Topic 2.1 Change in Arithmetic and Geometric Sequences (Daily Video 1)

AP Precalculus

In this video, we will explore two different methods to express both arithmetic and geometric sequences and discuss which method is best in a given situation.

Let's REVIEW!

Arithmetic Sequences	Geometric Sequences
• A sequence of numbers in which the **difference** between any two consecutive numbers is a constant d. • The n^{th} term, a_n, in the sequence can be expressed by: $a_n = a_1 + d(n-1)$.	• A sequence of numbers in which the **ratio** between any two consecutive numbers is a constant r. • The n^{th} term, a_n, in the sequence can be expressed by: $a_n = a_1(r^{n-1})$.
Observe: $2, 7, 12, 17, 22, 27, \ldots$ The value of $d =$____. The first term $a_1 =$ _____ To get from the first term to the fourth term, we add _______ to the first term _______ times. Write an equation, in terms of d, for the fourth term. $a_4 = 2 +$ _________________	**Observe:** $2, 6, 18, 54, 162, \ldots$ The value of $r =$____. The first term $a_1 =$ _____ To get from the first term to the fifth term, we multiply _______ times the first term _______ times. Write an equation, in terms of r, for the fifth term. $a_5 = 2 \cdot$ _________________
The general (or generic) term a_n of an arithmetic sequence with a common difference d is given by $a_n = a_k +$_____________, where a_k is the k^{th} term.	The general (or generic) term a_n of a geometric sequence with a common ratio r is given by $a_n = a_k \cdot$ _____________, where a_k is the k^{th} term.

Let's look at an EXAMPLE!

*Note: The presenter incorrectly wrote a sum instead of a sequence.

Is the sequence $-5, -\frac{9}{2}, -4, -\frac{7}{2}, -3, \ldots$ arithmetic or geometric? Justify your answer. Use the general equation $a_n = a_k + d \cdot (n-k)$ to find the eighth term of the sequence. Show how you arrived at your answer.	Is the sequence $1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \ldots$ arithmetic or geometric? Justify your answer. Use the general equation $a_n = a_k \cdot r^{n-k}$ to find the eighth term of the sequence. Show how you arrived at your answer.

What should we take away?

We should be able to recognize whether a sequence is arithmetic or geometric and write an equation for a general term of both.

Topic 2.1 Change in Arithmetic and Geometric Sequences (Daily Video 2)

AP Precalculus

In this video, we will analyze functions that represent arithmetic and geometric sequences.

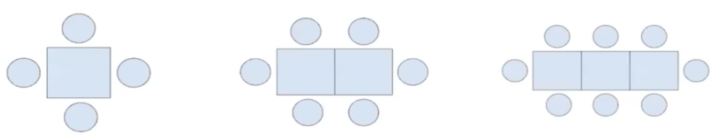
Let's REVIEW!

Arithmetic Sequences	Geometric Sequences
• A sequence of numbers in which the ____________ between any two consecutive terms is a constant. • A constant, often denoted d, can be _______ to any term to get the next term in the sequence. • Can be expressed, in general, as an equation: _______________________	• A sequence of numbers in which the ____________ of any two consecutive terms is a constant. • A constant, often denoted r, can be _____________ by any term to get the next term in the sequence. • Can be expressed, in general, as an equation: _______________________

Let's look at an EXAMPLE!

Example 1: Is the sequence arithmetic or geometric?

Four people can sit around a square table. If two tables are put together, 6 people can sit around them. If three tables are put together, 8 people can sit around them.

If we count the number of people around the table, we get the first few terms of the sequence: 4, 6, 8… 	This sequence is _______________ with $a_1 =$ ____ and $d =$ _____. Use the equation $a_n = a_k + (n - k)d$ to find the 8th term, a_8.

Example 2: Is the sequence arithmetic or geometric?

A ball is dropped from the roof of a house that is 27 feet above the ground. Each time the ball bounces, it reaches a height that is $\frac{1}{3}$ of the height from which it last fell.

The first few heights of the ball are 27, 9, 3… Use these terms to explain why this <u>cannot</u> be an arithmetic sequence.	This is a geometric sequence with a first term of $a_0 =$ ____ and $r =$ ____. Using $a_n = a_k(r^{n-k})$ find how high the ball will be after its 6th bounce.

What should we take away?

Equations can be written for arithmetic and geometric sequences by using a starting term and either a common difference (if _____________) or the common ratio (if ______________).

Topic 2.2 Change in Linear and Exponential Functions (Daily Video 1)

AP Precalculus

In this video, we will compare linear and exponential growth and practice representing these growth patterns with function formulas and graphs.

