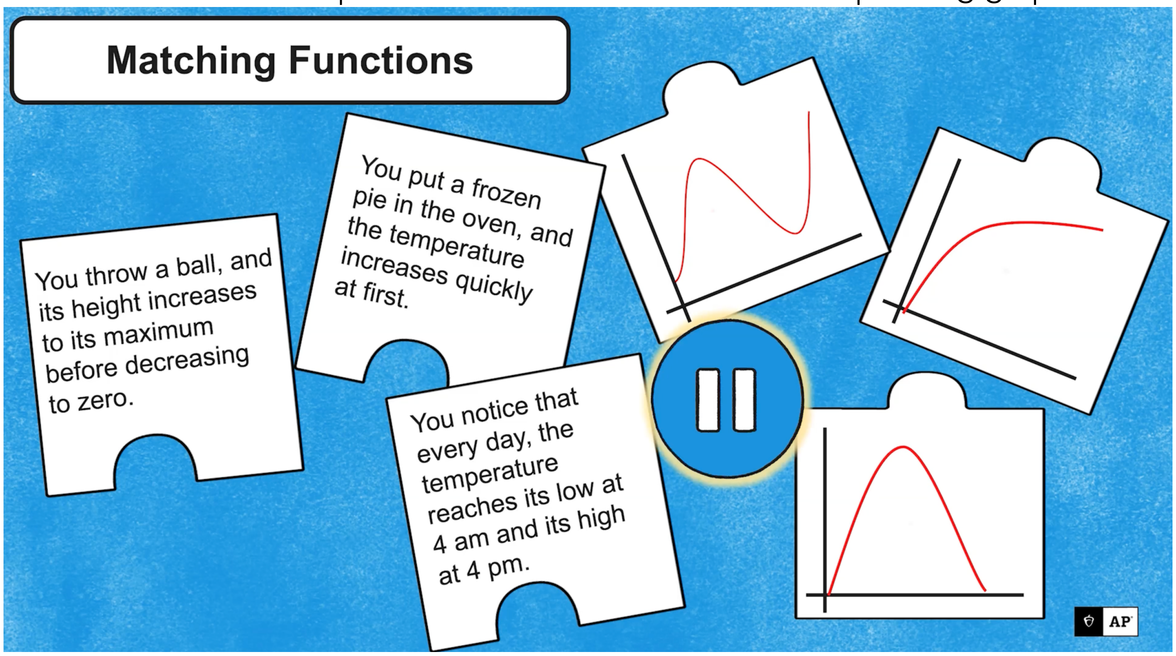
Topic 3.1 Periodic Phenomena (Daily Video 1)

AP Precalculus

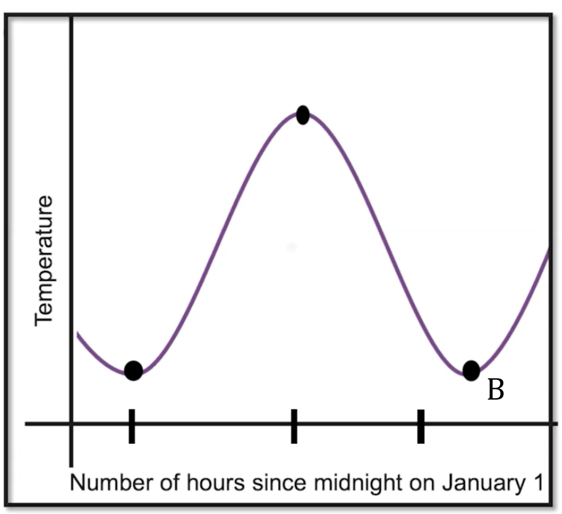
In this video, we will define periodic relationships and explore different contexts that are well-modeled with periodic functions.

Let's Review!

Match the verbal description of each function with its corresponding graph.



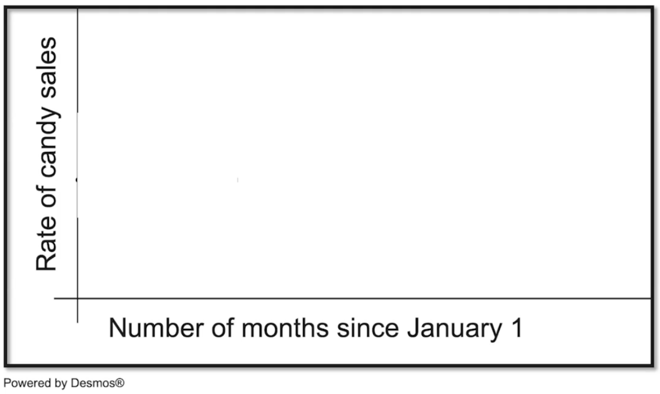
Patterns of Repeating Output Values

	You notice that every day, the temperature reaches its low at 4 am and its high at 4 pm. Every 24 hours, this pattern repeats. Label the values on the x-axis. The second minimum point (labeled B) occurs ____ hours after midnight on January 1. How many hours after midnight on January 1 will another high temperature occur?

A function is periodic if the output values ________ themselves over consecutive ______________ intervals of the domain. Why is the previous problem an example of a periodic function?

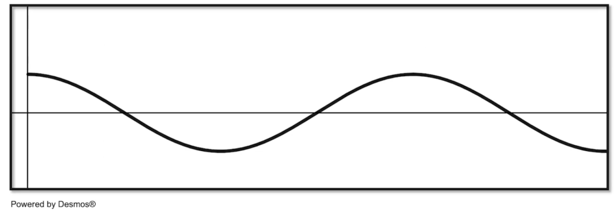
Let's Look at an EXAMPLE!

Candy sales remain steady for the first 8 months of the year, rise quickly until Halloween, and then decrease back to the previous sales rate in January.

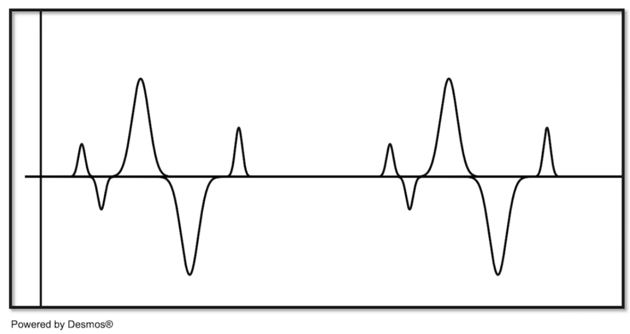


It is the ______________ of these functions over equal-sized intervals that makes them periodic.

The displacement of a buoy as it rises and falls with the ocean waves.



An electrocardiogram (EKG) recording of the electrical signal of your heartbeat over time.



What should we take away?

- A function is periodic if the ________ __________ of the function demonstrate a repeating pattern over successive, equal-length intervals of the domain.
- One _______ of a periodic function gives enough information to graph the function over any interval of the domain.

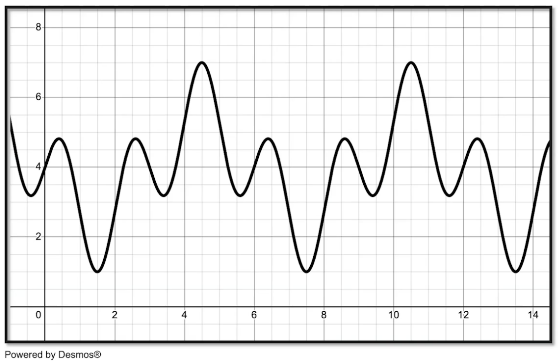
Topic 3.1 Periodic Phenomena (Daily Video 2)

AP Precalculus

In this video, we will describe the characteristics of a periodic function.

Let's look at an EXAMPLE!

Is this function periodic?



A function is periodic if the ________ values repeat themselves over consecutive __________ intervals of the domain.

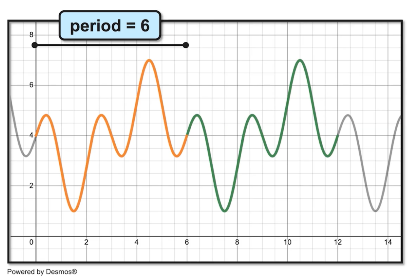
The pattern in the output values repeats every ____ units. The __________ is the length over which the pattern repeats itself.

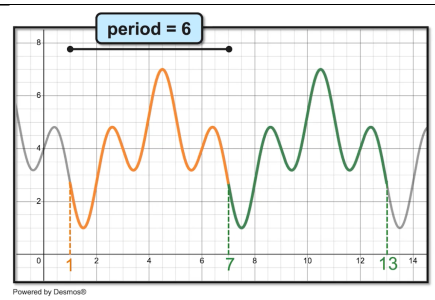
Color the graph to show the pattern between $x = 0$ and $x = 6$ repeats again between $x = 6$ and $x = 12$.

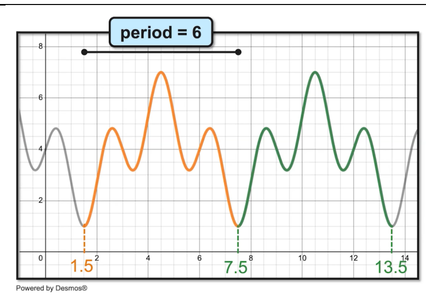
Definition: The period of a function is the smallest positive value k such that $f(x + k) = f(x)$ for all x in the domain.

- The period is always a ________ value.
- The period is the _________ interval of the domain over which the pattern of the output values starts to repeat.
- Any one period of the graph of the function will contain _____ the information about the pattern or the phenomena.

We can start at different locations on the graph and the period is the ________. For these graphs, the pattern repeats every six units no matter where we start along the x-axis measuring those six units.







The period can also describe local behavior of a function. Sketch on the graphs to show the function is increasing at $x = 8$, $x = 8 - 6$ and $x = 8 + 6$.

What should we take away?

- The period is the shortest interval of the domain over which the pattern in the output values starts to repeat.
- We can estimate period by investigating an input interval in which the pattern of the output values starts to repeat.
- Often it is not enough to consider when a particular output value repeats, and we must instead look for when the pattern of output values repeats.

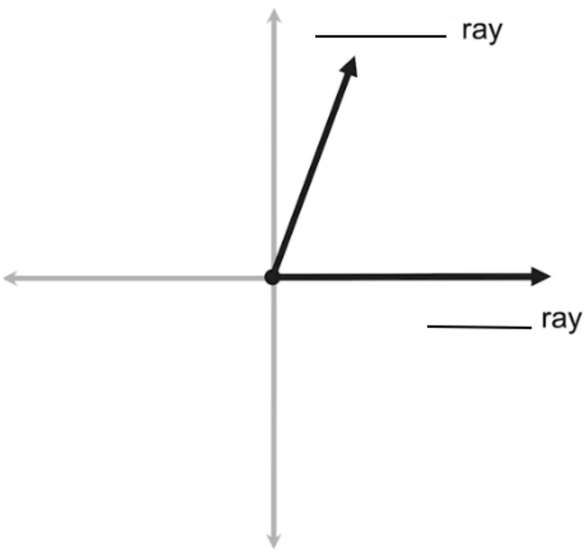
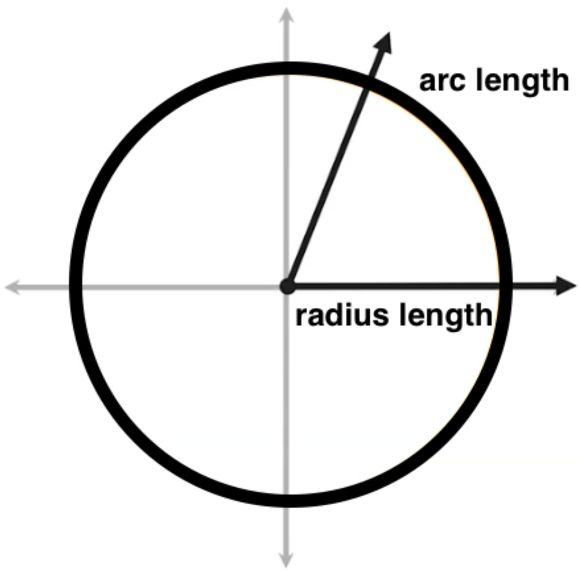
Topic 3.2 Sine, Cosine and Tangent (Daily Video 1)

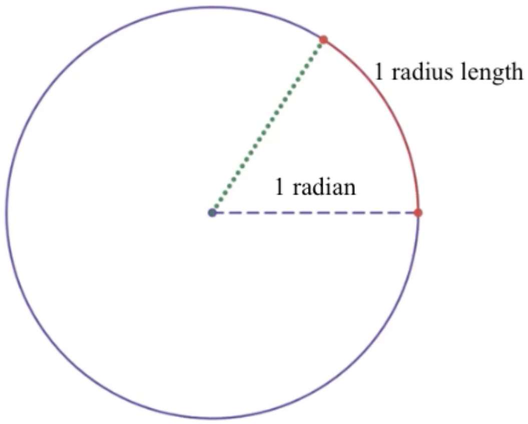
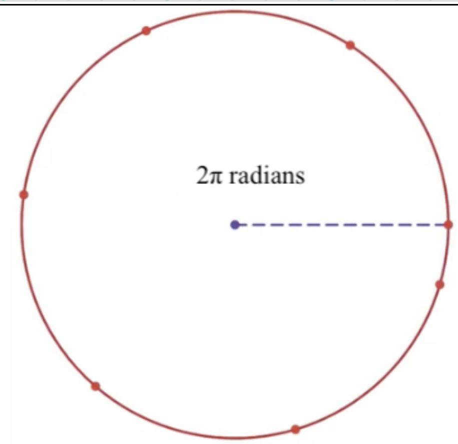
AP Precalculus

In this video, we will examine the radian as a unit of angle measure and explore relationships between the radian measure of an angle and other attributes of the angle.

