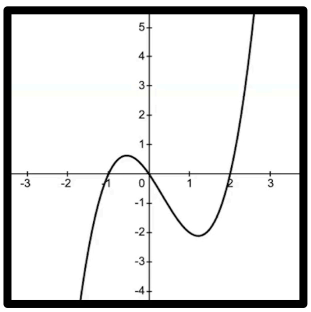
A) $f(x) = \dfrac{1}{x-3} + 2$

B) $f(x) = \dfrac{1}{x-2} + 3$

C) $f(x) = \dfrac{3x-5}{x-2}$

D) $f(x) = \dfrac{2x-5}{x-3}$

What is a possible equation of this function standard form?



Write the following function in standard form.

$$f(x) = \frac{2x-9}{x-3}$$

What should we take away?

We should be able to change ________________ functions and ________________ functions from ______________ form to ______________ form and vice versa.