Let's WARM UP!

Imagine two fast growing vines that sprouted from Jack's magical beans. When you begin measuring the vines, they are both 10 feet long. For the next 4 days, Vine A's length increases 50 feet each day, while Vine B's length doubles each day. The tables that follow show the changes in length for both vines.

Δt	Number of days since Vine A was 10 feet long. t	Vine A's length (in feet) $f(t)$	$\Delta f(t)$
	0	10	
1			
	1	60	
1			
	2	110	
1			
	3	160	
1			
	4	210	

Number of days since Vine B was 10 feet long t	Vine B's length (in feet) $g(t)$
0	10
1	20
2	40
3	80
4	160

Vine A's length on day t is $f(t) =$ __________

Whenever t changes by 1 day, Vine A's length changes by ____ feet.

Whenever t changes by Δt days, Vine A's length changes by ________ feet.

Vine B's length on day t is $g(t) =$ __________

Whenever t changes by 1 day, Vine B's length becomes ____ times as long.

Whenever t days have passed, Vine B's length is ___ times as long as its starting length.

Let's look at another EXAMPLE!

Which vine will be longer 3 days after the vines are 10 feet long ($t = 3$)?

Vine A	Vine B	Vine Length Comparison
$f(3) =$	$g(3) =$	Vine A is 160 – 80 = 80* feet longer * The video has an error here.

Which vine will be longer 30 days after the vines are 10 feet long ($t = 30$)?

Vine A	Vine B	Vine Length Comparison
$f(30) =$	$g(30) =$	

Anytime the growth factor is greater than 1, _______________ growth will always outgrow linear or polynomial growth.

Let's PRACTICE!

<table>
<tr><td colspan="2">If Vine B's length grows by only 5% each day (instead of 100% or doubling) will Vine B ever become larger than Vine A?

Let's compare their lengths after 180 days (about 6 months).</td></tr>
<tr><td>Linear growth (Vine A)
$f(180) =$______________ = ___________

After 180 days, Vine B is about _________ feet longer than Vine A.</td><td>Exponential Growth (Vine B)
The growth factor is ________
$g(t) =$ ________________
$g(180) =$ _______________ $\approx$ ________</td></tr>
</table>

What should we take away?

- Linear functions have a constant ________ of change.
- Exponential functions have a constant __________ of change.
- If a function is growing exponentially (growth factor > 1), equal changes in the independent variable produce larger and larger changes in the dependent variable. In the long run, exponential growth (growth factor > 1), will always outpace _________ and other __________________ functions.

Topic 2.2. Change in Linear and Exponential Functions (Daily Video 2)

AP Precalculus

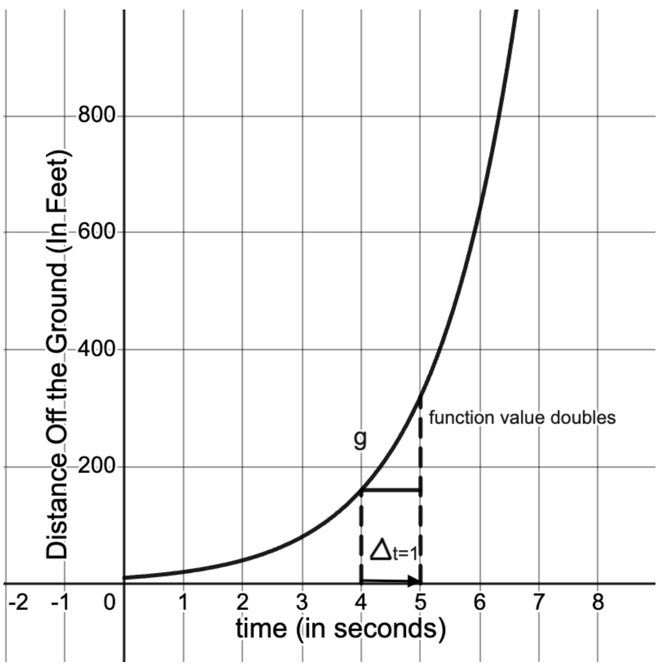
In this video, we will explore patterns in linear and exponential growth by contrasting ideas of a constant percent change with a constant amount of change.

Let's WARM-UP!

Recall that: If two quantities are changing together at a constant rate of change, ______ changes in one quantity results in a _____________ change in the other.

Let's REVIEW!

Let's examine the graph of g, which models the distance of a rocket from the ground (initially 10 feet off the ground), if its distance off the ground is doubling every second since liftoff.