Let's REVIEW!

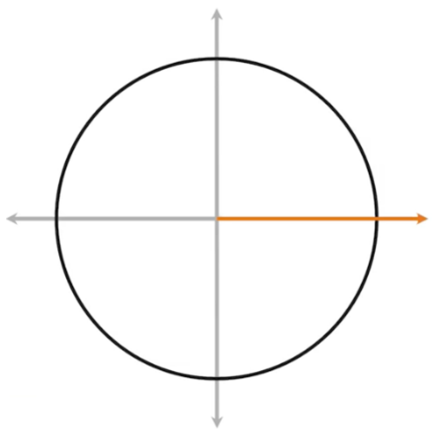
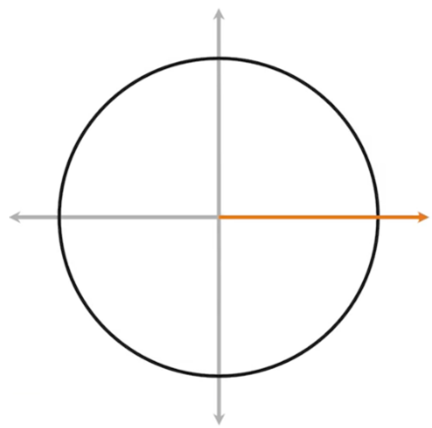
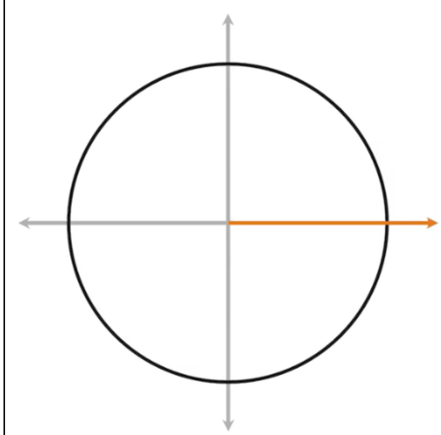
- An angle is a geometric object that consists of two rays with a ____________ ____________.
- When we measure an angle, we are measuring the amount of __________ between the two rays.

<u>Angles in the Coordinate Plane</u> An angle is in standard position when… • The angle's vertex is at the ________. • One of the angle's rays lies along the positive __________. • We use positive values to indicate the amount of rotation between the angle's rays in the _______________ direction.	
<u>Measuring Amounts of Rotation</u> Radian measure = Radian measure gives the number of ________ of the radius length that fit into the arc length.	

An angle with a measure of ____________ opens (or rotates) an entire circle.

Let's PRACTICE!

Draw an angle in standard position that has a measure of 0.5 radians.	Draw an angle in standard position that has a measure of π radians.	Draw an angle in standard position that has a measure of $\frac{\pi}{3}$ radians.
		

What should we take away?

- An angle is in _____________ position when its vertex is at the origin, one of the angle's rays lies along the positive x-axis, and positive values are used to denote the amount of rotation in a counterclockwise direction.
- A radian is a unit of angle measure that relates the arc length cut off by the angle's rays to the ____________ length of the circle.
- An angle has a measure of 1 radian when the arc length cut off by the angle's rays is equal to the length of the ____________.
- An angle that rotates a full circle has a measure of _____ radians.

Topic 3.2 Sine, Cosine and Tangent (Daily Video 2)

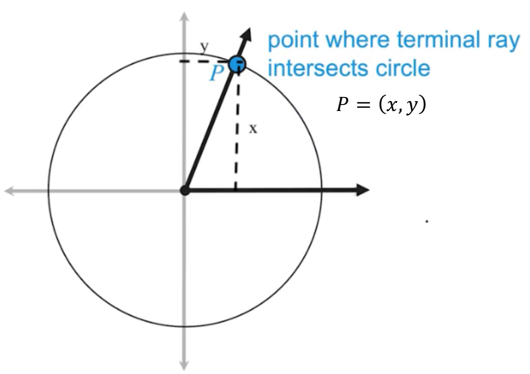
AP Precalculus

In this video, we will define sine, cosine, and tangent and investigate how to utilize the functions to describe points on the unit circle.

Let's REVIEW!

Radian measure =

When measuring in radians, θ gives the number of copies of the radius length that fit into the arc length. We are measuring the arc length with a ruler the length of the radius.



<u>Horizontal Displacement</u>

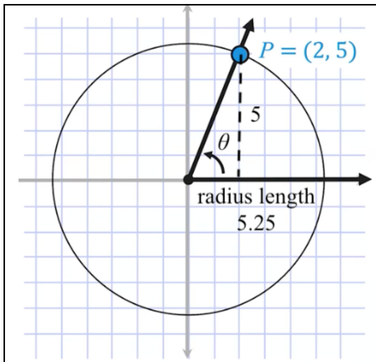
The x-coordinate of point P gives the horizontal displacement of the point from the y-axis.

Horizontal displacement can be:

- Positive (point is to the ________ of the y-axis)
- Negative (point is to the ________ of the y-axis)

Vertical displacement can be:

- Positive (point is to the ________ of the x-axis)
- Negative (point is to the ________ of the x-axis)

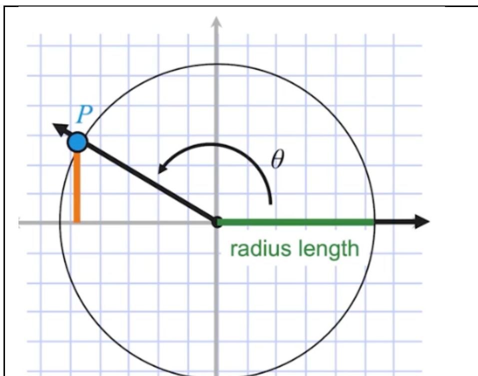


<u>Measuring Vertical Displacement: Sine Ratio</u>

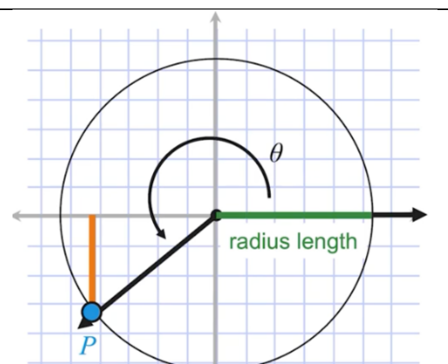
2 = _____________ displacement right of y-axis

5 = _____________ above x-axis

$\sin(\theta) =$ $\qquad\qquad$ $\sin(\theta) = \frac{5}{5.25} \approx 0.952$



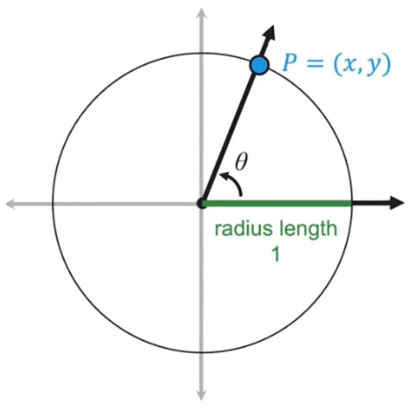
sin is positive when the terminal ray intersects the circle above the x-axis



sin is negative when the terminal ray intersects the circle below the x-axis

Properties of $\sin(\theta)$

- The value of $\sin(\theta)$ varies as the angle measure, θ, varies.
- The value of $\sin(\theta)$ is always between ____ and _____. In other words, $-1 \le \sin(\theta) \le 1$.



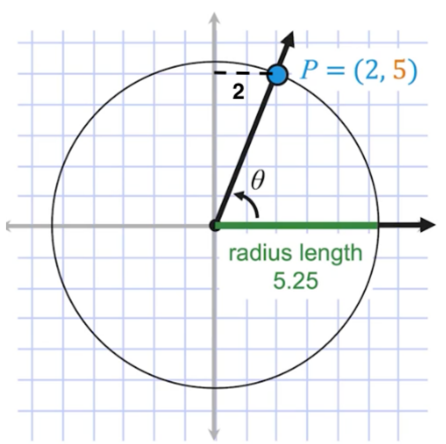
A unit circle has a radius of _____.

On a unit circle, the sin ratio is $\sin(\theta) = \frac{y}{1} = y$.

This means the _____________ of P is the value of $\sin(\theta)$.

On a unit circle, the cos ratio is $\cos(\theta) = \frac{x}{1} = x$.

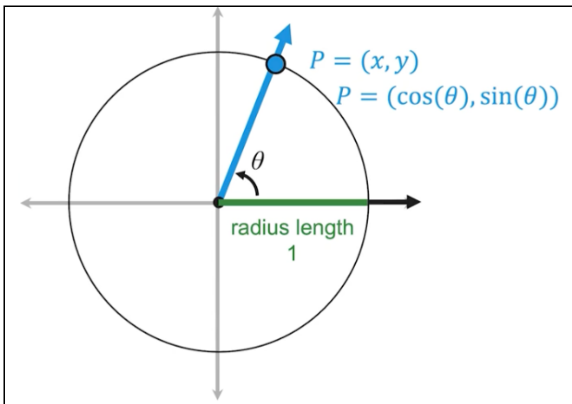
This means the _____________ of P is the value of $\cos(\theta)$.



Measuring Horizontal Displacement: Cos Ratio

$\cos(\theta) =$

$\cos(\theta) = \frac{2}{5.25} \approx 0.381$



The tangent ratio is the same as the ________ of the terminal ray.

$\tan(\theta) =$

On a unit circle, the tan ratio is $\tan(\theta) =$

What should we take away?

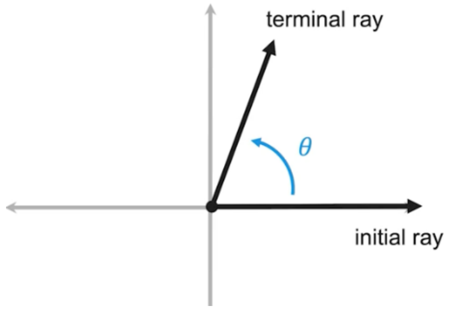
- The sin ratio is the vertical displacement of point *P* from the x-axis to the radius of the circle. In a unit circle, sin(θ) is the value of the y-coordinate of point *P*.
- The cos ratio is the horizontal displacement of point *P* from the y-axis to the radius of the circle. In a unit circle, cos(θ) is the value of the x-coordinate of point *P*.
- Tan(θ) is the ratio of the vertical displacement of point *P* from the x-axis to the horizontal displacement of point *P* from the y-axis. In the unit circle, tan(θ) is the ratio of sin(θ) to cos(θ).

Topic 3.3 Sine and Cosine Function Values (Daily Video 1)

AP Precalculus

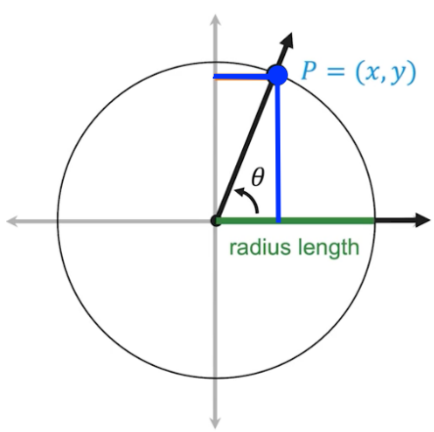
In this video, we will learn how to determine the coordinates of a point where the terminal ray of an angle intersects a circle.

Let's REVIEW!



An angle is in standard position when…

- The vertex is at the __________
- One of the rays lies on the positive ________
- A positive angle is measured with a _________________ rotation.



The sine ratio is…

$$\sin(\theta) = \frac{\qquad}{radius\ length} = \frac{y}{\quad}$$

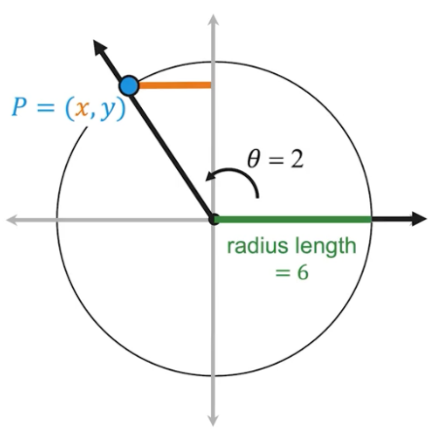
In the unit circle, the y-coordinate of P is the value of $\sin(\theta)$.

The cosine ratio is…

$$\cos(\theta) = \frac{\qquad}{radius\ length} = \frac{x}{\quad}$$

In the unit circle, the x-coordinate of P is the value of $\cos(\theta)$.

Let's look at an EXAMPLE!



Determining Coordinates: Suppose an angle with a measure of 2 radians is inscribed in a circle with a radius of 6 units. What are the coordinates where the terminal ray intersects the circle?

