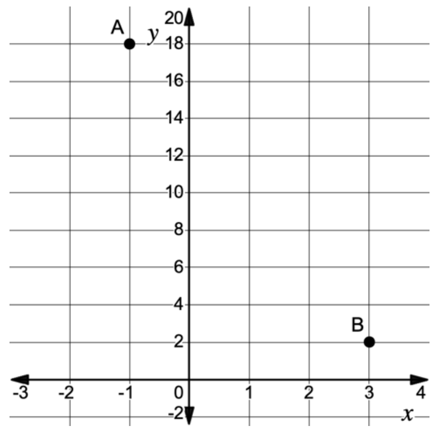
Sample answers

Chili Pepper Function Challenge

Points A and B are shown below on the coordinate plane. The challenges below should be solved **without a calculator.** Choose which challenges you want to complete so that you earn a total of 15 chili peppers. Use Desmos to verify your equation satisfies the challenge.



Challenge 1:
Write an equation for a linear function f that passes through A and B.

$$f(x) = 18 - 4(x+1)$$

Challenge 2:
Write an equation for your function f from Challenge 1 in an alternate, but equivalent, form.

$$f(x) = -4x + 14$$

Challenge 3:
Write an equation for an exponential function g that passes through A and B.

$$\frac{2}{18} = \frac{1}{9} \qquad r^4 = \frac{1}{9} \Rightarrow r = \left(\frac{1}{9}\right)^{1/4}$$

$$g(x) = 18\left(\frac{1}{9}^{1/4}\right)^{x+1}$$

Challenge 4:
Write at least three equivalent forms for your function g from Challenge 3.

$$\frac{1}{9}^{1/4} = \left(\frac{1}{9}^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(\frac{1}{3}\right)^{1/2} = \sqrt{\frac{1}{3}} = \frac{\sqrt{3}}{3}$$

$$g(x) = 18\left(\frac{\sqrt{3}}{3}\right)^{x+1}$$

$$g(x) = 2\left(\sqrt{\frac{1}{3}}\right)^{x-3}$$

$$g(x) = 6\left(\frac{\sqrt{3}}{3}\right)^{x-1}$$

$$g(x) = 6\sqrt{3}\left(\frac{\sqrt{3}}{3}\right)^{x}$$

$$g(x) = 18\left(\frac{1}{3}\right)^{\frac{x+1}{2}}$$

$$g(x) = 18\left(\frac{1}{3}\right)^{\frac{1}{2}x+\frac{1}{2}}$$

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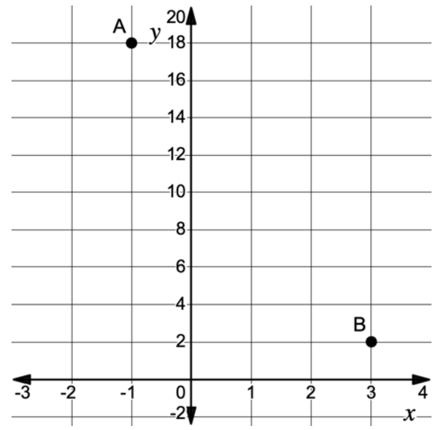
Challenge 5:
Write an equation for a quadratic function h that passes through A and B where $\lim_{x \to \infty} h(x) = \infty$. How many such functions are possible?



$(-1, 18)$ $(3, 2)$

$18 = a - b + c$
$2 = 9a + 3b + c$

$16 = -8a - 4b$

$\frac{8a + 16}{-4} = b$

$-2a - 4 = b$

Infinitely many

inf. many solutions
$a = 1, b = -6$
$c = 11$

$h(x) = x^2 - 6x + 11$
or $h(x) = 2x^2 - 8x + 8$

Challenge 6:
Write an equation for a quadratic function k that has a maximum at point A and passes through point B.

$a(x+1)^2 + 18 = k(x)$

$2 = a(3+1)^2 + 18$ $-16 = 16a \Rightarrow a = -1$

$k(x) = -(x+1)^2 + 18$

Challenge 7:
Write an equation for your function k from Challenge 6 in an alternate, but equivalent, form.

$k(x) = -x^2 - 2x + 17$

Challenge 7:
Write an equation for a piecewise function s where one sub-function passes through A and the other sub-function passes through B. Be sure to give the domain restriction of each sub-function.

$$s(x) = \begin{cases} 3x + 21, & x \le 0 \\ 2, & x > 0 \end{cases}$$

Challenge 8:
Write an equation for a sinusoidal function j that has a maximum at point A and a minimum at point B.

midline : $y = 10$ amp $= 8$ period $= 2(4) = 8$ max at $x = -1$

$\frac{2\pi}{8} = \frac{\pi}{4} = b$

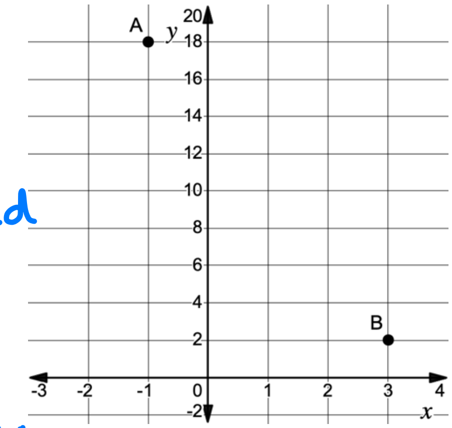
$j(x) = 8\cos\left(\frac{\pi}{4}(x+1)\right) + 10$

Challenge 9:
Write an equation for a sinusoidal function m that has a midline passing through A and a minimum at point B.

midline: $y = 18$ amp $= 16$ period $= 4(4) = 16$ max at $x = -5$
mid at $x = -1$

$y = -16\sin\left(\frac{\pi}{8}(x+1)\right) + 18$

Challenge 10:
Rewrite your equation for function j in Challenge 8 using a different trig function.



$$j(x) = 8\cos\left(\frac{\pi}{4}(x+1)\right) + 10$$

sine function is cosine function shifted right $\frac{1}{4}$ of a period

$$j(x) = 8\sin\left(\frac{\pi}{4}(x+1+2)\right) + 10$$
$$= 8\sin\left(\frac{\pi}{4}(x+3)\right) + 10$$

Challenge 11:
Write an equation for an even function d that passes through A and B.

$$d(x) = \begin{cases} 26 + 8x & x \le -1 \\ 18 & -1 < x < 1 \\ 26 - 8x & x \ge 1 \end{cases}$$

Challenge 13:
Write an equation for a rational function w that passes through A and B.

$$\frac{a}{bx+c} = w(x) \qquad \frac{a}{-b+c} = 18 \qquad \frac{a}{3b+c} = 2$$

$$18(-b+c) = a(3b+c)$$

$$-24b = -16c \qquad c = \frac{3}{2}b \qquad c=9,\ b=6 \text{ would work}$$

$$\boxed{w(x) = \frac{54}{6x+9}}$$

$$\frac{a}{-6+9} = 18 \Rightarrow a = 54 \qquad \frac{a}{3(6)+9} = 2 \Rightarrow a = 54 \checkmark$$

Challenge 14:
Write an equation for a cubic function z that has a relative maximum at point A and a relative minimum at point B. (You can investigate with Desmos for this one!)

$$z(x) = 1.5\left(\frac{x^3}{3} - x^2 - 3x\right) + 15.5$$

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