What is the function for the distance the rocket is off the ground after t seconds?

$g(t) =$ ______________

When t days have passed, the rocket's distance from the ground is _____ times as far as its starting length.

The distance off the ground for another rocket is modeled by the function $f(t) = 15(4^t)$.

The rocket's distance from the ground ______________ or increases by _____% every second.

Describe the distance off the ground in terms of the number of seconds, t, since liftoff.

$g(t) = 5(1.3^t)$ The rocket is initially ____ feet off the ground and its distance from the ground increases by ___% or grows by a factor of _____ every second.	$h(t) = 12(1.07^t)$ The rocket is initially ____ feet off the ground and its distance from the ground increases by ___% or grows by a factor of _____ every second.

Let's look at an EXAMPLE!

Another rocket is 30 feet off the ground when it lifts from its launching site. Its distance from the ground $g(t)$ doubles every 3 seconds.

Show how to find the 1-unit growth factor, *b*.

What is the function that gives the height of the rocket after t seconds?

*Note: Video states t years, instead of seconds.

Number of seconds since the rocket lifted off t	Distance off the ground $g(t)$
0	30
1	?
2	?
3	60
6	120

(+ 1 between consecutive t values; × b between consecutive $g(t)$ values from 0 to 3, × 2 from 0 to 3 and from 3 to 6)

Let's PRACTICE!

Exploring Growth Patterns: Water levels in lakes in the western United States are dropping. If the recorded water level of a lake in this region was 1215 feet in 2015 and 1042 in 2020, what methods might you use to project what the water level will be in 2035?

Let t represent the number of years since 2015. Let l represent the water level of the lake in feet.

Using the points (0, 1215) and (5, 1042) show how to find the linear and exponential growth rates.

Δt	Number of years, t, since 2015	Water level l (in feet)	Δl
	0	1215	
1			
1			
1			
1			
1	5	1042	
15	20	?	

What should we take away?

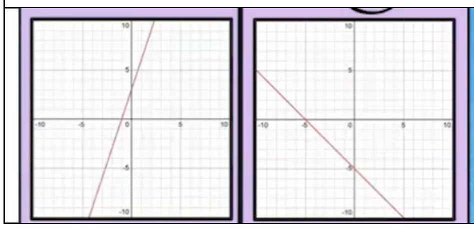
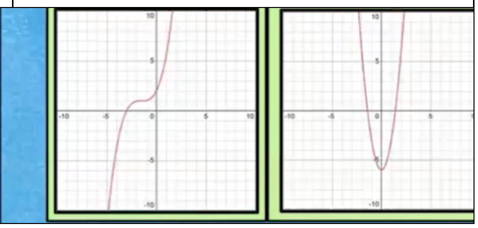
- If two quantities are related _________ the changes in the two quantities are ___________ and the corresponding changes in the two quantities' values remain _________ as the two quantities' values vary together.
- If two quantities are related _____________ the value of the dependent quantity changes by a constant factor b for every one unit increase in the value of x.
  The value of the dependent quantity gets larger if ____________ and the value of the dependent quantity gets smaller if ________.

Topic 2.3 Exponential Functions (Daily Video 1)

AP Precalculus

In this video, we will learn the key characteristics of an exponential function, including general form, growth/decay, and domain/range.

Let's WARM-UP!

Linear Function	Exponential Function	Neither
		

Based on the pictures, write a sentence or two to explain how exponential functions look compared to non-exponential functions.

Let's REVIEW!

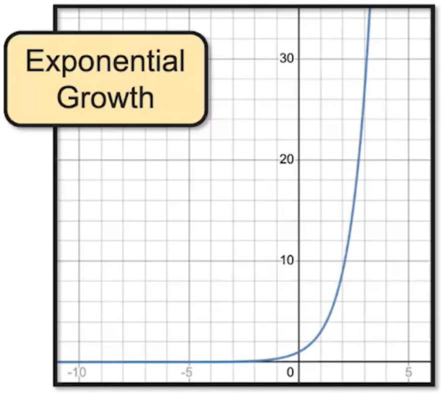
An exponential function has the form _______________, where __________ and ________.	
What is the base of each of the following functions? $f(x) = 2^x$ base = _____ $f(x) = \left(\frac{1}{2}\right)^x$ base = _____ $f(x) = 3e^{2x}$ base = _____ e is called ________________and is about _______.	Science and economics applications often involve exponential functions. Example: If bacterial doubles every hour, you would have _____ bacterial after x hours, written as $f(x) =$ ____.