Using $\cos(\theta) = \frac{x}{r}$

$\cos(2) =$ _____

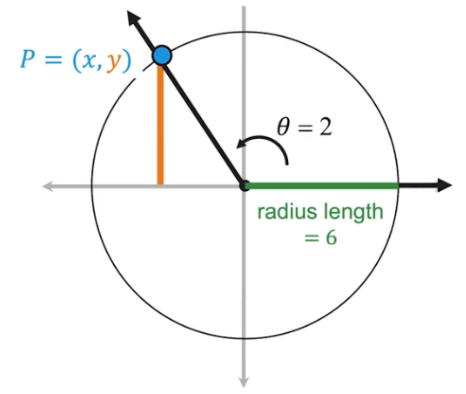
$6 * \cos(\_\_) =$ ____

$6 * -0.416 \approx x$

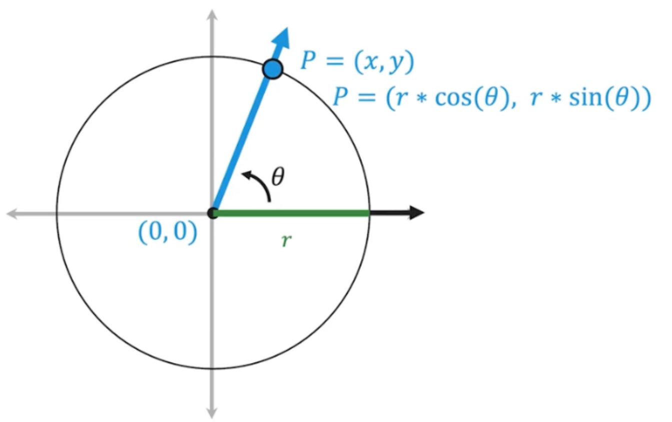
_________ $\approx x$

Make sure your calculator is in radian mode

***Note:** On the AP Exam you should not use a rounded off answer ($\cos(2) = -0.416$) in a later calculation. Your answer should be $x = 6\cos(2) \approx -2.4968\ldots$, which to 3 decimal places is $x \approx -2.497$ or -2.496.

	Using $\sin(\theta) = \frac{y}{r}$ $\sin(2) =$ ______ $6 * \sin($___$) =$ ____ $6 * 0.909 \approx y$ ________ $\approx y$	The coordinates of point P are $P =$ (________, ________) ***Note:** See note above. Your answer should be $y = 6\sin(2) \approx 5.4557 \ldots$, which to 3 decimal places is $y \approx 5.455$ or 5.456.

Coordinates of Points in a Plane: If point $P = (x, y)$ is on the terminal side of an angle in standard position with radius, r, and angle measure of θ, then the coordinates of the point can be written in terms of r and θ as $P = (r * \cos(\theta), r * \sin(\theta))$.

	The terminal ray contains the origin and the point P. The slope of the ray is calculated as follows: $\tan(\theta) = \frac{r * \sin(\theta) - 0}{\quad}$ $\tan(\theta) = \frac{r * \sin(\theta)}{\quad} = \frac{\quad}{\cos(\theta)}$ $\tan(\theta) =$

What should we take away?

- The coordinates (__________, __________) describe the location where the terminal ray of an angle with measure θ intersects a circle with radius r.
- The slope of the terminal ray, $\tan(\theta)$, is always given by the ratio __________.

Topic 3.3 Sine and Cosine Function Values (Daily Video 2)

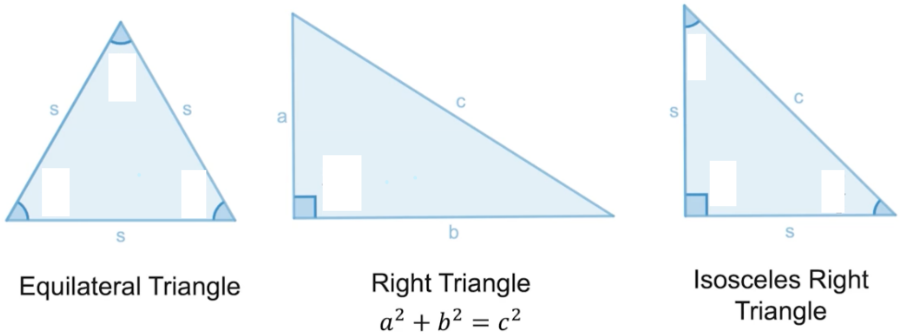
AP Precalculus

In this video, we will learn how to use the geometries of isosceles and equilateral triangles to determine the exact values of the sine and cosine functions for certain angle measures.

Let's REVIEW!

If point $P = (x, y)$ is on the terminal side of an angle in standard position with radius, r, and angle measure of θ, then the coordinates of the point can be written in terms of r and θ as
$P = ($__________, __________$)$.

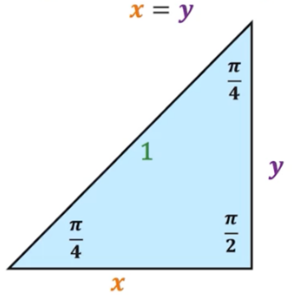
Label each angle of the special triangles in terms of radians.



Coordinates at $\frac{\pi}{4}$

Right Isosceles Triangle

$x = y$



If the radius is 1, what are the values of x and y?
For simplicity, let's call x and y both s.

$s^2 + s^2 =$ _____

$2s^2 =$ _____

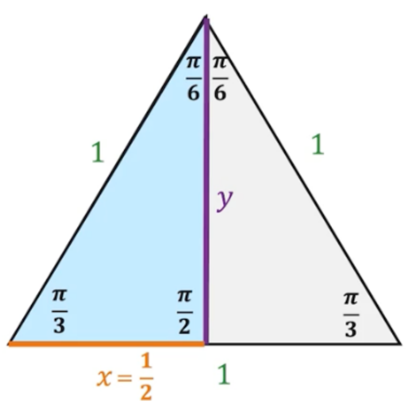
$s^2 =$ _____

$s = \pm\sqrt{\frac{1}{2}} =$ ______

On the unit circle, the coordinates of $P = ($______, ______$)$.

Coordinates at $\frac{\pi}{3}$

Half of an Equilateral Triangle



$x = \frac{1}{2}$, because it is half of the length of the side of the equilateral triangle. Using the Pythagorean Theorem to solve for y:

$x^2 + y^2 = 1^2$

_____ $+ y^2 = 1^2$

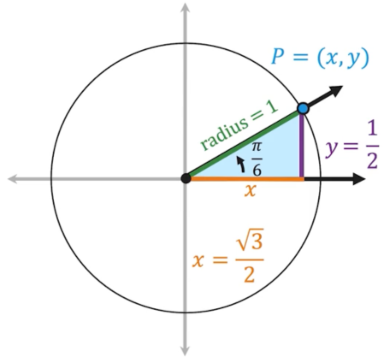
_____ $+ y^2 = 1^2$

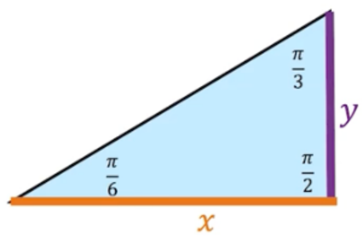
$y^2 =$ ____ $\qquad y = \pm\sqrt{\frac{3}{4}} =$ ______

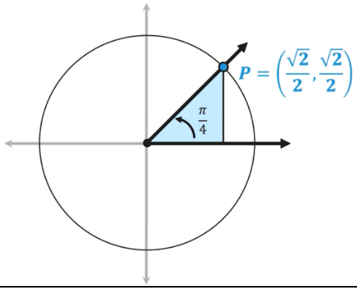
On the unit circle, the coordinates of $P = ($______, ______$)$.

Coordinates at $\frac{\pi}{6}$

These values can be determined based on the values we used for the previous triangle.



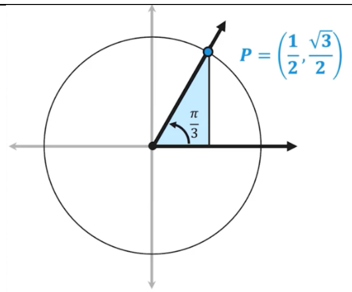




In a unit circle
$(x, y) = (\cos(\theta), \sin(\theta))$

$$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \left(\cos\left(\frac{\pi}{4}\right), \sin\left(\frac{\pi}{4}\right)\right)$$

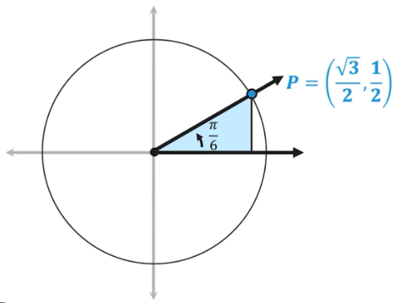
$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\quad$ $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$



In a unit circle
$(x, y) = (\cos(\theta), \sin(\theta))$

$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) = \left(\cos\left(\frac{\pi}{3}\right), \sin\left(\frac{\pi}{3}\right)\right)$$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ $\quad$ $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$



In a unit circle
$(x, y) = (\cos(\theta), \sin(\theta))$

$$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) = \left(\cos\left(\frac{\pi}{6}\right), \sin\left(\frac{\pi}{6}\right)\right)$$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ $\quad$ $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

What should we take away?

The exact values of $\sin(\theta)$ and $\cos(\theta)$ for angle measures of $\theta = \frac{\pi}{4}, \frac{\pi}{3}$, and $\frac{\pi}{6}$:

$$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \qquad \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \qquad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \qquad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

Topic 3.4 Sine and Cosine Function Graphs (Daily Video 2)

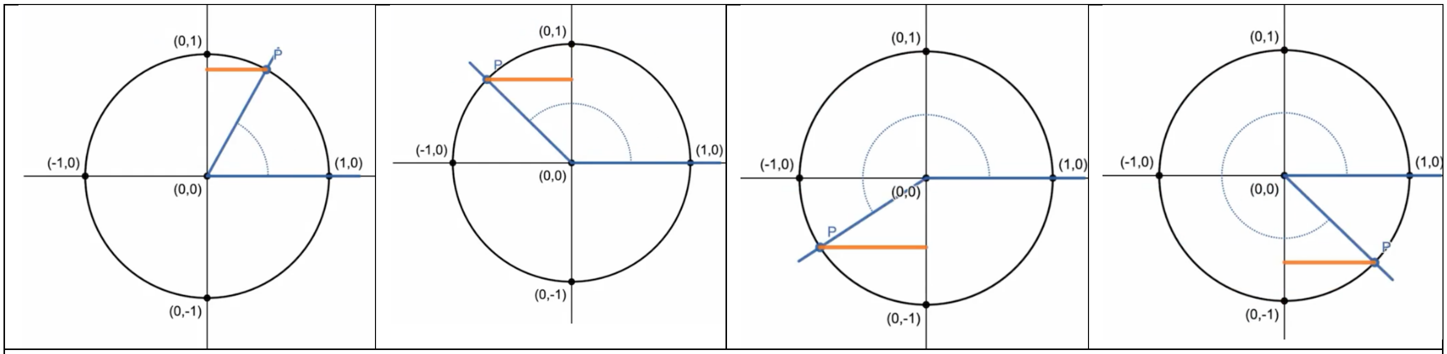
AP Precalculus

In this video, we will construct a graph of the cosine function and explore the properties of this function.

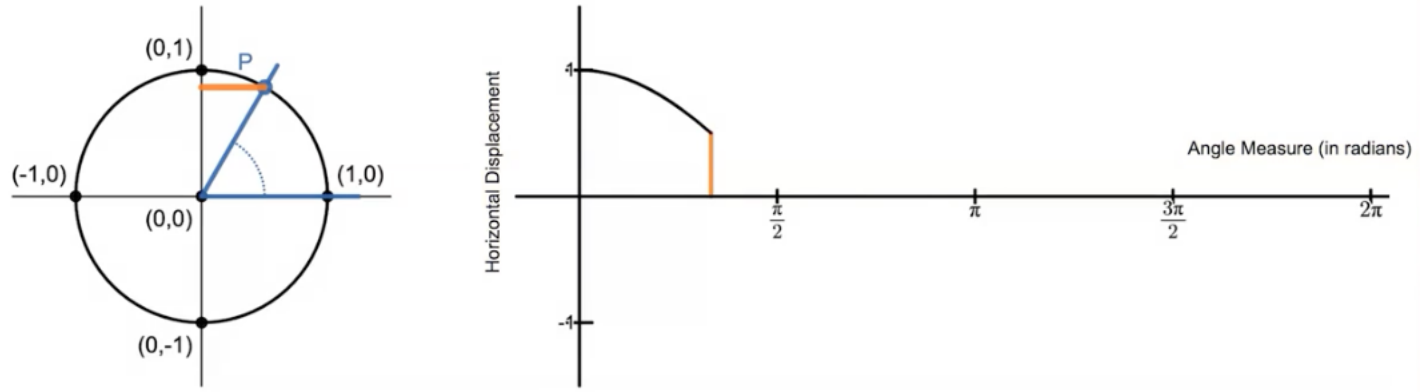
Let's REVIEW!

In a unit circle, the x-coordinate of a point on the circle, P, is the value of ______________.

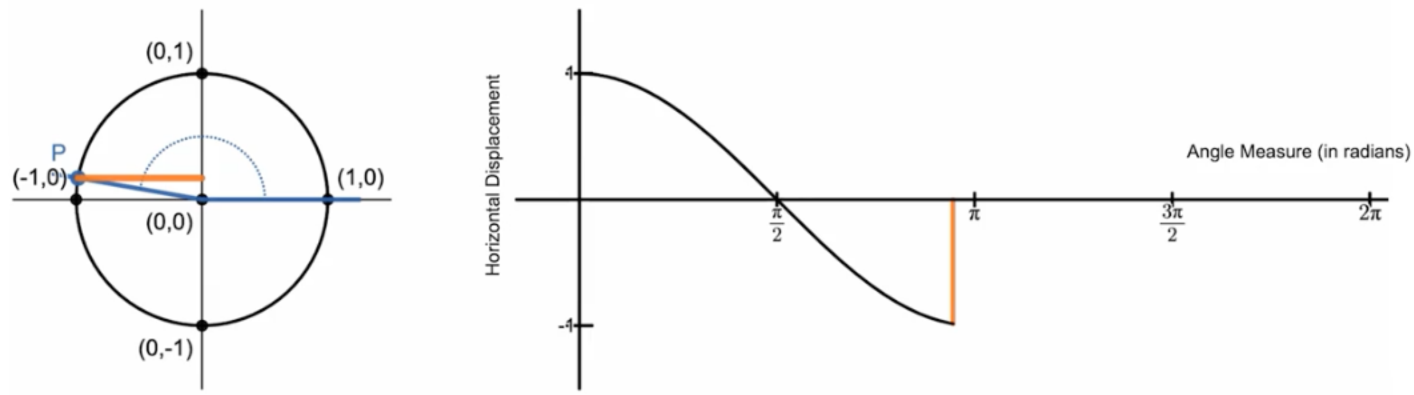
How the value of $\cos(\theta)$ changes as the angle measure, θ, changes?



