

Function Features

A function  $f$   
is concave up if...



Function Features

A function  $f$   
is concave down if...



Function Features

A function  $f$   
is increasing on an  
interval if...



Function Features

A function  $f$   
is decreasing on an  
interval if...



Function Features

Average rate of change  
of  $f$  on the interval  $[a, b]$



Function Features

The slope of a function  
at any given point  
gives...



Function Features

A positive rate of change  
indicates that the function  
output is...



Function Features

A negative rate of change  
indicates that the function  
output is...



Function Features

Point of inflection



Function Features

One-to-one function



<p>The rates of change of <math>f</math> are decreasing</p>	<p>The rates of change of <math>f</math> are increasing</p>
<p>As the input values increase, the output values always decrease.</p> <p>OR</p> <p>For all <math>a</math> and <math>b</math> in the interval, if <math>a &lt; b</math>, then <math>f(a) &gt; f(b)</math>.</p>	<p>As the input values increase, the output values always increase.</p> <p>OR</p> <p>For all <math>a</math> and <math>b</math> in the interval, if <math>a &lt; b</math>, then <math>f(a) &lt; f(b)</math>.</p>
<p>The rate of change of the function at that input</p>	$\frac{f(b) - f(a)}{b - a}$
<p>Decreasing</p>	<p>Increasing</p>
<p>Function where each input has a unique output (no repeated outputs)</p>	<p>Point on the graph of a function where the concavity changes, indicating a maximum or minimum rate of change</p>

Function Features

A relative minimum occurs when a function  $f$ ...



Function Features

A relative maximum occurs when a function  $f$ ...



Function Features

Absolute minimum



Function Features

Absolute maximum



Function Features

Multiplicity



Function Features

A polynomial of degree  $n$  has...



Function Features

If  $x = a$  is a real zero of a polynomial with an odd multiplicity, then...



Function Features

If  $x = a$  is a real zero of a polynomial with an even multiplicity, then...



Function Features

Odd function













Function Features

Even function



Changes from increasing to decreasing	Changes from decreasing to increasing
The greatest output of a function	The least output of a function
<ul style="list-style-type: none"> <li>• Exactly <math>n</math> complex zeros (real or imaginary)</li> <li>• Constant <math>n</math>th differences</li> <li>• At most <math>n - 1</math> extrema</li> </ul>	The number of times a factor occurs in a polynomial function
The graph of the polynomial is tangent to the $x$ -axis at $x = a$ .	The graph of the polynomial passes through the $x$ -axis at $x = a$ .
$f(-x) = f(x)$	$f(-x) = -f(x)$

<p>Function Features</p> <p>End behavior of a polynomial <math>f</math> with an even degree and a negative leading coefficient</p> 	<p>Function Features</p> <p>End behavior of a polynomial <math>f</math> with an odd degree and a positive leading coefficient</p> 
<p>Function Features</p> <p>End behavior of a polynomial <math>f</math> with an odd degree and a negative leading coefficient</p> 	<p>Function Features</p> <p>End behavior of a polynomial <math>f</math> with an even degree and a positive leading coefficient</p> 
<p>Function Features</p> <p>If a rational function, <math>f</math>, has a horizontal asymptote at <math>y = b</math>, then...</p> 	<p>Function Features</p> <p>To determine the end behavior of a rational function...</p> 
<p>Function Features</p> <p>A rational function has a zero at <math>x = a</math> if...</p> 	<p>Function Features</p> <p>A rational function has a hole at <math>x = a</math> if...</p> 
<p>Function Features</p> <p>A rational function has a vertical asymptote at <math>x = a</math> if...</p> 	<p>Function Features</p> <p>For rational functions, a slant asymptote occurs when...</p> 

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

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$$\lim_{x \rightarrow \infty} f(x) = -\infty$$

$$\lim_{x \rightarrow -\infty} f(x) = \infty$$

Analyze the ratio of leading terms

The ratio of leading terms is a constant,  $b$ ,  
 $\lim_{x \rightarrow \infty} f(x) = b$ , and  
 $\lim_{x \rightarrow -\infty} f(x) = b$

$x = a$  is a zero of the numerator AND the denominator

$x = a$  is a zero of the numerator but NOT the denominator

The degree of the numerator is exactly one more than the degree of the denominator

$x = a$  is a zero of the denominator but NOT the numerator

## Function Features

If a rational function,  $f$ , has a vertical asymptote at  $x = a$ , then  $\lim_{x \rightarrow a^-} f(x) = \underline{\hspace{2cm}}$  and  $\lim_{x \rightarrow a^+} f(x) = \underline{\hspace{2cm}}$ .



## Function Features

If a rational function,  $f$ , has a hole at  $(a, L)$  then  $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = \underline{\hspace{2cm}}$ .



