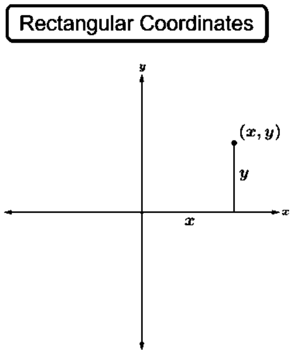
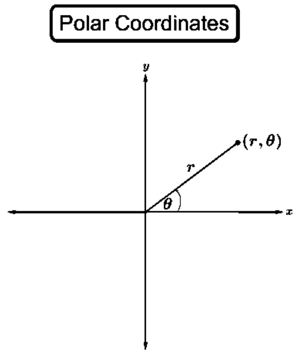
Topic 3.13 Trigonometry and Polar Coordinates (Daily Video 1)

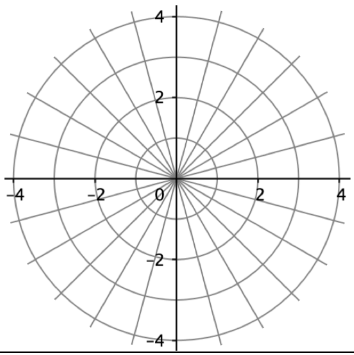
AP Precalculus

In this video, we will introduce and discuss the purpose of polar coordinates, as well as practice converting from polar to rectangular coordinates.

What Are Polar Coordinates?







Representing Points in Polar Coordinate

There are multiple ways to represent the same point in polar coodinates!

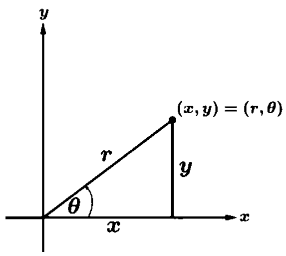
A full turn is _____ and a half turn is _____ radians.

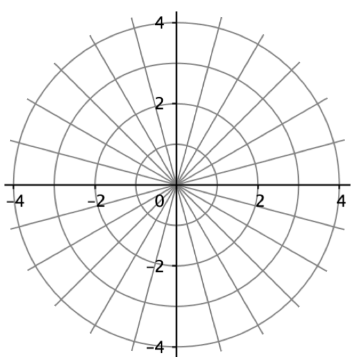
1. $\theta \pm 2\pi$ and r stays the same
2. $\theta \pm \pi$ and $r \rightarrow -r$

Let's look at an EXAMPLE!

a) Plot the polar coordinate $\left(3, \frac{\pi}{3}\right)$ on the grid provided.

b) Write the polar coordinate two additional ways.





Converting From Polar To Rectangular

$\cos(\theta) = \frac{\_\_\_\_}{r}$ $\qquad$ $\sin(\theta) = \frac{\_\_\_\_}{r}$

$x =$ ____________ $\qquad$ $y =$ ____________

Let's look at an EXAMPLE!

Convert the following polar coordinates to rectangular coordinates.

a) $\left(3, \frac{\pi}{3}\right)$

b) $\left(-2, \frac{3\pi}{2}\right)$

What should we take away?

1. Polar coordinates can be written in different ways and represent the same point.
2. We can convert from polar coordinates to rectangular coordinates using the conversion:

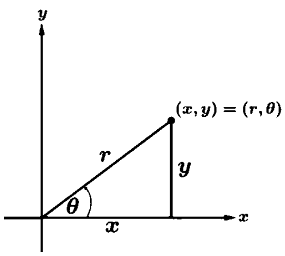
$x =$ ____________ $\qquad$ $y =$ ____________

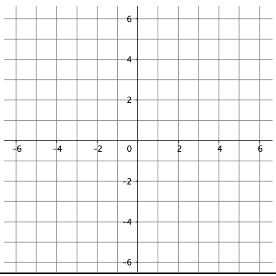
CALC MEDIC

Topic 3.13 Trigonometry and Polar Coordinates (Daily Video 2)

AP Precalculus

In this video, we will learn to convert from rectangular coordinates to polar coordinates, as well as how to represent complex numbers in both rectangular and polar coordinates.