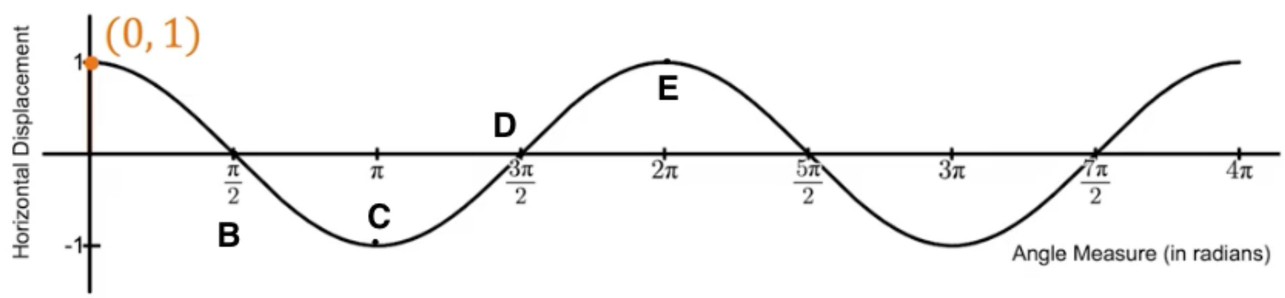
The horizontal displacement shown on the unit circle has the same length as the segment above the x-axis.



Here the segment is below the x-axis. This is because the horizontal segment on the unit circle is to the left of the y-axis, which means the horizontal displacement is _________.



On the graph of the cosine function, label the points B, C, D and E with their coordinates.



What should we take away?

- Tracking the horizontal displacement from the y-axis of a point P on the unit circle as the angle measure varies gives the graph of the _________ function.
- The range of the cosine function is [-1, 1], because the point is never more than 1 unit from the __________.
- The __________ of the cosine function is all real numbers, representing all possible angle measures.
- The cosine function is _________________.

Math Medic

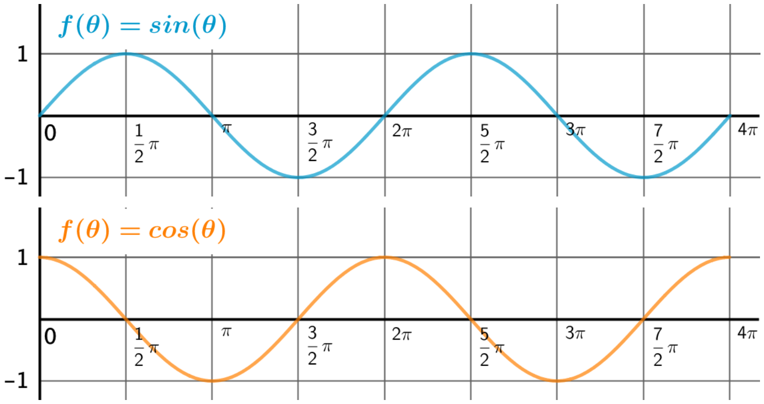
Topic 3.5 Sinusoidal Functions (Daily Video 1)

AP Precalculus

In this video, we will establish the properties of the sinusoidal functions $\boldsymbol{f(\theta) = \sin(\theta)}$ and $\boldsymbol{f(\theta) = \cos(\theta)}$, and we will relate these properties back to the unit circle.

Let's REVIEW!

You can use the website www.geogebra.org/m/tyaseusf to explore the relationships between the graphs of the sine function and cosine function with the vertical and horizontal displacement on the unit circle. Here are two periods of the sin and cos functions.



- When the cosine graph is shifted ______ units to the right, it is the same as the sine graph. This means $f(\theta) = \sin(\theta) = \cos($__________$)$.

- When the sine graph is shifted ______ units to the left, it is the same as the cosine graph. This means $f(\theta) = \cos(\theta) = \sin($__________$)$.

- In other words, there is a phase shift of ____ units between the sine and cosine graphs.

Let's Look at an EXAMPLE!

- The **period** of the graphs of $f(\theta) = \sin(\theta)$ and $f(\theta) = \cos(\theta)$ is ____ radians because that is one revolution around the unit circle.

- The **frequency** of the graphs of $f(\theta) = \sin(\theta)$ and $f(\theta) = \cos(\theta)$ is the reciprocal of the period. So, it is ______.

- The **amplitude** of the graphs of $f(\theta) = \sin(\theta)$ and $f(\theta) = \cos(\theta)$ is ___, because it is the distance from the midline to the maximum or minimum on the graph.

- The **midline** of the graphs of $f(\theta) = \sin(\theta)$ and $f(\theta) = \cos(\theta)$ is _____ and it splits the graph horizontally down the middle.

- The **symmetry** of the graph of $f(\theta) = \sin(\theta)$ is rotationally symmetric about the orgin and that means it has _____ symmetry. In other words, $\sin(-\theta) = -\sin(\theta)$.

- The **symmetry** of the graph of $f(\theta) = \cos(\theta)$ is reflectively symmetric about the y-axis it has _____ symmetry. In other words, $\cos(-\theta) = \cos(\theta)$.

- As input values increase, the graphs of sine and cosine oscillate between concave _____and concave _______.

- As input values increase, the graphs of sine and cosine oscillate between ___________ and ______________.

What should we take away?

- The graphs of $y = \sin(\theta)$ and $y = \cos(\theta)$ have
  - period 2π and frequency of $\frac{1}{2\pi}$
  - amplitude $|a| = 1$ and midline $y = 0$.
- The graph of $y = \sin(\theta)$ has odd symmetry and the graph of $y = \cos(\theta)$ has even symmetry.
- As input values increase, the graphs of sinusoidal functions oscillate between increasing and decreasing and concave up and concave down.
- The properties of the sine and cosine graphs are tied into positions of a point on the unit circle.

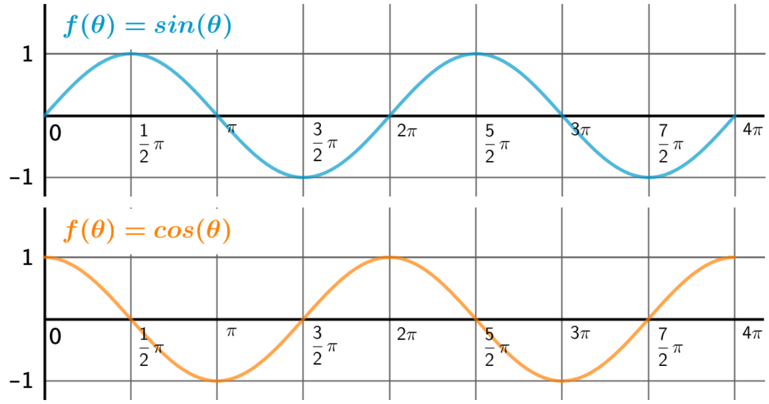
Topic 3.5 Sinusoidal Functions (Daily Video 2)

AP Precalculus

In this video, we will determine the properties of sinusoidal functions based on their graphs.

Let's REVIEW!

Look at your notes to refresh your memory about the following features of the sine and cosine graphs: period & frequency, amplitude & midline, symmetry, concavity and increasing/decreasing.



Let's PRACTICE!

1) The period of $y = \sin(\theta)$ and $y = \cos(\theta)$ is _______.

   This is because a point rotates through a ______ radian angle around the _______ before repeating itself.

2) The range of $h(t) = \sin(t)$ and $j(t) = \cos(t)$ is ___________.

   This is because the ___________ coordinates of all points on a unit circle are between (and include) ___ and ___.

3) If you know the frequency of a sinusoid, then you can also determine the period. True or false? Justify your answer.

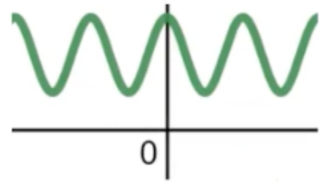
   _______, because the period and frequency are ___________ of each other.

4) Is it possible for a sinusoid to have neither even nor odd symmetry? Justify your answer.

   _____, because the graph could be transformed. (Sketch an example.)

5) Because of its even symmetry, this curve was created with the cosine function, not the sine function. True or False? Justify your answer.

__________, because the graphs can be made identical with a____________ translation (phase shift).



6) We know that $\cos(\theta) = \sin\left(\theta + \frac{\pi}{2}\right)$ for all angles θ. Which of the following is also true for all θ?

(a) $\cos(\theta) = \sin\left(\theta - \frac{\pi}{2}\right)$

(b) $\cos(\theta) = \sin\left(\theta + \frac{3\pi}{2}\right)$

(c) $\cos(\theta) = \sin\left(\theta + \frac{5\pi}{2}\right)$

(d) $\cos(\theta) = \sin\left(\theta + \frac{7\pi}{2}\right)$

Note that angles of $\theta + \frac{\pi}{2}$ and ___________ differ by 2π. And this means the angles are _______________.

What should we take away?

- The unit circle is useful for helping to understand the parent graphs of $y = \sin(\theta)$ and $y = \cos(\theta)$.
- We can extend our knowledge to answer questions about all sinusoids, including those that have been transformed.

Topic 3.6 Sinusoidal Function Transformations (Daily Video 1)

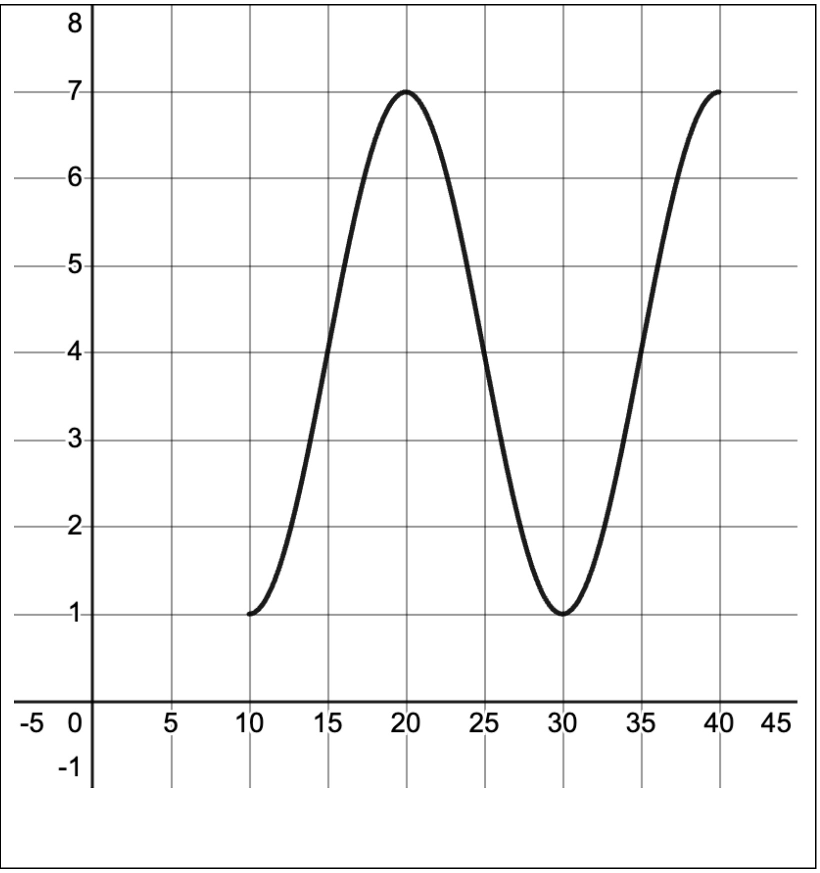
AP Precalculus

In this video, we will identify and apply function transformations (e.g., translations and dilations, both horizontal and vertical) to sinusoidal function equations.

Let's REVIEW!

Parent Function	Transformed Function
$y = x^2$ These same transformations can be applied to sinusoidal functions!	$y = -2\left(\frac{1}{5}(x-1)^2\right) + 7$ Reflection over x-axis Vertical Dilation by 2 Horizontal Dilation by 5 Horizontal Translation by +1 (right) Vertical Translation by +7 (up)

Let's Look at an EXAMPLE!

Period Frequency Amplitude Midline Symmetry Sin or Cos? Equation	

What should we take away?

- Much of the new vocabulary we used to describe sinusoids relates to transformation vocabulary from earlier in this course.
- By relating the properties of a transformed sinusoid to the properties of a parent/basic sinusoid $y = \sin(\theta)$ or $y = \cos(\theta)$, we can write a function equation for a given sinusoid graph.

Topic 3.6 Sinusoidal Function Transformations (Daily Video 2)

AP Precalculus

In this video, we will write the sinusoidal function equation for a provided graph by determining and applying the necessary transformations to a parent sinusoidal function.