Let's REVIEW!

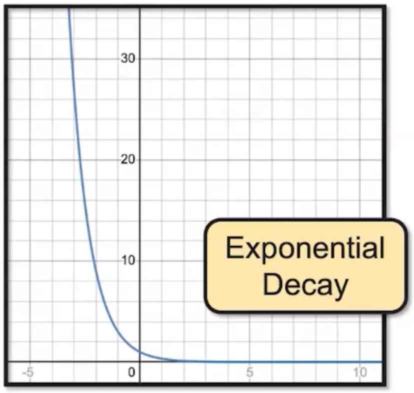
Exponential Growth	Exponential Decay
• A quantity ___________slowly in the beginning, then there is _________ increase. • Used to model population growth, compound interest, doubling time, etc. • The graph is always ________________. • The base, b, is always greater than ____. • The domain is __________. • The range is ________.	• A quantity ___________ rapidly in the beginning and then there is a ______ decrease. • Used to model population decay, find half-life, etc. • The graph is always ________________. • The base, b, is between ____ and ____. • The domain is __________. • The range is _______

Both exponential growth and exponential decay can be written in the form $f(x) = ab^x$, where a is the initial value.

Graph $f(x) = 3^x$



Graph $g(x) = \left(\frac{1}{3}\right)^x$ or $g(x) = (3)^{-x}$



Let's PRACTICE!

A function that has exponential growth has a base (multiplier) that is ____________.

A) Equal to 1 B) between 0 and 1 C) greater than 1 D) less than 0

Given $f(x) = 3^x$, choose the best representation of the domain.

A) $(0, \infty)$ B) $(-\infty, \infty)$ C) $(-\infty, 0)$ D) (-3, 3)

Given $f(x) = 2^x$, choose the best representation of the range.

A) $(0, \infty)$ B) $(-\infty, \infty)$ C) $(-\infty, 0)$ D) (-2, 2)

What should we take away?

General form

- $f(x) = b^x$, where $b > 0$ and $b \neq 1$

Growth and decay

- **Growth**: $f(x) = a \cdot b^x$, where $b > 1$
- **Decay**: $f(x) = a \cdot b^x$, where $0 < b < 1$

Domain and range

- **Domain**: $(-\infty, \infty)$
- **Range**: $(0, \infty)$

Topic 2.4 Exponential Function Manipulation (Daily Video 1)
AP Precalculus

In this video, we will learn the properties of exponential expressions and how to use them to rewrite the expressions in equivalent forms.

Let's WARM-UP!

Explain why $(-7)^4$ is not equivalent to $-(7)^4$. Use the words base and power in your response.

Let's Look at an EXAMPLE!

Complete each table and then write the general rules for exponents in the final row of the table.

Original Product	Exponential Form
$2^2 \cdot 2^3$	
$3^4 \cdot 3^2$	
$4^6 \cdot 4^2$	
$x^2 \cdot x^4$	
$x^a \cdot x^b$	

Original Product	Exponential Form
$\frac{3^5}{3^2}$	
$\frac{2^4}{2^1}$	
$\frac{5^{12}}{5^8}$	
$\frac{4^7}{4^5}$	
$\frac{x^4}{x^2}$	
$\frac{x^a}{x^b}$	

Original Product	Exponential Form
$(2^2)^3$	
$(4^5)^3$	
$(x^3)^4$	
$(2 \cdot 3)^3$	
$(3 \cdot 4)^3$	
$(xy)^3$	
$(x^a)^b$	
$(xy)^b$	

Let's PRACTICE!

Use the properties of exponents to rewrite each expression in simplified form. Show how you determined your answer.

1. $2m^4n^2 \cdot 4nm^2$

2. $\frac{4x^3y^4}{3xy^3}$

3. $2x(x^4y^4)^4$

4. $x^2y^4 \cdot xy^2$

5. $\frac{xy^3}{4xy}$

6. $\left(\frac{(2x)^3}{x^3}\right)^2$

7. $(2u^3v^4)^2$

8. $\frac{2a^2b^2a^7}{(ba^4)^2}$

9. $\left(\frac{2ba^7 \cdot 2b^4}{ba^2 \cdot 3a^3b^4}\right)$

What should we take away?

Exponential Properties used to rewrite equivalent expressions:

Product:

Power:

Quotient:

Topic 2.4 Exponential Function Manipulation (Daily Video 2)

AP Precalculus

In this video, we will learn equivalent representations of exponential expressions, including negative and fractional exponents.