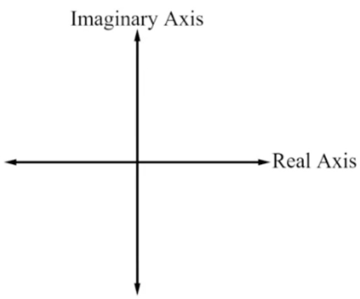
Converting From Rectangular To Polar

$x^2 + y^2 =$ ____________ $\tan\theta = \frac{\quad}{\quad}$

Let's look at an EXAMPLE!

Convert the following rectangular coordinates to polar coordinates.

a) $(5,5)$ b) $(1, -\sqrt{3})$



Complex Numbers

Rectangular Coordinates $(a, b) \rightarrow a + bi$

Polar Coordinates $(r, \theta) \rightarrow$ ____________________________

Write the complex number $-3i$ using polar coordinates.

Convert $z = 4\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$ to rectangular coordinates.

What should we take away?

1. We can convert from rectangular coordinates to polar coordinates:

   $x^2 + y^2 =$ ____ $\tan\theta = \frac{\quad}{\quad}$

2. We can write complex numbers in both rectangular and polar forms:

   $(a, b) \rightarrow$ __________ $(r, \theta) \rightarrow$ _________________________

Pro Tip
Always sketch out the point first! This will help ensure your answer is reasonable.

Topic 3.14 Polar Function Graphs (Daily Video 1)

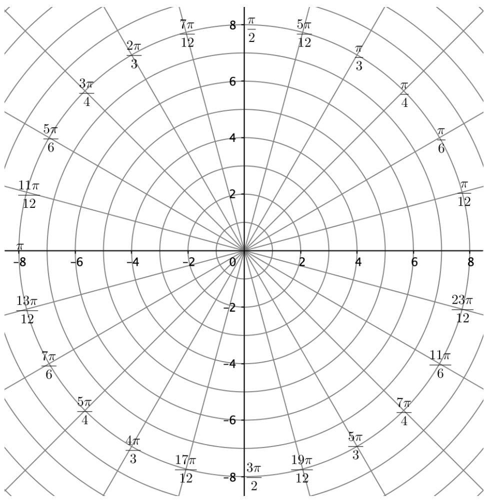
AP Precalculus

In this video, we will introduce how polar functions may be graphed by looking at several examples of basic polar graphs and practicing creating polar curves .

Functions	
Rectangular	**Polar**
$y = f(x)$	$r = f(\theta)$

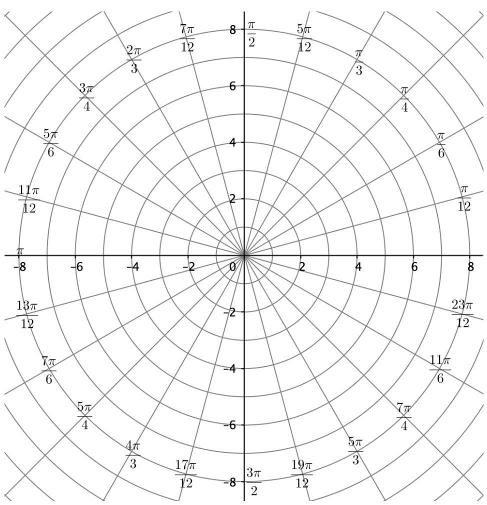
Important note: For polar functions, the dependent variable is given first! $(x, y) \rightarrow (r, \theta)$

Let's look at an Example!