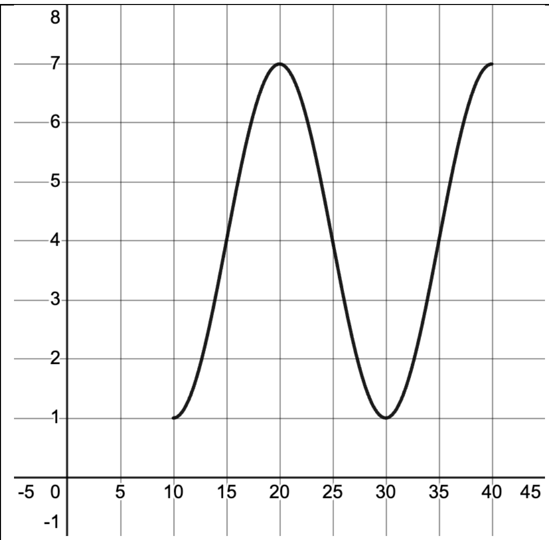
Let's REVIEW!

For the given sinusoid graph, write a function equation.

$P = 20 \qquad F = \frac{1}{20} \qquad |a| = 3$

Vertical translation = 4 even

What are 3 different equations to represent this function?



Explore https://www.geogebra.org/m/vtpv6wmp to see how nested parentheses are applied to a parent function.

Let's PRACTICE!

For the given sinusoid graph, write a function equation. A minimum lies on $x = 0.2$.

Period

Frequency

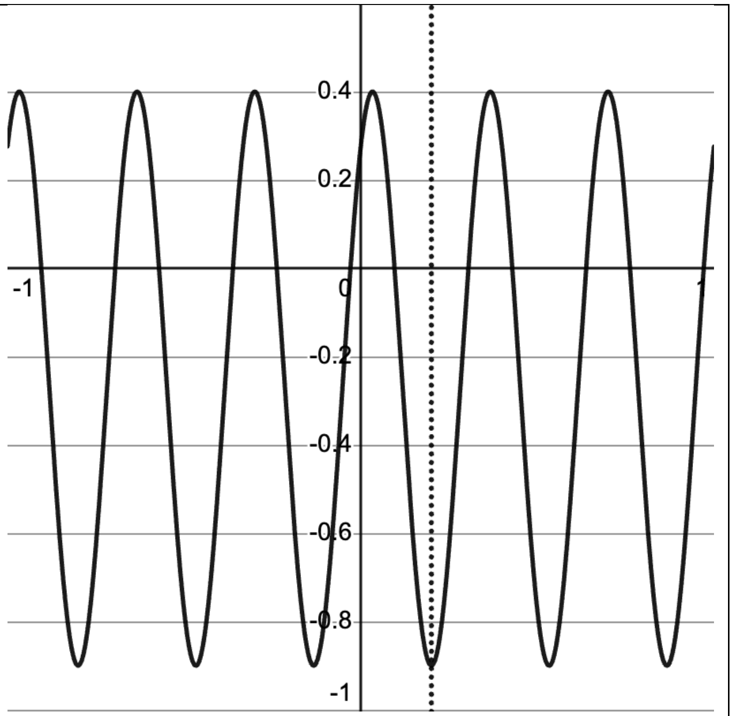
Amplitude

Midline

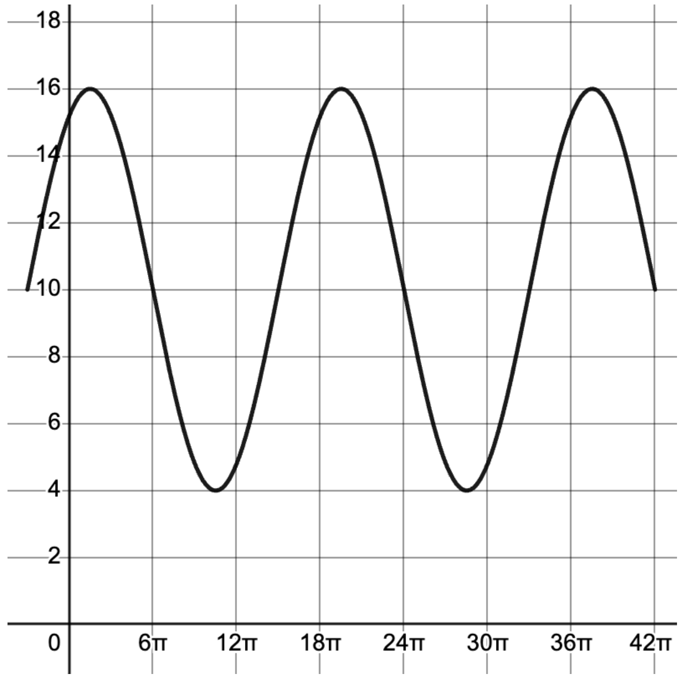
Symmetry

Sin or Cos?

Equation



Let's PRACTICE!

For the given sinusoid graph, write a function equation.	

What should we take away?

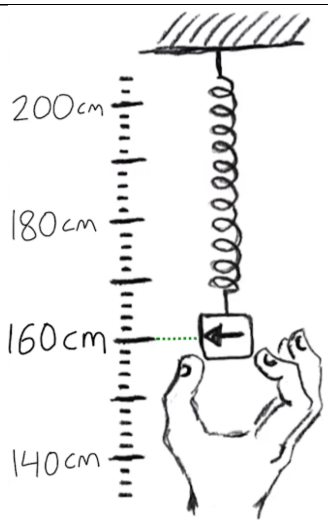
- Given properties about a sinusoid (numerically or graphically), we can write a sinusoid equation that models it.

Topic 3.7 Sinusoidal Function Context and Data Modeling (Daily Video 1)

AP Precalculus

In this video, we will use numerical data describing a periodic phenomenon to construct a sinusoidal function model.

Let's Look at an EXAMPLE!



A mass hangs from the end of a spring and rests 174 cm above the ground.

You pull the mass downward until it is 160 cm above the ground, and you start a stopwatch at the exact time you let go of the mass. The mass oscillates up and down, completing 20 full cycles every minute.

What an equation, $h(t)$, that models the height of the mass above the ground as a function of time, t, in seconds.

Assume simple harmonic motion.

- Rests _____ cm above the ground.
- Low point at ____ cm above the ground at t = 0 seconds.
- ____ full cycles every minute or 60 seconds.

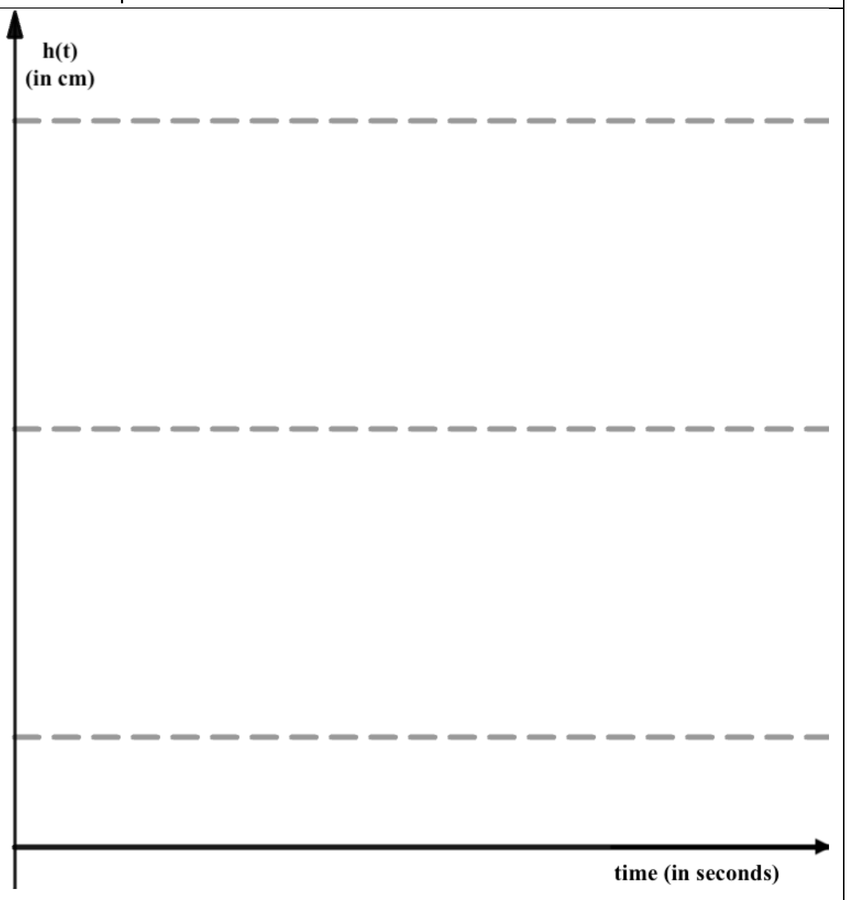
Frequency =

Period =

$h(t) =$ ___________________

Why did the instructor in the video not need a phase shift in the equation?

Describe how you could determine when the weight is at 180 cm above the ground.



What should we take away?

Given numerical and/or graphical data about a periodic phenomenon, we can construct a sinusoidal function model for it.

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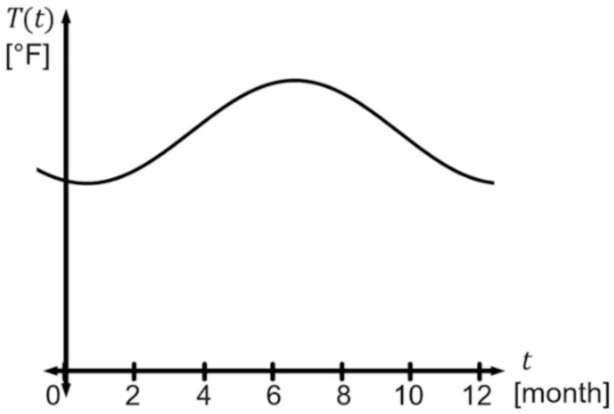
Topic 3.7 Sinusoidal Function Context and Data Modeling (Daily Video 2)

AP Precalculus

In this video, we will continue practicing the construction of sinusoidal function models from provided numerical data.

Let's PRACTICE!

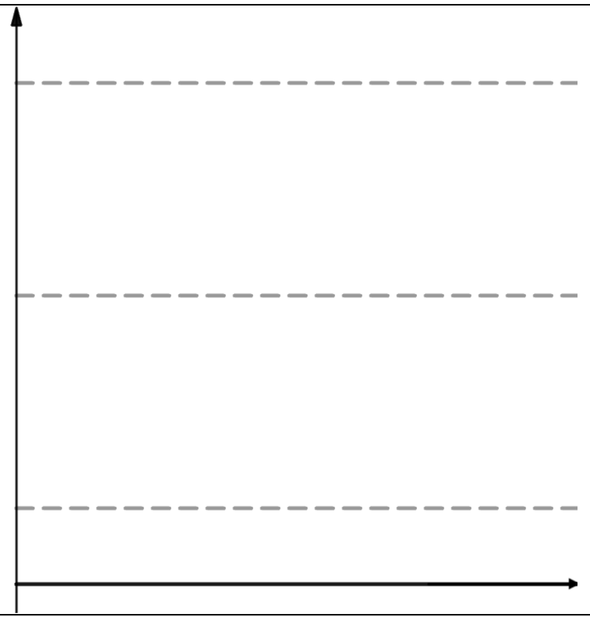
Example 1: A sinusoidal function, $T(t)$, models average New Orleans temperatures (in ^{0}F) as a function of time, t, in months after January 1. According to the model, the highest average temperature of 85^0 F occurs about three weeks into July ($t = 6.7$).



The lowest average temperature of 55^{0}F occurs about half a year earlier. Write an equation for $T(t)$.

According to the model, what is the average temperature on the last day of October ($t = 10$)?

Example 2: A Ferris wheel with a 34-foot diameter rotates at a constant angular speed, taking 50 seconds to complete one full rotation. The lowest position a rider will ever reach is 12 feet off the ground. At time $t = 20$ seconds, Bill the kitten is at the position indicated in the figure, traveling in the indicated direction.



Write an equation for B(t), Bill's height off the ground, as a function of time.

What is Bill's height when $t = 0$?

Topic 3.8 The Tangent Function (Daily Video 1)

AP Precalculus

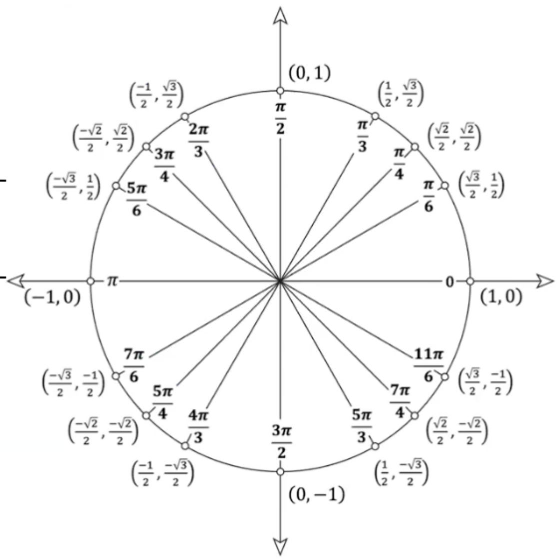
In this video, we will introduce a third trigonometric function with a strong relationship to sine, cosine and the slope created by the terminal ray of an angle.

Let's Review!

Recall **sine** and **cosine** as defined with triangles and the unit circle.

If $r = 1$, then $\sin\theta =$ ____________

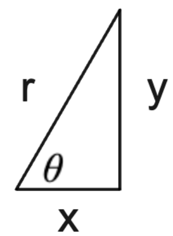
If $r = 1$, then $\cos\theta =$ ____________



Definition of tangent

$f(\theta) = \tan\theta = \frac{\quad}{\quad}$

$\tan\theta = \frac{y/r}{x/r} = \frac{\quad}{\quad}$



We can think of tangent in three significant ways:

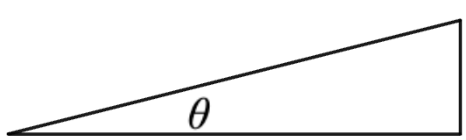
- As the ____________ of the ray created by θ
- As the ratio of the y and x coordinates from the ________ ________
- As the ratio of the ________ and ____________ of θ

The Period of the Tangent Function

The tangent function repeats values every half-circle. It has a period of ____.