Let's Look at an EXAMPLE!

Zero Property of Exponents:

Explain why $x^0 = 1$.

Note: 0^0 is ________________.

A negative exponent conveys the number of times to multiply the ____________ of the base.

Example: $7^{-2} =$ _________

Write each expression with a single, positive exponent.

a. 2^{-1}	b. $\left(\frac{1}{3}\right)^{-1}$	c. x^{-3}	d. $(2+4x)^{-2}$

Rational (Fractional) Exponents

$x^{\frac{1}{n}} =$ _______ $\qquad$ $x^{\frac{m}{n}} =$ _______

a. $8^{\frac{2}{3}}$	b. $16^{\frac{1}{4}}$	c. $4^{\frac{3}{2}}$	d. $100^{-\frac{3}{2}}$

Let's PRACTICE!

Use the properties of exponents to rewrite each expression in simplified form. Note: Answers should contain only positive exponents.

1. $\left(a^{\frac{1}{2}}b^{\frac{1}{2}}\right)^{-1}$	2. $\left(x^{\frac{5}{3}}y^{-2}\right)^{0}$	3. $\frac{a^2b^0}{3a^4}$
4. $\frac{2x^{\frac{1}{2}}y^{\frac{1}{3}}}{2x^{\frac{4}{3}}y^{-\frac{7}{4}}}$	5. $\frac{y^0}{\left(x^{\frac{4}{3}}y^{-1}\right)^{\frac{1}{3}}}$	6. $\left(\frac{y^{1/3}y^{-2}}{\left(x^{\frac{5}{3}}y^{3}\right)^{-\frac{3}{2}}}\right)^{\frac{3}{2}}$

What should we take away?

- ☐ Zero/negative exponents

- ☐ Fractional exponents

Topic 2.5 Exponential Function Context and Data Modeling (Daily Video 1)

AP Precalculus

In this video, we will explore how proportional growth patterns suggest an exponential model.

Let's REVIEW!

The general form of an exponential equation is: ______________

- For every one-unit increase in the x-value, the y-value increases by a common _______ or proportion.
- A _____________ sequence can be modeled by an exponential equation with only integer values of x.

Interpreting the Parts of an Exponential Equation: $y = a \cdot b^x$	What is e?
• a is the _________ value and graphically it is the same as the y-value when x is ________ • b describes how the graph increases (or decreases). b is the growth rate if _______ and it is the decay rate if ___________. • x describes the number of times the growth or decay factor is applied.	• It is also known as Euler's number. • It is _______________ (like π) and it is approximately ___________. • e is used to represent __________ growth.

Let's Look at an EXAMPLE!

A small business is growing at a fast rate, but it is unclear whether its growth follows an exponential model. The business's revenue in its first year of operation was \$150,000. In year 2, its revenue was \$223,000. In year 3, its revenue was \$310,000. Is the business's growth over its first three years exponential?
Compare the proportional change in the y-values over equal-width intervals of x-values. The ratios are _______________ and _______________. Are the growth rates equal? YES or NO (circle one) Is the growth rate exponential? YES or NO (circle one)

Let's look at an EXAMPLE!

The population in Mali was 7.09 million in 1980. Since that time, the population has increased, on average, 2.07% per year. Write an exponential equation that models the population in Mali since 1980.	
$a =$_______ million $b =$ _______	The equation is ______________, where y is ________________(in millions) and x is _________________________.
Based on this model, what would we expect the population to be in 1994? Why must $x = 14$? How close was the prediction?	

Let's look at an EXAMPLE!

You invested $11,235 five years ago. Interest is compounded continuously at a rate of 3%. What is this investment worth today?

What should we take away?

- The parts of an exponential equation have meanings in terms of the graph and in terms of the real-world context.
- e is an irrational number and it is about 2.71828.

Topic 2.5 Exponential Function Context and Data Modeling (Daily Video 2)

AP Precalculus

In this video, we will explore how different forms of equivalent exponential functions reveal different characteristics about particular exponential patterns.

Let's REVIEW!

Interpreting the Parts of an Exponential Equation: $y = a \cdot b^x$
• a is the initial value and graphically it is the same as the y-value when x is zero. • b describes how the graph increases (or decreases). b is the growth rate if $b > 1$ and it is the decay rate if $0 < b < 1$. • x describes the number of times the growth or decay factor is applied.

Let's look at an EXAMPLE!

Two Ways to Describe a Growth Rate

- To compare two growth rates, they need to be on the same ____________ of the dependent variable.
- Compare 1% interest compounded each month to 12% compounded each year.