θ	$r(\theta) = 3\cos(\theta)$
0	
$\pi/6$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$5\pi/6$	
π	

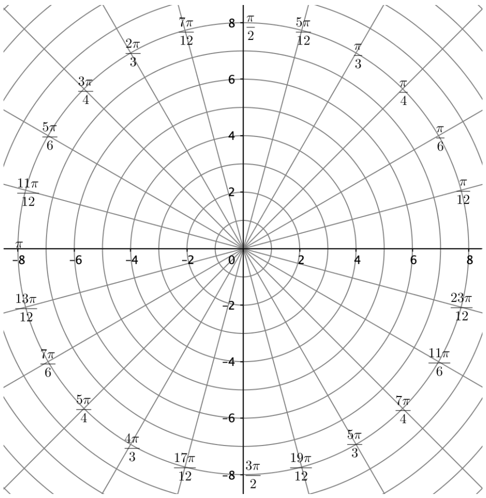
Graph $r(\theta) = 3\sin(2\theta)$



θ	$r(\theta)$
0	
$\pi/6$	
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	

θ	$r(\theta)$
π	
$7\pi/6$	
$5\pi/4$	
$4\pi/3$	
$3\pi/2$	
$5\pi/3$	
$7\pi/4$	
$11\pi/6$	
2π	

Graph $r(\theta) = 4\cos(3\theta)$

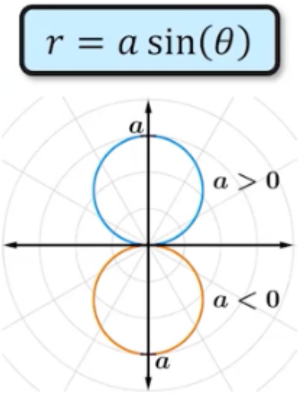


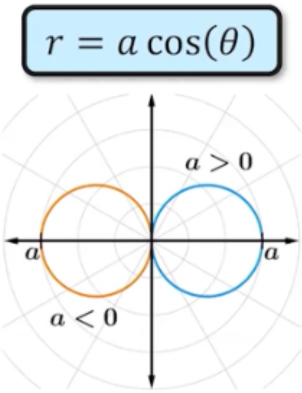
θ	$r(\theta)$
0	
$\pi/12$	
$\pi/6$	
$\pi/4$	
$\pi/3$	
$5\pi/12$	
$\pi/2$	

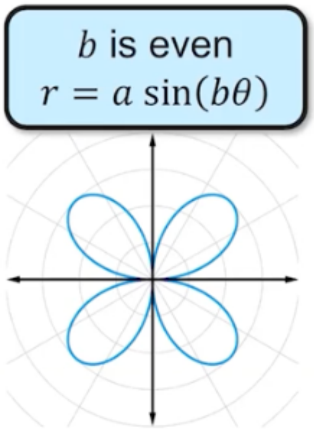
θ	$r(\theta)$
$7\pi/12$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
$11\pi/12$	
π	

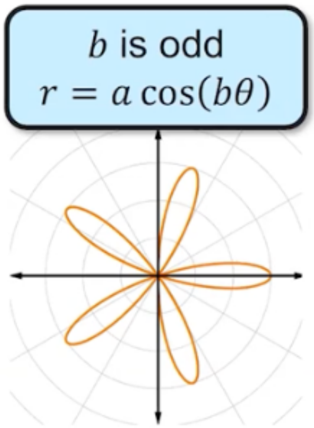
What should we take away?

1. It is helpful to recognize common polar functions.
2. Many polar curves have symmetry, which is helpful when graphing.









Note: If b is even, the polar curve will have $2b$ petals. If b is odd, the polar curve will have b petals.

Topic 3.14 Polar Function Graphs (Daily Video 2)

AP Precalculus

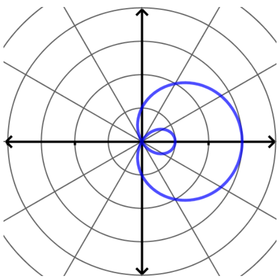
In this video, we will continue to develop our understanding of polar function graphs while identifying common characteristics found in many of these graphs .