Let's look at an EXAMPLE!

A plane is climbing and rises ½ mile for every 2 miles it travels parallel to the ground. What is the tangent of the angle of observation for an observer located on the ground? Label the diagram and show the calculation of the tangent ratio.



What should we take away?

- $f(x) = tan\ x$ can be evaluated in several ways.
  - The tangent of the angle a ray makes with the horizontal is the _________ of the ray.
  - The tangent of an angle is the ratio of the ___ and ___ coordinates at that angle on the unit circle.
  - $f(x) = \tan x$ can be rewritten or evaluated using $f(x) = \frac{\quad}{\quad}$.
- The period of the tangent function is a ________________, or ____ units.

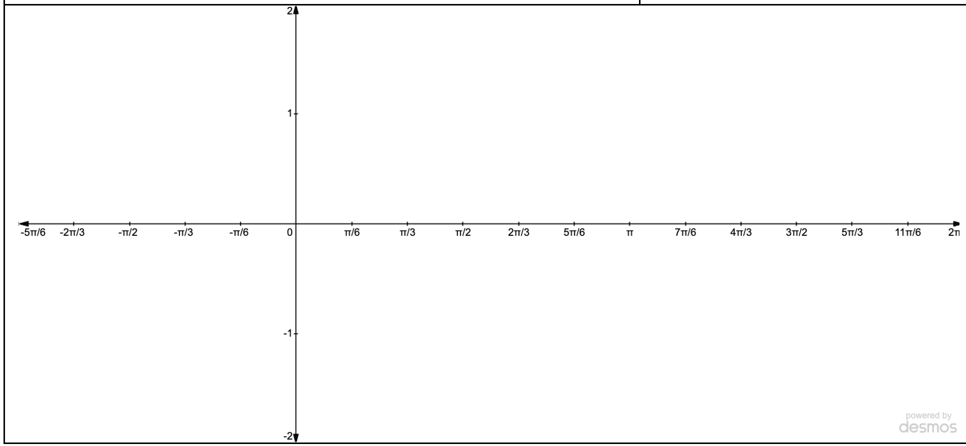
Topic 3.8 The Tangent Function (Daily Video 2)

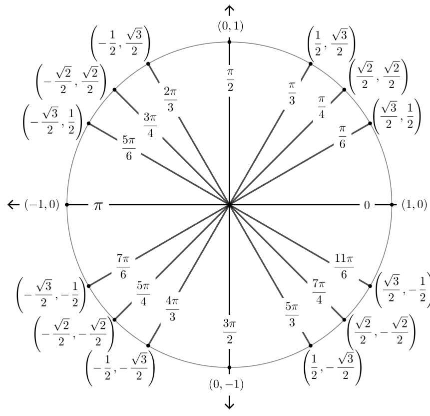
AP Precalculus

In this video, we will explore graphs of the tangent function with transformations and restrictions on their domains.

Let's WARM UP!

Evaluate		Plot the ordered pairs $(\theta, \tan\theta)$ on the axes below. Recall that he period of $f(x) = \tan x$ is π units.
$\tan 0$	$\tan\frac{2\pi}{3}$	Describe what is happening to the slope of the ray as x approaches $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.
$\tan\frac{\pi}{6}$	$\tan\frac{3\pi}{4}$	
$\tan\frac{\pi}{4}$	$\tan\frac{5\pi}{6}$	
$\tan\frac{\pi}{3}$	$\tan\pi$	How is this related to evaluations of $f(x) = \tan x$?
$\tan\frac{\pi}{2}$		





Transformations

$$y = a\tan(b(x - h)) + k$$

- h shifts the graph __________ and k shifts the graph ______________. The new "center" of the graph is at the point _________.
- The graph will have its inflection at $y =$ _____ and vertical asymptotes will be at $h \pm$ **half of** the period length. *Note: video error: The length of one period is distance between two vertical asymptotes. "…vertical asymptotes will be at $h \pm$ **half of** the period length."
- a is the __________ stretch or shrink.
- b affects the ___________ of the function.

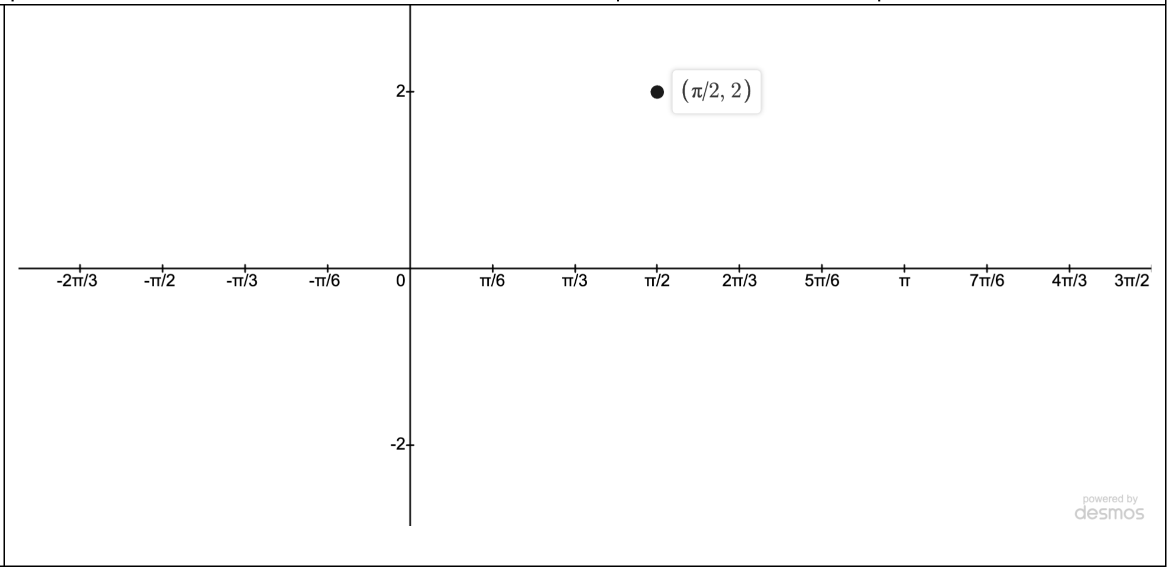
Let's look at an EXAMPLE!

Transformations: Shifts/Translations

What is the horizontal shift and vertical shift of $y = \tan\left(x - \frac{\pi}{2}\right) + 2$? Sketch the graph below, including the asymptotes. Note: The new "center" is already plotted on the graph.

Horizontal:

Vertical:



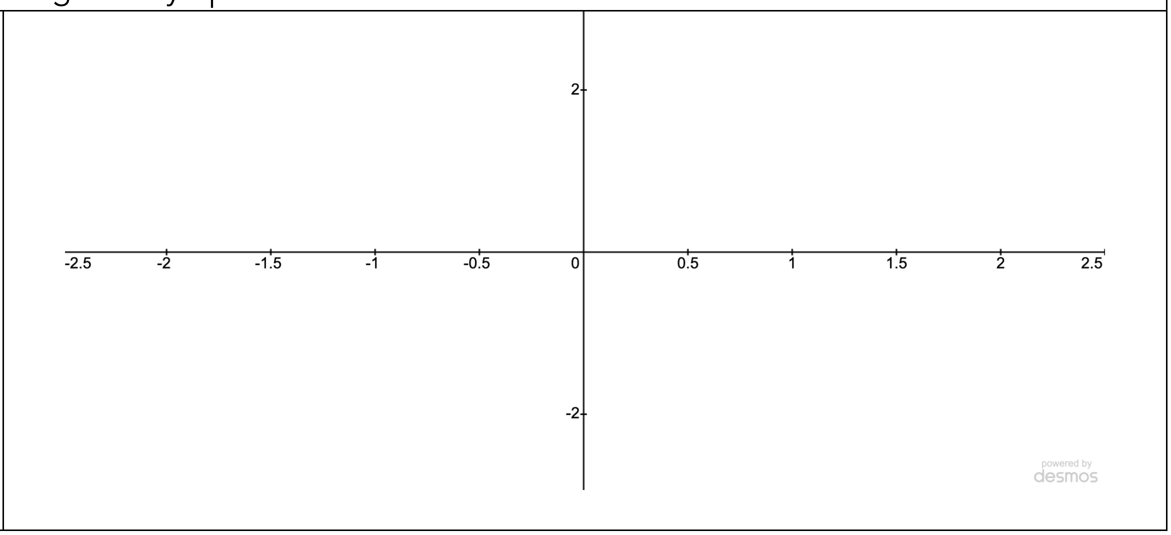
Let's PRACTICE!

Horizontal and Vertical Stretches

How does the graph of $y = \frac{1}{2}\tan(\pi x)$ compare to the graph of the parent function? Sketch the graph below, including the asymptotes.

$a = \frac{1}{2}$

$b = \pi$



What should we take away?

The old rules for transformations still apply.

- h is the ________________ shift
- k is the ____________ shift
- a affects the ___________ heights of function values
- b affects the period (new period = ——)

The new "center" is at (h, k). Asymptotes are a ______________ to the left and right of (h, k).

Topic 3.9 Inverse Trigonometric Functions (Daily Video 1)

AP Precalculus

In this video, we will use the concept of inverse functions to investigate the inverses of the sine, cosine, and tangent functions.

Let's REVIEW!

Suppose you are given $f(3) = 4, f(4) = 7,$ and $f(7) = 10$.

Predict: $f^{-1}(7) =$________ Think: If the output value of f is 7, what is the input value?

$f^{-1}(4) =$________ Think: If the output value of f is 4, what is the input value?

Let's look at an EXAMPLE!

The inverse trigonometric function will find a specific __________ related to a given __________.

$\tan^{-1}(1) =$ ________ because $\tan$ ______ $= 1$.

$\sin^{-1}(x) = \arcsin x$ is the inverse of $\sin(x)$

$\cos^{-1}(x) = \arccos x$ is the inverse of $\cos(x)$

$\tan^{-1}(x) = \arctan x$ is the inverse of $\tan(x)$

Let's PRACTICE!

$\cos^{-1}\left(\frac{1}{2}\right)$

$\arctan\left(\frac{1}{\sqrt{3}}\right)$

$\arcsin\left(\frac{\sqrt{2}}{2}\right)$

$\sin^{-1}(0.3)$

What should we take away?

Trignometric functions find __________ that go with angles.

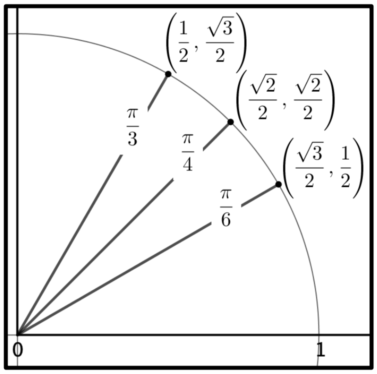
Inverse trignometric functions find _________ that go with ratios.

We really need to know our unit circle for special angles.

$\sin(\theta) = y - \text{coordinate}$

$\cos(\theta) = x - \text{coordinate}$

$\tan(\theta) = \frac{y}{x}$



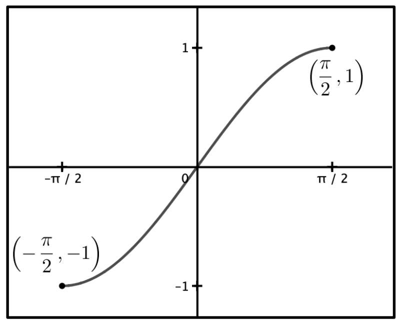
Topic 3.9 Inverse Trigonometric Functions (Daily Video 2)

AP Precalculus

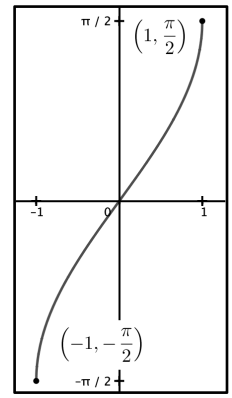
In this video, we will further explore inverse trigonometric functions and consider restrictions on their domains.

Let's REVIEW!

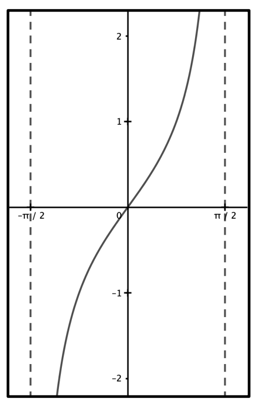
If we restrict $f(x) = \sin x$ to $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, we get this graph:



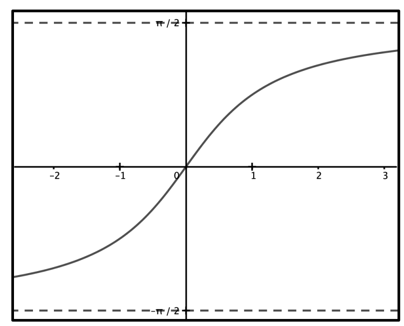
Then $y = \arcsin x$ is a function and looks like this:



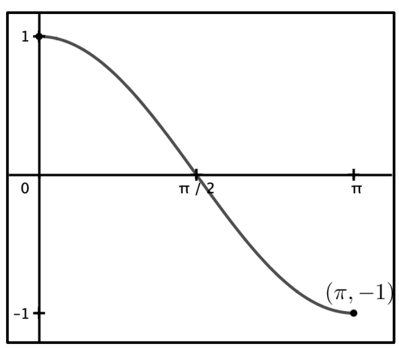
If we restrict $f(x) = \tan x$ to $-\frac{\pi}{2} < x < \frac{\pi}{2}$, we get this graph:



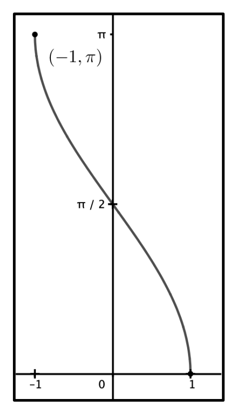
Then $y = \arctan x$ is a function and looks like this:



If we restrict $f(x) = \cos x$ to $0 \leq x \leq \pi$, we get this graph:



Then $y = \arccos x$ is a function and looks like this:



Let's PRACTICE!

