Solutions to Practice Question 4: Symbolic Manipulations

(A)	(i)	$g(x) = \dfrac{\tan x\,(1 - \sin^2 x)}{\cos x}$ $g(x) = \dfrac{\tan x\,(\cos^2 x)}{\cos x}$ $g(x) = \dfrac{\sin x}{\cos x} \cdot \cos x$ $g(x) = \sin x$	1 pt
	(ii)	$h(x) = \log_3(x^2) + 10\log_9 x$ $h(x) = \log_3(x^2) + 10 \cdot \dfrac{1}{2}\log_3 x$ $h(x) = 2\log_3 x + 5\log_3 x$ $h(x) = 7\log_3 x$, where $x > 0$	1 pt
(B)	(i)	$j(x) = e$ $\dfrac{e^4}{\sqrt{e^x}} = e$ $\dfrac{e^4}{e^{\frac{1}{2}x}} = e$ $e^{\left(4 - \frac{1}{2}x\right)} = e^1$ $4 - \dfrac{1}{2}x = 1$ $-\dfrac{1}{2}x = -3$ $x = 6$	1 pt
	(ii)	$k(x) = \dfrac{2\pi}{3}$ $\cos^{-1}(5x) = \dfrac{2\pi}{3}$ $5x = -\dfrac{1}{2}$ $x = -\dfrac{1}{10}$	1 pt

(C)	$m(x) = 6\csc\left(x - \frac{\pi}{6}\right) = 12$ $\csc\left(x - \frac{\pi}{6}\right) = 2$ $\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$ $x - \frac{\pi}{6} = \frac{\pi}{6} + 2\pi k$ or $x - \frac{\pi}{6} = \frac{5\pi}{6} + 2\pi k$ where k is an integer $x = \frac{\pi}{3} + 2\pi k$ or $x = \pi + 2\pi k$ where k is an integer	Solves $\sin\theta = \frac{1}{2}$ for $0 \le \theta \le 2\pi$ 1 pt General solution expression 1 pt