Additional Common Polar Graphs

Graphs of the form $r(\theta) = a \pm b\cos(\theta)$ and $r(\theta) = a \pm b\sin(\theta)$

Limacon

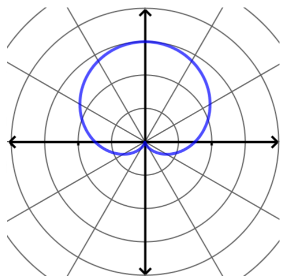
$\frac{a}{b} < 1$



$r(\theta) = 1 + 2\cos(\theta)$

Cardoid

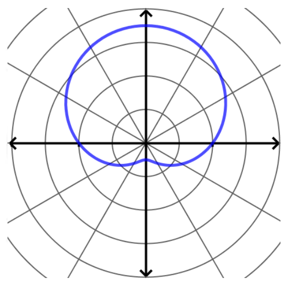
$\frac{a}{b} = 1$



$r(\theta) = 1.5 + 1.5\sin(\theta)$

Dimpled Cardoid

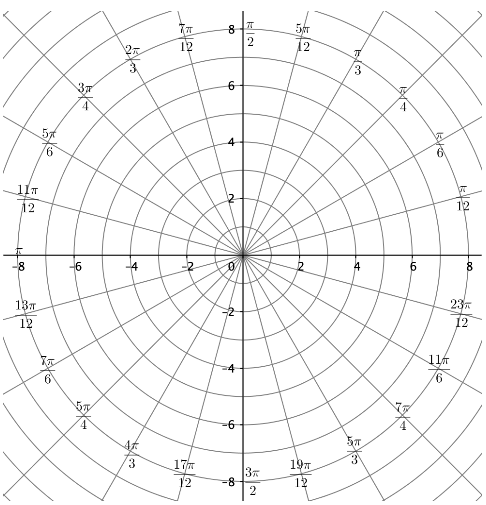
$1 < \frac{a}{b} < 2$



$r(\theta) = 2 + 1.5\sin(\theta)$

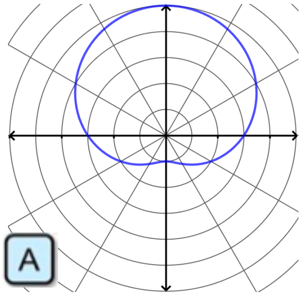
Note: The names of common polar graphs are not tested on the AP Exam.

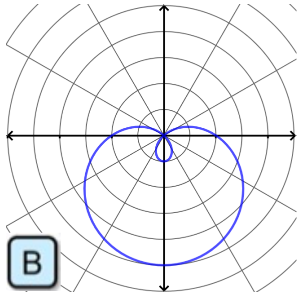
Let's look at an Example!

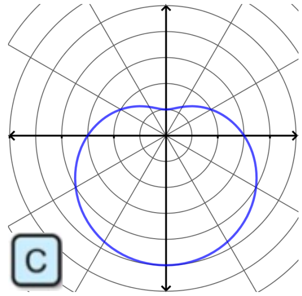


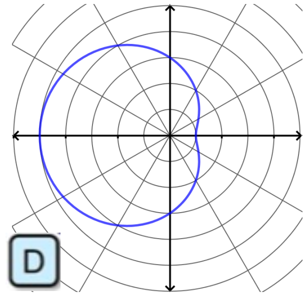
θ	$r(\theta) = 2 - 2\cos(\theta)$
0	
$\pi/6$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$5\pi/6$	
π	

Which of the following is the graph of $r(\theta) = 3 - 2\sin\theta$?

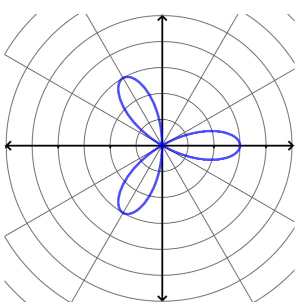
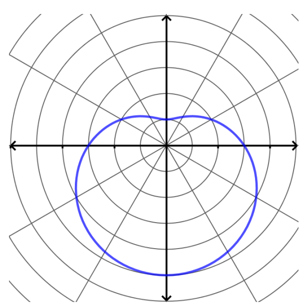
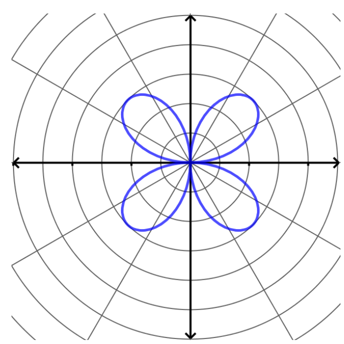








$r(0) =$ ____________ $r\left(\frac{\pi}{2}\right) =$ ____________

Symmetry about ...		
Polar Axis	$\theta = \frac{\pi}{2}$	Pole
$r(-\theta) = r(\theta)$	$-r(-\theta) = r(\theta)$	$-r(\theta) = r(\theta)$
		

Show that the polar function $r(\theta) = 2 - 2\cos(\theta)$ is symmetric about the polar axis.