## Function Features

A function  $f(x) = ab^x$  demonstrates exponential growth if...



## Function Features

A function  $f(x) = ab^x$  demonstrates exponential decay if...



## Function Features

Key features of  $y = \log_b x$  where  $b > 1$



## Function Features

Key features of  $y = b^x$  where  $b > 1$



## Properties and Identities

$$b^{x+c} =$$



## Properties and Identities

$$b^{x-c} =$$



## Properties and Identities

$$e^{a \ln b} =$$



## Properties and Identities

$$\log_b(1) =$$



$L$	$\pm\infty ; \pm\infty$
$0 < b < 1$	$b > 1$
<ul style="list-style-type: none"> <li>• Domain: all real numbers</li> <li>• Range: <math>y &gt; 0</math></li> <li>• Horizontal asymptote at <math>y = 0</math></li> <li>• Increasing and concave up over entire domain</li> </ul>	<ul style="list-style-type: none"> <li>• Domain: <math>x &gt; 0</math></li> <li>• Range: all real numbers</li> <li>• Vertical asymptote at <math>x = 0</math></li> <li>• Increasing and concave down over entire domain</li> </ul>
$\frac{b^x}{b^c}$	$b^x \cdot b^c$
$0$	$b^a$



Properties and Identities

$$\log_b(b) =$$



Properties and Identities

$$\log_b(mn)$$



Properties and Identities

$$\log_b\left(\frac{m}{n}\right)$$



Properties and Identities

$$\log_b m^k$$



Properties and Identities

Pythagorean  
Identities



Properties and Identities

$$\begin{aligned}\sec \theta &= \\ \csc \theta &= \\ \cot \theta &= \end{aligned}$$



Properties and Identities

$$\cos(\alpha \pm \theta)$$



Properties and Identities

$$\sin(\alpha \pm \theta)$$



Properties and Identities

$$\cos(2\theta)$$




Properties and Identities

$$\sin(2\theta)$$




$\log_b m + \log_b n$	$1$
$k \log_b m$	$\log_b m - \log_b n$
$\frac{1}{\frac{\cos \theta}{1}}$ $\frac{1}{\sin \theta}$ $\frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$	$\sin^2 \theta + \cos^2 \theta = 1$ $1 + \cot^2 \theta = \csc^2 \theta$ $\tan^2 \theta + 1 = \sec^2 \theta$
$\sin \alpha \cos \theta \pm \sin \theta \cos \alpha$	$\cos \alpha \cos \theta \mp \sin \alpha \sin \theta$
$2 \sin \theta \cos \theta$	$\cos^2 \theta - \sin^2 \theta$ $= 2 \cos^2 \theta - 1$ $= 1 - 2 \sin^2 \theta$


Polar Functions

Given  $(x, y)$  in Cartesian coordinates, determine polar coordinates,  $(r, \theta)$  


Polar Functions

Given  $(r, \theta)$  in polar coordinates, determine Cartesian coordinates,  $(x, y)$  


Polar Functions

A polar function  $r = f(\theta)$  is increasing if... 


Polar Functions

A polar function  $r = f(\theta)$  is decreasing if... 


Polar functions

The distance between a point on a polar function  $r = f(\theta)$  and the origin is increasing if... 


Polar functions

The distance between a point on a polar function  $r = f(\theta)$  and the origin is decreasing if... 


Describing Growth in Functions

A function is linear if over equal-length input intervals, output values \_\_\_\_\_ 


Describing Growth in Functions

A function is quadratic if over equal-length input intervals, output values \_\_\_\_\_ 

Describing Growth in Functions

A function is exponential if as input values change \_\_\_\_\_, output values change \_\_\_\_\_. 

Describing Growth in Functions

A function is logarithmic if as input values change \_\_\_\_\_, output values change \_\_\_\_\_. 

$x = r \cos \theta$ $y = r \sin \theta$	$r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1} \left( \frac{y}{x} \right)$ <p>(*Add <math>\pi</math> if angle is in Q2 or Q3)</p>
<p>As <math>\theta</math> increases, <math>r</math> decreases.</p>	<p>As <math>\theta</math> increases, <math>r</math> increases.</p>
<p><math>r</math> is positive and decreasing or <math>r</math> is negative and increasing. (<math> r </math> is decreasing)</p>	<p><math>r</math> is positive and increasing or <math>r</math> is negative and decreasing. (<math> r </math> is increasing)</p>
<p>Change by a constant second difference.</p>	<p>Change by a constant amount.</p>
<p>multiplicatively; additively</p>	<p>additively; multiplicatively</p>

Describing Growth in Functions

The average rates of change of a linear function are...



Describing Growth in Functions

The average rates of change of a quadratic function...



