



# Who's Who on Halloween?



Key

Amari, Francisca, Jamirea, Micaiah, and Senad live in the same neighborhood. On Halloween, they each go out trick-or-treating, but they each start at slightly different times, wear a different costume, and have different favorite candies they like to collect. For each derivative problem you solve, you'll receive a clue from one of the neighbors about what they saw that night. Can you figure out who's who on Halloween, including what time each person started trick-or-treating, what their favorite candy is, and what costume they were in?

		First Names					Favorite Candy					Costume				
		Amari	Francisca	Jamirea	Micaiah	Senad	M&Ms	Starbursts	Twix	Air Heads	Mounds	Chef	Banana	Ghost	Lumberjack	Pirate
Time	5:45	X		X	X	X	X	X	X		X		X	X	X	X
	6:00	X	X	X	X		X	X		X	X	X			X	X
	6:30	X	X	X		X		X	X	X	X		X	X	X	X
	6:45		X	X	X	X		X	X	X	X	X	X	X		X
	7:00	X	X		X	X	X	X	X		X	X		X	X	X
Costume	Chef	X		X	X	X	X	X		X		X	X	X	X	X
	Banana	X	X	X		X		X	X	X	X	X	X	X	X	X
	Ghost	X	X		X	X	X	X	X		X	X	X	X	X	X
	Lumberjack	X	X	X	X		X	X	X	X	X	X	X	X	X	X
	Pirate		X	X	X	X	X	X	X	X	X	X	X	X	X	X
Favorite Candy	M&Ms		X	X	X	X	X	X	X	X	X	X	X	X	X	X
	Starbursts	X	X	X		X	X	X	X	X	X	X	X	X	X	X
	Twix	X	X	X	X		X	X	X	X	X	X	X	X	X	X
	Air Heads	X		X	X	X	X	X	X	X	X	X	X	X	X	X
	Mounds	X	X		X	X	X	X	X	X	X	X	X	X	X	X

1. The person whose favorite candy is Air Heads started trick-or-treating before Senad.
2. The 5 people are the person whose favorite candy is Air Heads, the pirate, Micaiah, the person who started trick-or-treating at 7 PM and the person whose favorite candy is Twix.
3. Of the chef and Senad, one started trick-or-treating at 5:45 PM and the other has Twix as their favorite candy.
4. Francisca started trick-or-treating at 5:45 PM.
5. Jamirea is either the ghost or the lumberjack.
6. The ghost started trick-or-treating after the banana.
7. The person whose favorite candy is M&Ms is neither the ghost nor the banana.
8. The person who started trick-or-treating at 6:30 has Starbursts as their favorite candy.
9. The person who started trick-or-treating at 6:45 was not the banana.
10. The banana started trick-or-treating after Senad.

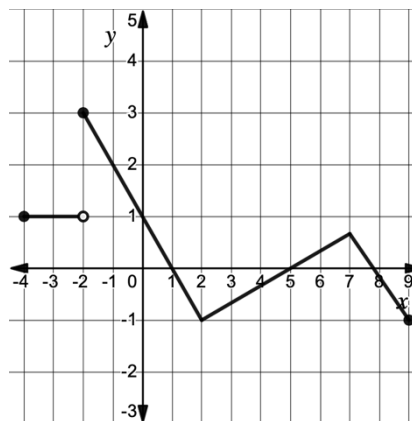
Key

1. The graph of  $y = f(x)$  is shown.

Evaluate  $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$

This means find  $f'(3)$

$$\boxed{\frac{1}{3}}$$



2. Find the derivative of each function.

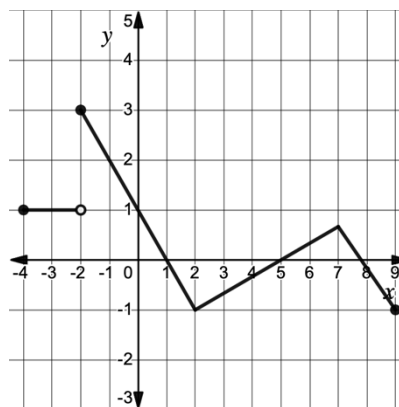
$f(x) = \ln(4x) \quad f'(x) = \frac{4}{4x} = \frac{1}{x}$

$g(x) = (3x - 5)^2 \quad g'(x) = 3 \cdot 2(3x - 5) = 6(3x - 5) = 18x - 30$

$h(x) = \sec x \quad h'(x) = \sec x \tan x$

3. The graph of  $y = f(x)$  is shown below for  $-4 \leq x \leq 9$ . For which value(s) of  $x$  is  $f$  continuous but not differentiable?