We must show that ______________________.

$r(-\theta) =$ ______________________________

$r(-\theta) =$ _________________________ $= r(\theta)$

Identity: Since cosine is even, we know that

$\cos(-\theta) =$ __________

What should we take away?

1. We can use symmetry to help when graphing certain polar curves.
2. Knowing common polar curves is helpful when graphing and describing polar functions.

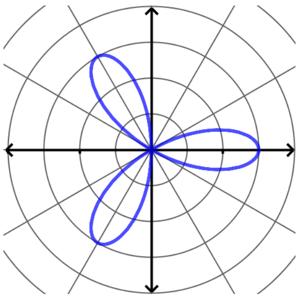
Topic 3.15 Rates of Change in Polar Functions (Daily Video 1)

AP Precalculus

In this video, we will learn how the distance to the origin of a polar graph may increase or decrease. Also, we will introduce the concept of relative extrema in polar functions.

Changes in the distance to the Origin

Consider the polar function $r(\theta) = 3\cos(3\theta)$ graphed below.



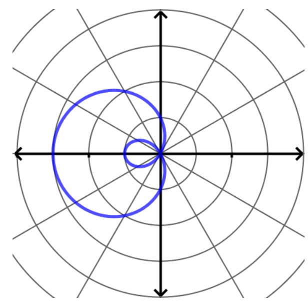
a) $\theta = \frac{\pi}{6}$ $r(\theta) =$ ______________

b) $\theta = \frac{\pi}{4}$ $r(\theta) =$ ______________

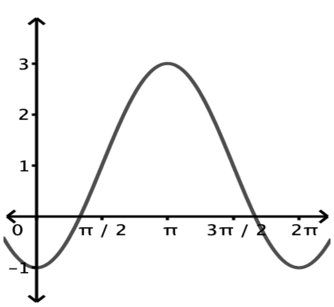
c) $\theta = \frac{\pi}{3}$ $r(\theta) =$ ______________

Distance to Origin	
Increasing	**Decreasing**
r is positive and increasing	r is positive and decreasing
r is negative and decreasing	r is negative and increasing

Let's look at an Example!



The graph of $r(\theta) = 1 - 2\cos(\theta)$ is shown in the figure to the right, along with selected values of r and θ in the table below. Determine if the polar curve is getting closer to the origin, further from the origin, or neither when $\theta = \frac{\pi}{6}$.



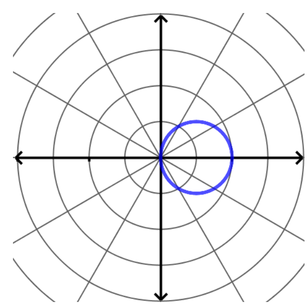
θ	$\pi/6$	$\pi/2$	π	$2\pi/3$
r	$1-\sqrt{3}$	1	3	2

The polar curve is getting ____________________ when $\theta = \frac{\pi}{6}$.

Give a reason for your answer.

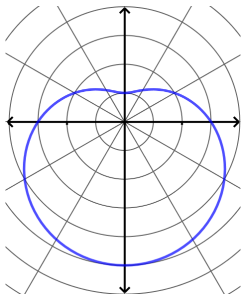
Relative Extrema

If a polar function $r = f(\theta)$ changes from increasing to decreasing or from decreasing to increasing, then the function has a ____________ ____________ corresponding to a point that is relatively furthest or closest to the origin.



Let's look at an Example!

Consider the curve $r(\theta) = -3 - 2\sin(\theta)$. Determine if the polar curve has a relative extremum on the following interval. If the curve does have a relative extremum, classify it as a relative maximum or relative minimum.