1. $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

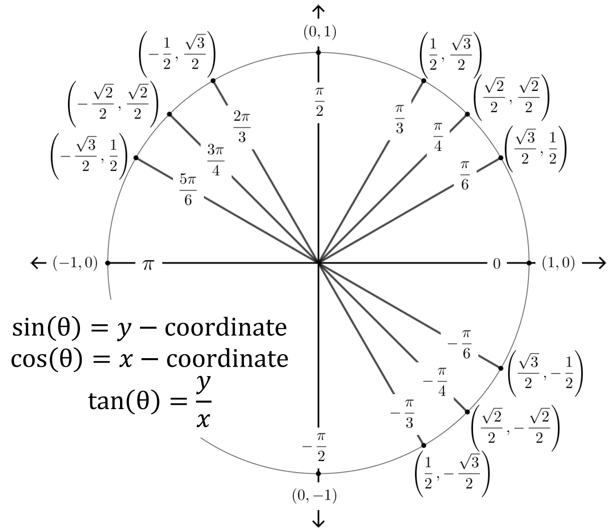
4. $\arccos\left(-\frac{1}{2}\right)$

2. $\tan^{-1}(-1)$

5. $\sin^{-1}(1)$

3. $\cos^{-1}(-1)$

6. $\arctan\left(-\frac{1}{\sqrt{3}}\right)$



What should we take away?

- For inverses to be functions, the original function must be ________________________.
- To make trigonometric functions one-to-one, the domain needs to be restricted:
  $\sin(x)$ is restricted to _______________

  $\tan(x)$ is restricted to _______________

  $\cos(x)$ is restricted to _______________
- We can use our knowledge of common ratios and the unit circle to determine the value of inverse trigonometric functions.

Topic 3.10 Trigonometric Equations and Inequalities (Daily Video 1)

AP Precalculus

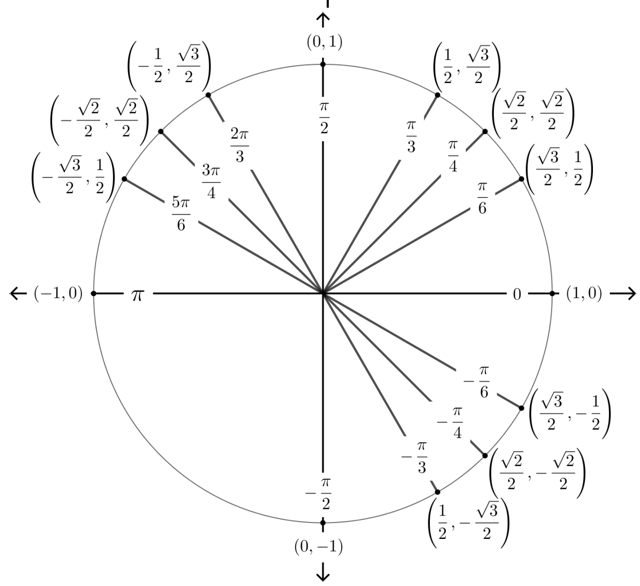
In this video, we will solve trigonometric equations and inequalities using inverse functions.

Let's REVIEW!

What is $\sin^{-1}\left(\frac{1}{2}\right)$?

What is $\cos\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$?

Unit Circle from Topic 3.9 video 2



Let's look at an EXAMPLE!

Find all solutions for θ using an inverse. Note: Presenter only found solutions on the interval $0 < \theta < \pi$.

$2\sin(2\theta) = \sqrt{3}$

$\sin(2\theta) =$ ______________

$\sin^{-1}(\sin(2\theta)) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

_________ $= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

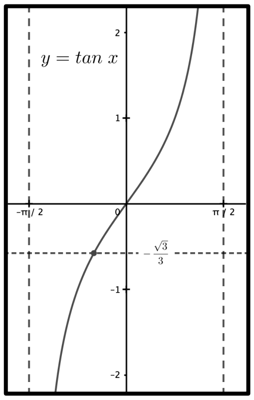
$2\theta =$ $\qquad\qquad$ $2\theta =$

$\theta =$ $\qquad\qquad$ $\theta =$

Find the solution for θ where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

$\tan(3\theta) > -\frac{\sqrt{3}}{3}$

One period of $\tan x$ is shown to the right.



$3\theta > \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

$\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) =$ _____

$-\frac{\pi}{6} < 3\theta < \frac{\pi}{6}$

_______ $< \theta <$ _______ or $\left(\text{—},\text{—}\right)$

One period of $\tan(3\theta)$ is ______ so all solutions would include

$-\frac{\pi}{3}$ $\qquad$ $+\frac{\pi}{3}$

$\left(\text{—},\text{—}\right) \cup \left(-\frac{\pi}{18}, \frac{\pi}{6}\right) \cup \left(\text{—},\text{—}\right)$

Rewrite the intervals using decimals.

($\qquad$, $\qquad$) ∪ ($\qquad$, $\qquad$) ∪ ($\qquad$, $\qquad$)

Let's PRACTICE! Let $3\cos(4\theta) = \frac{3}{2}$. Solve for θ where $0 \leq \theta \leq 2\pi$.

A) $\theta = \frac{\pi}{24}$ B) $\theta = \frac{\pi}{12}$ C) $\theta = -\frac{\pi}{24}$ D) $\theta = -\frac{\pi}{12}$

What should we take away?

Applying an inverse can be used to solve trigonometric equations and inequalities.

Topic 3.10 Trigonometric Equations and Inequalities (Daily Video 2)

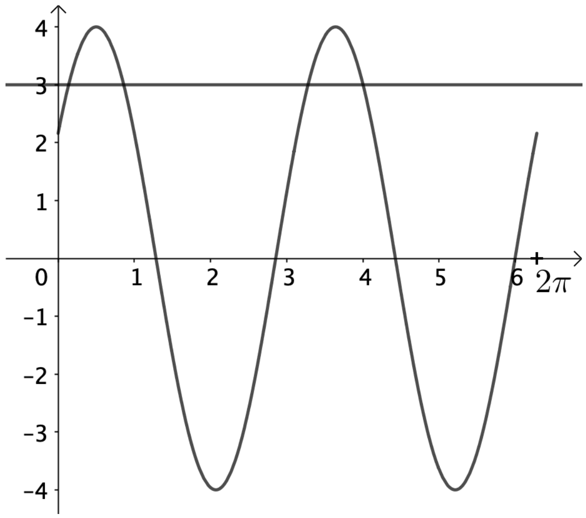
AP Precalculus

In this video, we will solve trigonometric equations and inequalities using inverse functions, with and without technology, to answer a question.

Let's look at an EXAMPLE!

Reminders: Make sure your calculator is in RADIAN mode and be sure to write your solutions to three decimal places.

Where is $4\cos(2x-1)=3$? Restrict your answers to $0 \le x < 2\pi$.



Where is $4\cos(2x-1)>3$? Restrict your answers to $0 \le x < 2\pi$.

Let $2=\sin^2 x - \sin x$. Solve for x, where $0 \le x < 2\pi$, without using a calculator. Show how you arrived at your answer.

What should we take away?

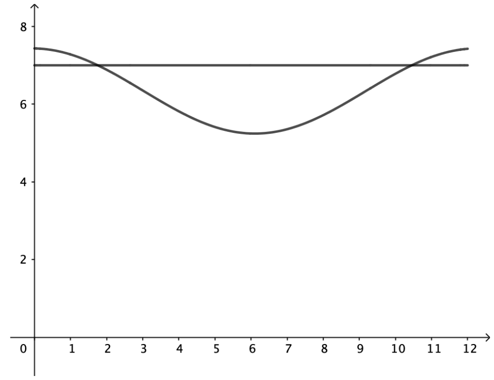
- Trigonometric equations and inequalities can be solved analytically or graphically.
- Solutions to trignometric inequalites must be written in interval notation.

Topic 3.10 Trigonometric Equations and Inequalities (Daily Video 3)

AP Precalculus

In this video, we will continue solving trigonometric equations and inequalities and explore models in context with reasonable solutions.

Modeling the Sunrise



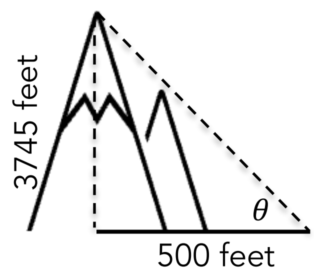
In a certain town in the United States, the time of sunrise can be modeled by the equation

$$y = 1.096\sin(0.510x + 1.600) + 6.339$$

where x represents the month (January = 1, December = 12), and y represents the time of the sunrise. If you woke up everyday at sunrise, during what months would you have to wake up before 7:00 AM? Show how you arrived at your answer.

Angle of Elevation

The peak of a mountain is 3,745 feet high. If you are standing 500 feet from the mountain's base, what is the measure, in radians, of the angle of elevation of the mountain's peak? Show how you arrived at your answer.



Let's PRACTICE!

What is $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta$?

A) $-\frac{\pi}{3}$ B) $-\frac{2\pi}{3}$ C) $\frac{\pi}{3}$ D) $\frac{2\pi}{3}$

Solve the inequality: $\cos(2x) > -\frac{1}{2}$ for $0 \le x \le \pi$.

A) $\left(0, \frac{\pi}{3}\right)$ B) $\left[0, \frac{\pi}{3}\right)$

C) $\left(\frac{2\pi}{3}, \pi\right]$ D) $\left[0, \frac{\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \pi\right]$

Let $y = 2x\cos(0.3x - 9) + 5$ represent the height of a kite bobbing in the wind, where x represents the number of minutes the kite has been in the air and y is how high the kite is, in feet. After how many minutes will the kite first reach 20 feet?

A) -18.409 min B) 7.941 min C) 25.682 min D) 34.318 min

What should we take away?

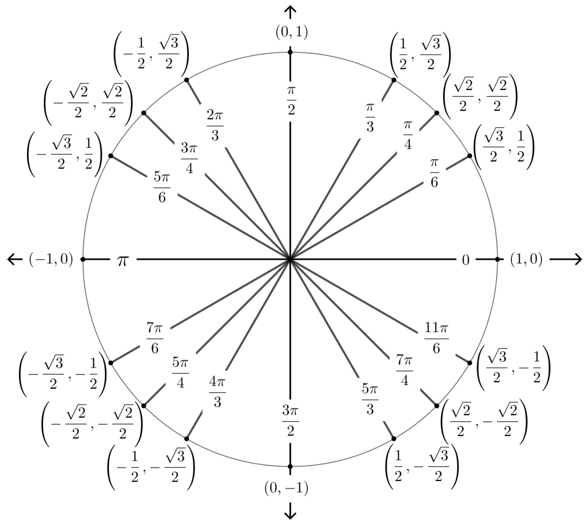
Trigonometric equations and inequalities can be solved analytically or graphically.

Topic 3.11 The Secant, Cosecant, and Cotangent Functions (Daily Video 1)

AP Precalculus

In this video, we will introduce the reciprocal functions of the sine, cosine, and tangent functions.

Let's look at an EXAMPLE!



The cosecant function:

$$f(\theta) = \csc\theta = \frac{1}{\sin\theta} = \frac{r}{y}$$

$$\csc\frac{\pi}{6} =$$

$$\csc\frac{3\pi}{4} =$$

The secant function:

$$f(\theta) = \sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$$

$$\sec\frac{\pi}{6} =$$

$$\sec\frac{3\pi}{4} =$$

The cotangent function:

$$f(\theta) = \cot\theta = \frac{1}{\tan\theta} = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$$

$$\cot\frac{\pi}{6} =$$

$$\cot\frac{3\pi}{4} =$$

Let's PRACTICE!

$\csc\frac{\pi}{3} =$	$\cot\frac{11\pi}{6} =$
$\sec\frac{3\pi}{4} =$	$\sec 1.2 =$

What should we take away?

- Cosecant, secant and cotangent are reciprocals of sine, cosine and tangent, respectively.

$f(\theta) = \csc\theta = \frac{1}{\_\_\_\_} = \frac{r}{\_\_\_\_}$ $\quad$ $f(\theta) = \sec\theta = \frac{1}{\_\_\_\_} = \frac{r}{\_\_\_\_}$ $\quad$ $f(\theta) = \cot\theta = \frac{1}{\_\_\_\_} = \frac{\_\_\_\_}{\_\_\_\_} = \frac{\_\_\_\_\,\theta}{\_\_\_\_\,\theta}$

Topic 3.11 The Secant, Cosecant, and Cotangent Functions (Daily Video 2)

AP Precalculus

In this video, we will further explore the secant, cosecant, and cotangent functions by analyzing their graphs, domains, and ranges.

Let's WARM UP!