Trig Functions

$\tan \theta$  gives the \_\_\_\_\_ of the terminal ray of  $\theta$ .



Trig Functions

Domain and range of  $y = \arcsin x$



Trig Functions

Domain and range of  $y = \arccos x$



Trig Functions

Domain and range of  $y = \arctan x$



Trig Functions

$f(x) = \tan x$  has vertical asymptotes at...



Trig Functions

$f(x) = \cot x$  has vertical asymptotes at ...



Trig Functions

Determine the amplitude, period, midline, and phase shift of  $f(x) = a \sin(b(x - c)) + d$



Trig Functions

$y = \tan(bx)$  has a period of...



<p>Are changing at a constant rate OR follow a linear pattern</p>	<p>Constant</p>
<p>Domain: <math>[-1,1]</math> Range: <math>\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]</math></p>	<p>Slope</p>
<p>Domain: <math>(-\infty, \infty)</math> Range: <math>\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)</math></p>	<p>Domain: <math>[-1,1]</math> Range: <math>[0, \pi]</math></p>
<p><math>x = \pi k</math>, where <math>k</math> is an integer</p>	<p><math>x = \frac{\pi}{2} + \pi k</math>, where <math>k</math> is an integer</p>
<p><math>\frac{\pi}{b}</math></p>	<p>Amplitude = <math> a </math> Period = <math>\frac{2\pi}{b}</math> Midline: <math>y = d</math> Phase shift: <math>c</math> units to the right</p>

Trig Functions

Key features of  
 $y = \sin x$



Trig Functions

Key features of  
 $y = \cos x$



Transformations

$$f(x) + c$$



Transformations

$$f(x - c)$$



Transformations

$$cf(x)$$



Transformations

$$f(cx)$$



Transformations

$$-f(x)$$



Transformations

$$f(-x)$$



Miscellaneous

Pascal's Triangle



Miscellaneous

What does  
the constant  $e$   
represent?



<ul style="list-style-type: none"> <li>• Domain: all real numbers</li> <li>• Range: <math>[-1, 1]</math></li> <li>• Period: <math>2\pi</math></li> <li>• Amplitude: 1</li> <li>• Midline: <math>y = 0</math></li> <li>• Passes through <math>(0, 1)</math></li> </ul>	<ul style="list-style-type: none"> <li>• Domain: all real numbers</li> <li>• Range: <math>[-1, 1]</math></li> <li>• Period: <math>2\pi</math></li> <li>• Amplitude: 1</li> <li>• Midline: <math>y = 0</math></li> <li>• Passes through <math>(0, 0)</math></li> </ul>
<p>Horizontal translation  <math>c</math> units to the right if <math>c &gt; 0</math>  or  <math>c</math> units to the left if <math>c &lt; 0</math></p>	<p>Vertical translation  <math>c</math> units up if <math>c &gt; 0</math>  or  <math>c</math> units down if <math>c &lt; 0</math></p>
<p>Horizontal dilation by a  factor of <math>\frac{1}{c}</math></p>	<p>Vertical dilation by a  factor of <math>c</math></p>
<p>Reflection over the  <math>y</math>-axis</p>	<p>Reflection over the  <math>x</math>-axis</p>
<p>The base rate of growth  for all continually  growing processes  <math>e \approx 2.718</math></p>	<p style="text-align: center;"> 1  1 1  1 2 1  1 3 3 1  1 4 6 4 1  1 5 10 10 5 1  1 6 15 20 15 6 1  1 7 21 35 35 21 7 1 </p>



Miscellaneous

A positive residual indicates that the predicted value is an

\_\_\_\_\_.



Miscellaneous

A negative residual indicates that the predicted value is an

\_\_\_\_\_.



Miscellaneous

Explicit rule for  $n$ th term of a geometric sequence given common ratio  $r$ , and the  $a_k$  term



Miscellaneous

Explicit rule for  $n$ th term of an arithmetic sequence given common difference  $d$ , and the  $a_k$  term



Miscellaneous

Residual



Miscellaneous

A model is considered appropriate for a data set if the residual plot...



Miscellaneous

Error (in a model)



Miscellaneous

$f$  and  $g$  are inverse functions if...



Miscellaneous

If the  $y$ -axis is logarithmically scaled, then...



Miscellaneous

In a semi-log plot where the  $y$ -axis is logarithmically scaled, exponential functions will appear



Overestimate	Underestimate
$a_n = a_k + d(n - k)$	$a_n = a_k \cdot r^{n-k}$
Appears without pattern	Actual value - Predicted value
$f(g(x)) = g(f(x)) = x$	Predicted value - Actual value
linear	Equal-sized increments on the y-axis represent proportional changes in the output variable