1% per month for 12 months for x years

12% per year for x years

Circle the greater growth rate.

Convert the Following Growth Rates (Round to the nearest ten-thousandth.)

- A certain type of bacteria is growing at a rate of 1% per day. What is the equivalent *weekly* rate of growth?
- A student's childcare business grew 30% over the course of a year. What is the equivalent *monthly* rate?

Why is the exponent for this problem a fractional exponent?

- A nation's GDP is forecasted to grow at a rate of 2.5% per week over the next year. What is the equivalent annual rate?

Why do we subtract 1 from each of the above calculations?

Rewriting a Function with a Different Growth Rate

The population of a certain type of bacteria can be modeled by the equation $f(n) = 300 \cdot 2^n$, where $f(n)$ represents the number of bacteria after n days.

- What does the 300 represent in this context?
- How many bacteria are present after 7 days?
- Rewrite the function so the growth rate is weekly (w) instead of daily (d).

What should we take away?

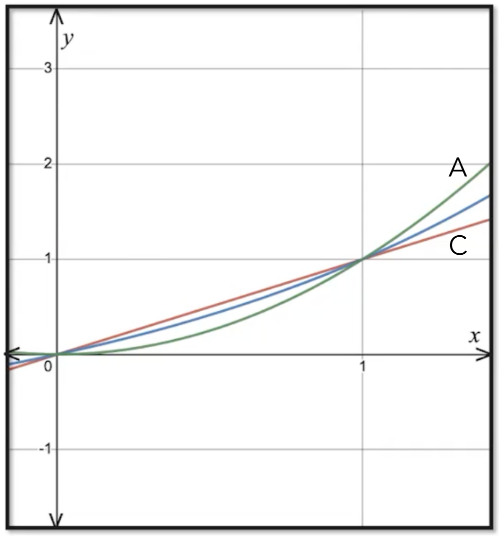
- There are multiple ways to write the same exponential model.
- Given growth rates can be written over different time intervals.

Topic 2.6 Competing Function Model Validation (Daily Video 1)

AP Precalculus

In this video, we will explore patterns in data that reflect how linear, exponential, and quadratic models differ.

Let's WARM UP!



Three functions are graphed on the left. They are:

$$f(x) = x$$
$$f(x) = 2^x - 1$$
$$f(x) = x^2$$

Which graph goes with which function?

What is making it difficult to tell these graphs apart?

- ☐ It may be very difficult to tell what type of function best models a set of data with two points.
- ☐ The minimum number of points needed to assess which model is best is _______.
- ☐ Why can't we always rely on negative value for x to identify which function is best for a graph?

Rates of Growth for Different Functions

Linear: A ____________ means the y-values increase by the same amount for equal values of x.	Exponential: The y-values increase ________________.	Quadratic: The y-values can ______ and the difference between the differences of the y-values is ___________.

Let's look at an EXAMPLE!

What type of model is represented by the data? Explain.

Show the calculation of the differences.

t	$f(t)$
2	6.4
4	4.2
6	2
8	–0.2

Show the calculation of the ratios.

x	y
0	0.5
1	2
2	8
3	32

Let's look at an EXAMPLE!

What type of model is represented by the data? Explain.

x	y
–6	5
–4	–1
–2	–3
0	–1

What y-values repeat? ______

Compute the first differences and second differences for the table to the left.

Based on the data in the table, why is the quadratic model the best choice?

Let's look at an EXAMPLE!

What type of model is represented by the data?

Show the calculation of the differences of the differences.

x	y
0	1.5
1	16.6
2	21.9
3	17.4
4	3.1

What should we take away?