$x = 2$  and  $x = 7$



4. Selected values of  $f, g$ , and their derivatives are given in the table.

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-3	10	1	-1	3
-1	4	-2	-3	0
2	1	-5	0	-2
5	-2	-3	2	8
8	-5	11	7	-3.5
11	5	8	13	1

Let  $h(x) = f(x) \cdot g(x)$ . Find  $h'(8)$ .

$$\begin{aligned} h'(8) &= f(8) \cdot g'(8) + g(8) \cdot f'(8) \\ &= -5 \cdot (-3.5) + 11 \cdot 7 \\ &= 94.5 \end{aligned}$$

5. Let  $j(x) = 5x^3 - kx^2 + 10x + m$  for some constants  $k$  and  $m$ . If  $j(2) = -10$  and  $j'(2) = 6$ , find the values of  $k$  and  $m$ .

$$j'(x) = 15x^2 - 2kx + 10$$

$$j'(2) = 15(2)^2 - 2k(2) + 10 = 70 - 4k = 6 \Rightarrow k = 16$$

$$j(2) = 5(2)^3 - 16(2)^2 + 10(2) + m = -10$$

$$= 40 - 64 + 20 + m = -10 \quad m = -6$$

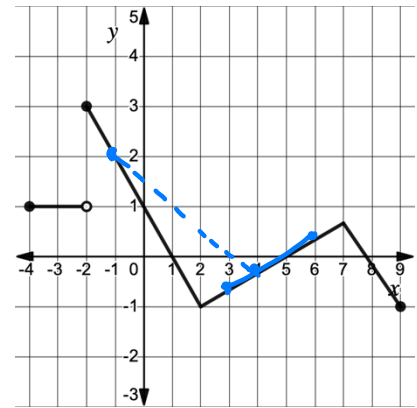
6. The graph of  $y = f(x)$  is shown below. Order the following from least=1 to greatest=4.

1  $\lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x + 1} = -1$

4 The average rate of change of  $f$  on the interval  $[3, 6]$   $> 0$

2  $\frac{f(4) - f(-1)}{5}$  neg but less steep

3  $f'(-3) = 0$



7. Selected values of  $f$ ,  $g$ , and their derivatives are given in the table.

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-3	10	1	-1	3
-1	4	-2	-3	0
2	1	-5	0	-2
5	-2	-3	2	8
8	-5	11	7	-3.5
11	5	8	13	1

Let  $k(x) = \frac{g(x)}{f(x)}$ . Find  $k'(-1)$ .

$$k'(-1) = \frac{f(-1) \cdot g'(-1) - f'(-1) \cdot g(-1)}{(f(-1))^2} = \frac{4(0) - (-3)(-2)}{4^2} = \frac{-6}{16} = \boxed{\frac{-3}{8}}$$

8. If  $f(x) = \frac{\cos^2 x}{\sin x}$ , find  $f'(x)$ .

$$f(x) = \frac{1 - \sin^2 x}{\sin x} = \csc x - \sin x$$

$$f'(x) = \boxed{-\csc x \cot x - \cos x}$$

9. Selected values of  $f$ ,  $g$ , and their derivatives are given in the table.

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-3	10	1	-1	3
-1	4	-2	-3	0
2	1	-5	0	-2
5	-2	-3	2	8
8	-5	11	7	-3.5
11	5	8	13	1

Let  $j(x) = f(g(x))$ . Find  $j'(5)$ .

$$j'(5) = g'(5) \cdot f'(g(5)) = 8 \cdot f'(-3) = 8 \cdot (-1) = -8$$

10. Let  $f(x) = x^2 + 5x$  and let  $g$  be a function so that  $g'(x) = 3$  for all  $x$ .

If  $h(x) = f(g(x))$  and  $h'(x) = 18x + 57$ , write an equation for  $g(x)$ .

$g(x)$  is linear with a slope of 3.  $g(x) = 3x + b$

$$f'(x) = 2x + 5$$

$$h'(x) = g'(x) \cdot f'(g(x)) = 3 \cdot [2(3x + b) + 5] = 18x + 57$$

$$= 18x + 6b + 15 = 18x + 57$$

$$6b = 42$$

$$b = 7$$

$$\boxed{g(x) = 3x + 7}$$