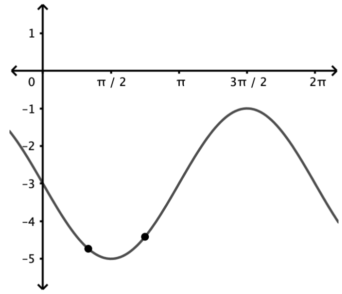


On the interval $\left[\frac{\pi}{3}, \frac{3\pi}{4}\right]$ $\qquad r\left(\frac{\pi}{3}\right) =$ ________

$r\left(\frac{3\pi}{4}\right) =$ ________

The graph of r has a relative ______________

on the interval $\left[\frac{\pi}{3}, \frac{3\pi}{4}\right]$. Justify your answer.



What should we take away?

1. We can determine whether a polar curve is getting closer or further from the origin based on the behavior of the function.
2. Using the behavior of a polar function, we can often determine when the graph reaches a relative minimum or relative maximum distance from the origin.

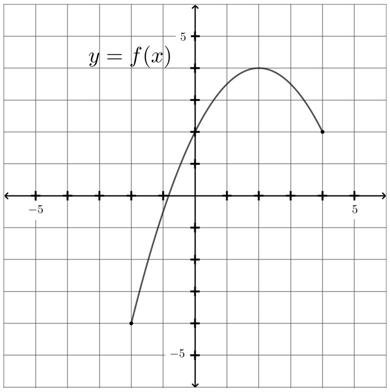
Distance to Origin	
Increasing	**Decreasing**
r is positive and ___________	r is ________ and decreasing
r is ________ and decreasing	r is negative and ___________

Topic 3.15 Rates of Change in Polar Functions (Daily Video 2)

AP Precalculus

In this video, we will learn how that the average rate of change of a polar function helps us understand how r values change as θ changes.

Let's REVIEW! Average Rate of Change



AROC of $f(x)$ over the interval $[a, b]$

$\text{AROC} =$ ______________

Calculate the average rate of change of $f(x)$ over the interval $[-2,4]$. Show your work.

Sketch the "linear path" to verify that your answer above is correct.

Average Rate of Change: Polar Functions

Average rate of change of $r(\theta)$ over the interval $[\theta_1, \theta_2]$ is $\text{AROC} =$ ______________.

Let's look at an Example!

Selected values of θ and r are shown in the table to the right. Find the average rate of change of $r(\theta)$ over the interval $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$. Show all your work.

θ	$\pi/6$	$\pi/2$	π	$7\pi/6$
r	$-2-\sqrt{3}$	-3	7	$5-2\sqrt{3}$

Write an interpretation of the AROC calculated above.

Interpretation: The AROC tells us that r is ___________ by approximately $\frac{3(-1+\sqrt{3})}{\pi}$ units per radian over the interval ___________.

Approximating Values of Polar Functions

Selected values of θ and r are shown in the table to the right. The AROC of $r(\theta)$ over the interval $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ is _________ .

θ	$\pi/6$	$\pi/3$
r	-2	-3

We can use the AROC of $r(\theta)$ to estimate values for r inside the interval.

$$r(\theta_2) \approx r(\theta_1) + \text{AROC} \cdot (\theta_2 - \theta_1)$$

Use the AROC of $r(\theta)$ over the interval $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ to approximate $r\left(\frac{\pi}{4}\right)$ using the values in the table to the right. Show all your work.

θ	$\pi/6$	$\pi/3$
r	-2	-3

$r(\theta_2) \approx r(\theta_1) + \text{AROC} \cdot (\theta_2 - \theta_1)$

$r\left(\frac{\pi}{4}\right) \approx$ _______ + _______ $\cdot$ (   )

$r\left(\frac{\pi}{4}\right) \approx$ _________

What should we take away?

1. We can find the average rate of change of polar functions in a similar way to how we find average rate of change of rectangular functions.

$$\text{AROC} = \frac{r(\theta_2) - r(\theta_1)}{\theta_2 - \theta_1}$$

2. The average rate of change of a polar function can be used to approximate values of polar functions.

$$r(\theta_2) \approx r(\theta_1) + \text{AROC} \cdot (\theta_2 - \theta_1)$$