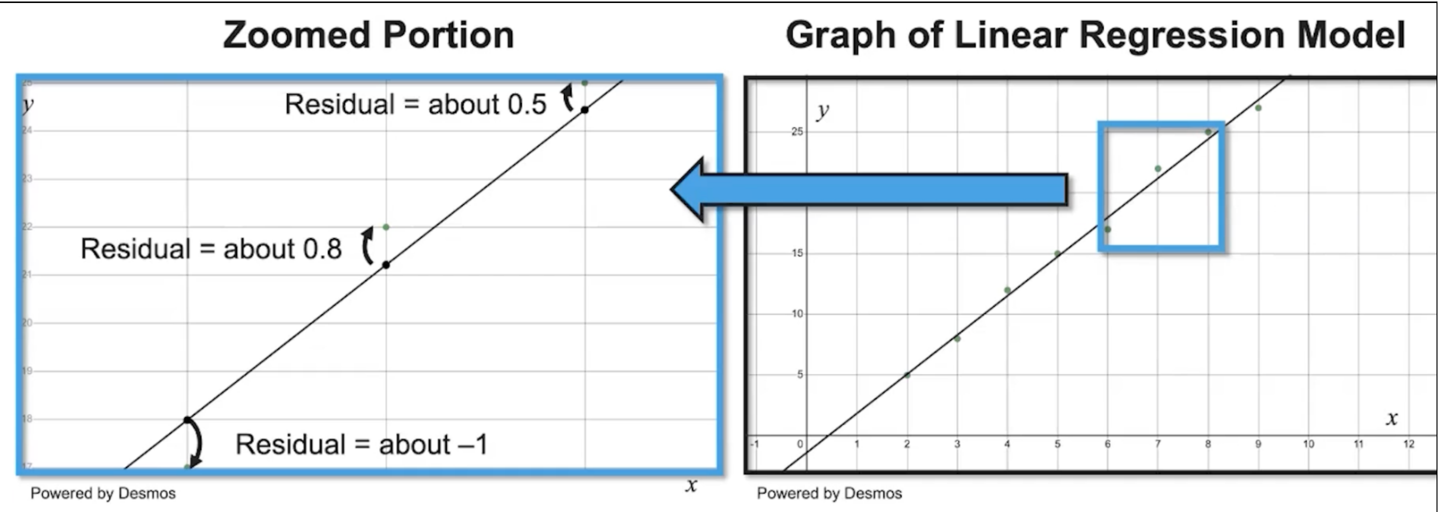
- ☐ At least ___ points are needed to assess which model is best.
- ☐ If the y-values are repeated, a quadratic model <u>may</u> be best.
- ☐ A _____________ model is best if the differences between y-values over equal intervals of x is a constant.
- ☐ A ________________ model is best if the differences between the differences of the y-values over equal intervals of x is a constant.
- ☐ An __________________ model is best if the proportions between the y-values over equal intervals of x is a constant.

Topic 2.6 Competing Function Model Validation (Daily Video 2)

AP Precalculus

In this video, we will explore what a residual plot reveals about a given model.

What is a residual?



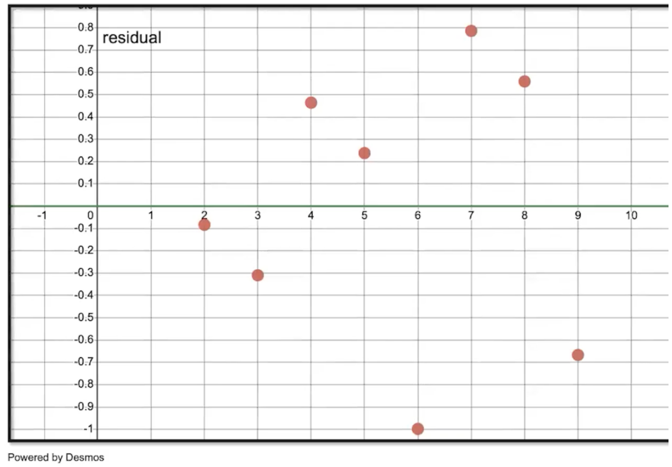
Residual = y-value from the data point – y-value from the ______________.

If the residual is negative, is the point above or below the line? _____________

If the residual is positive, is the point above or below the line? ______________

Table of Some Residual Values and a Residual Plot

x	y	y-value on regression line	residual	point on residual plot
2	5	5.08333	5 − 5.08333 = **−0.08333**	(2, −0.08333)
3	8	8.30952	8 − 8.30952 = **−0.30952**	(3, −0.30952)
4	12	11.53571	12 − 11.53571 = **0.46429**	(4, 0.46429)

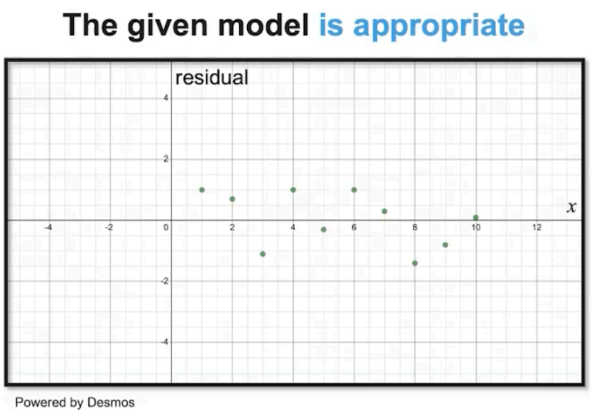


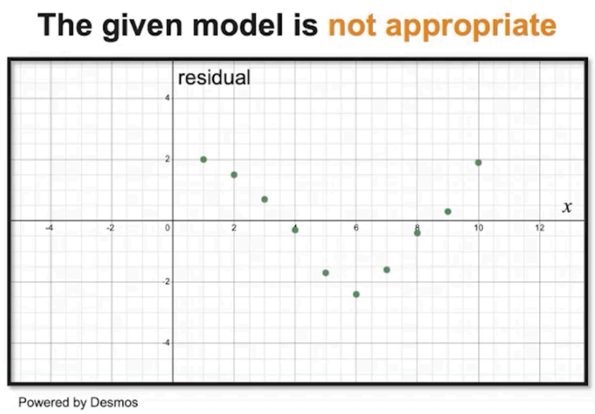
point on residual plot
(2, −0.08333)
(3, −0.30952)
(4, 0.46429)
(5, 0.2381)
(6, −0.98809)
(7, 0.78572)
(8, 0.55953)
(9, −0.66666)

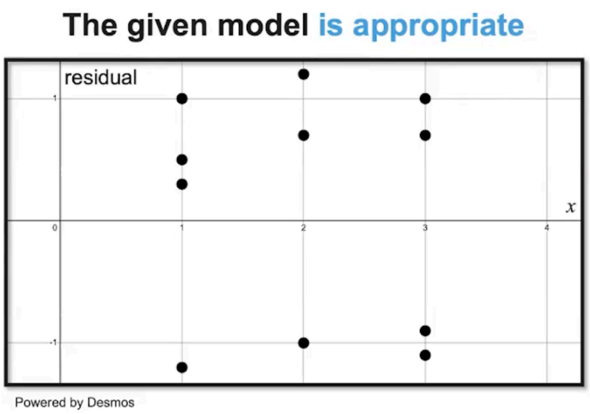
The ordered pair for a point on a residual plot is (x, _______________).

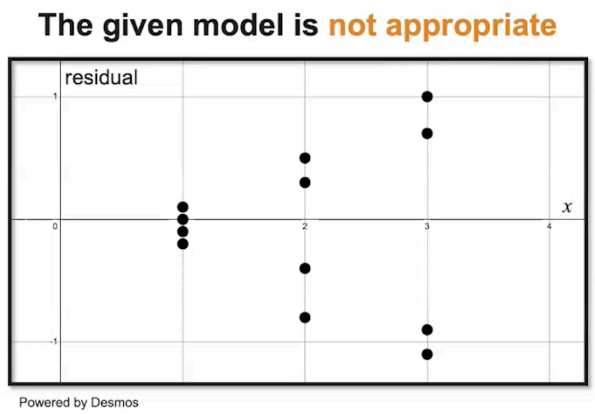
Is the model appropriate?

- If there is a pattern, the given model _________ appropriate.
- If there is not a pattern, the given model _______ appropriate.



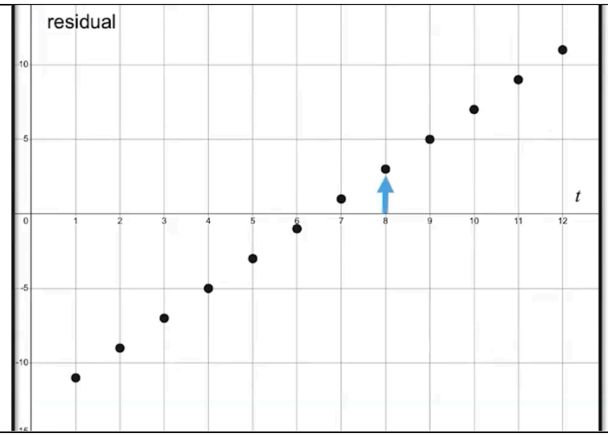






Let's look at an EXAMPLE!

Is the model appropriate? A social media expert is studying how a particular meme is spreading. They can see how many times the meme has been viewed each hour on their platform. After creating an exponential function, $f(t)$, to model the number of views after t hours, they look at 12 hours of data and see the residual plot shown here. What can be said of the exponential model based on this residual plot?



What should we take away?

- A residual plot shows how different the actual y-values are from what the model predicts.
- A residual plot has x-values for the x-axis and the value of the residuals for the y-axis.
- A residual plot shows which values are most different from what the model predicts.
- If the residuals are randomly distributed, a given model is appropriate.
- If the residuals follow a pattern, a given model is not appropriate.