Solve for $-\pi < x \leq 2\pi$

$\sin x = 0$ $\qquad\qquad$ $\cos x = 0$ $\qquad\qquad$ $\tan x = 0$

Let's REVIEW!

Vertical Asymptotes usually occur when the denominator of a function that is a ratio is zero.
Where can we predict asymptotes for secant, cosecant, and cotangent on the interval $[-\pi, 2\pi]$?

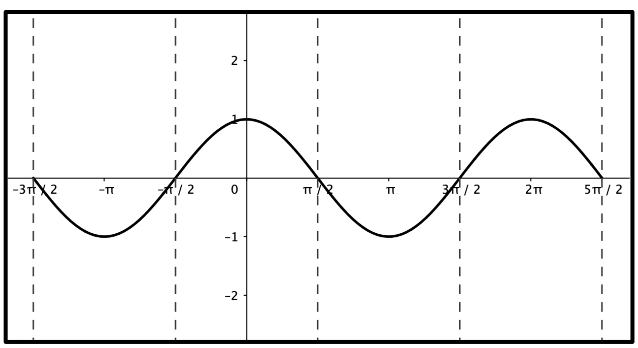
$f(x) = \sec x = \dfrac{1}{\_\_\_\_\_}$ $\qquad$ vertical asymptotes at $x =$

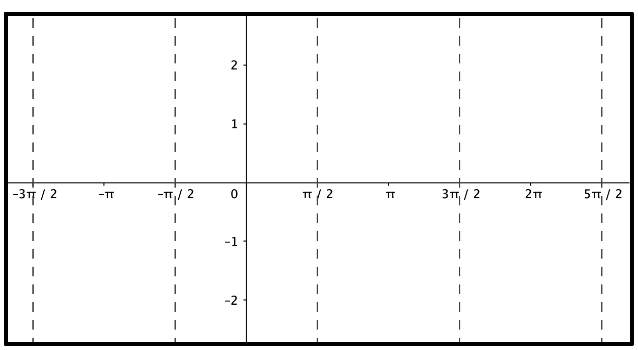
$f(x) = \csc x = \dfrac{1}{\_\_\_\_\_}$ $\qquad$ vertical asymptotes at $x =$

$f(x) = \cot x = \dfrac{1}{\_\_\_\_\_} = \dfrac{\_\_\_\_\_}{\_\_\_\_\_}$ $\qquad$ vertical asymptotes at $x =$

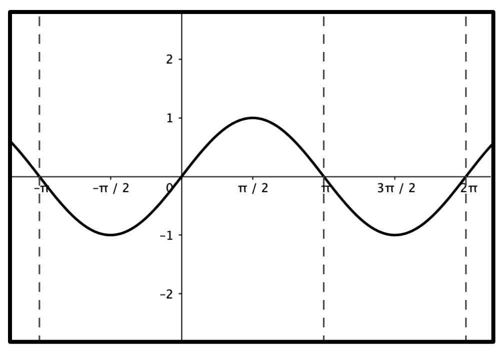
Let's look at an EXAMPLE!

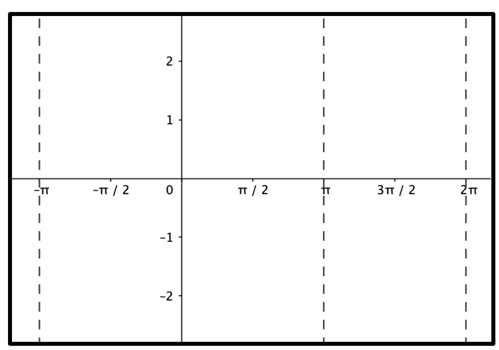
Sketch a graph of $f(x) = \sec x$ from the graph of $\cos x$.





Sketch a graph of $f(x) = \csc x$ from the graph of $\sin x$.

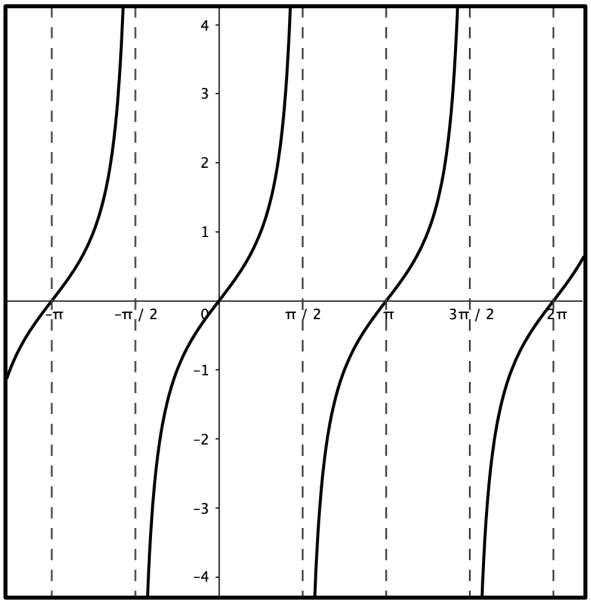


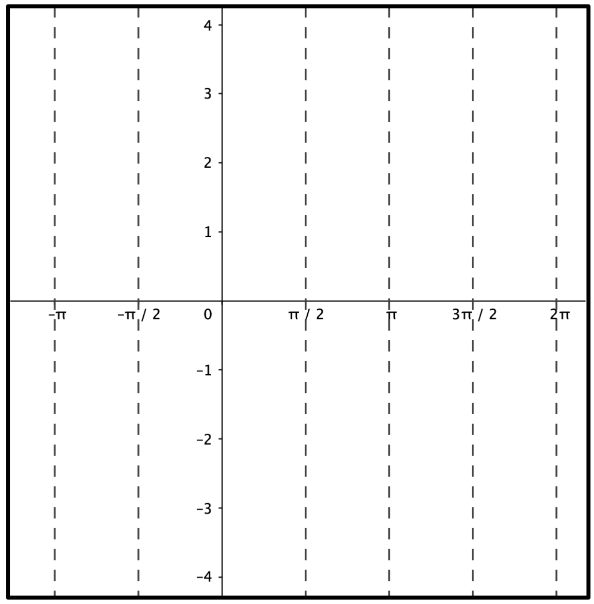


Sine and cosine have ranges of ________

Secant and cosecant have ranges of __________________

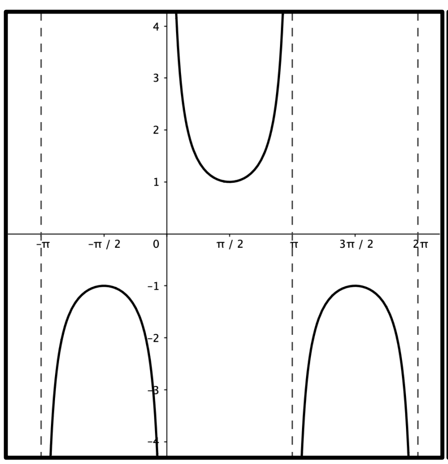
Sketch a graph of $f(x) = \cot x$ from the graph of $\tan x$.



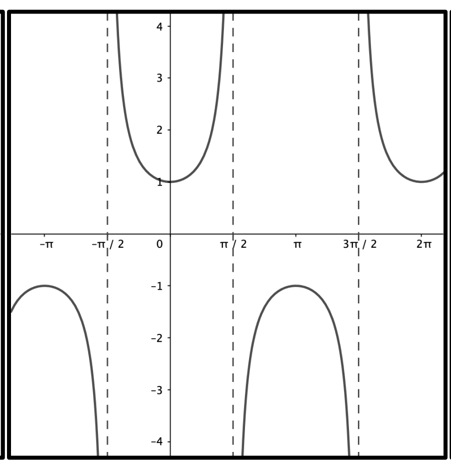


What should we take away?

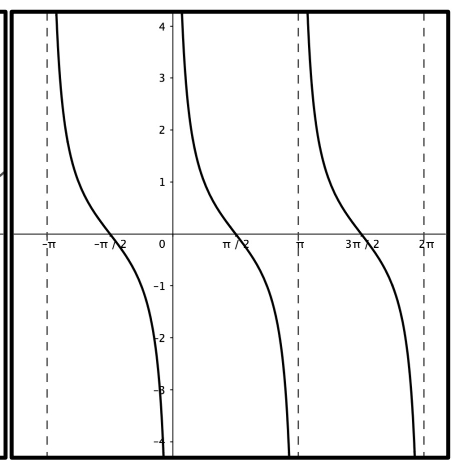
secant



cosecant



cotangent

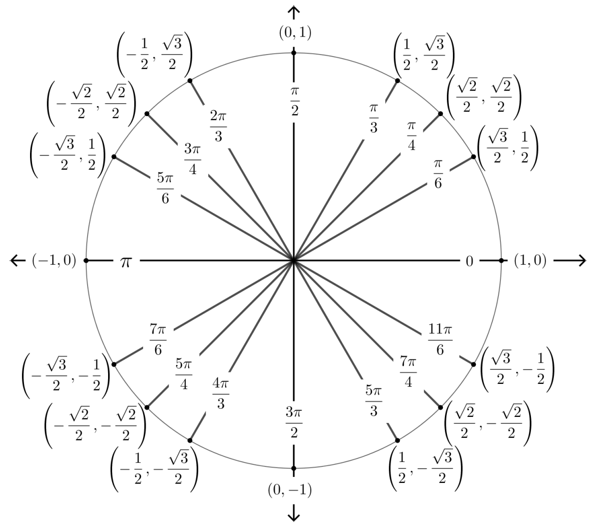


- We can use the zeros of cosine, sine, and tangent to predict the ____________ asymptotes of secant, cosecant, and cotangent.
- The tangent graph is always increasing between consecutive asymptotes so the cotangent is always _____________ between consecutuve asymptotes.

Topic 3.12 Equivalent Representations of Trigonometric Functions (Daily Video 1)

AP Precalculus

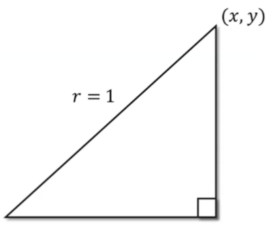
In this video, we will explore the Pythagorean identities and use them to write equivalent trigonometric expressions.



Let's REVIEW!

On the unit circle $(x, y) =$ (______, ______)

The Pythagorean theorem can be applied to a right triangle at a point on the unit circle.



$x^2 + y^2 =$ ______

Or in terms of sine and cosine we get the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$.

Let's look at an EXAMPLE!

Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to write the identity for $\tan^2\theta$.	Simplify $\sin\theta + \cos\theta \cdot \cot\theta$	Simplify $\cot^2 x - \cot^2 x \cdot \cos^2 x$

Let's PRACTICE!

Simplify $\dfrac{1}{\tan^2\theta + 1}$	Simplify $\cos\theta\,(\sec\theta - \cos\theta)$
A) $\cot^2\theta$	A) $\cot^2\theta$
B) $\sin^2\theta$	B) $\sin^2\theta$
C) $\cos^2\theta$	C) $\cos^2\theta$
D) $\csc^2\theta$	D) $\csc^2\theta$

What should we take away?

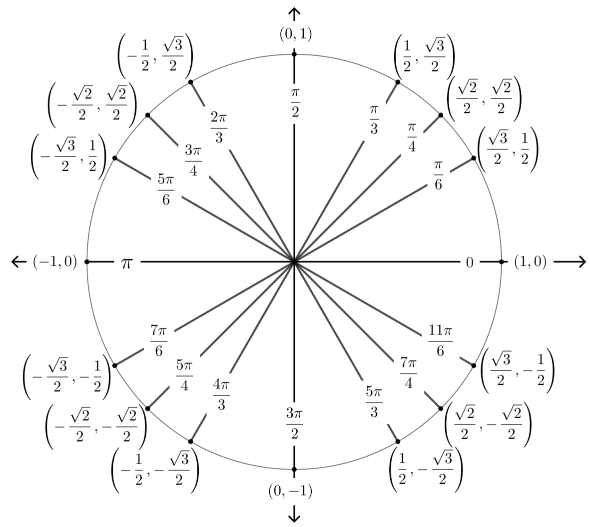
There are two Pytagorean identities $\sin^2\theta +$ ______ $= 1$ and $\tan^2\theta =$ ______ $- 1$ that can used to simplify trigonometric expressions.

Topic 3.12 Equivalent Representations of Trigonometric Functions (Daily Video 2)

AP Precalculus

In this video, we will explore sum, difference, and double-angle identities of sine and cosine.

Let's REVIEW!



$\tan\theta =$ $\csc\theta =$

$\sin^2\theta + \cos^2\theta =$ $\sec\theta =$

Verify $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ using $\alpha = \frac{\pi}{6}$ and $\beta = \frac{\pi}{3}$.

Verify $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ using $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{2}$.

Let's Look at an EXAMPLE! **Let's PRACTICE!**

Sine Double-Angle Identities	Cosine Double-Angle Identities	Verify
$\sin(2\alpha) = \sin(\alpha+\alpha)$	$\cos(2\alpha) = \cos(\alpha+\alpha)$	$\csc^2\alpha = \dfrac{\csc\alpha}{\sin\alpha}$

What should we take away?

$\sin(\alpha+\beta) =$ ______________________ $\cos(\alpha+\beta) =$ ______________________

$\sin(\alpha-\beta) =$ ______________________ $\cos(\alpha-\beta) =$ ______________________

$\sin(2\alpha) =$ ______________________ $\cos(2\alpha) =$ ______________________

*Note: Difference identities were not verified in the video. Hint: Write the difference as the sum of the opposite, $(\alpha-\beta) = (\alpha+(-\beta))$. Then use the fact that sine is an odd function, which means $\sin(-\beta) = -\sin\beta$, and cosine is an even function, which means $\cos(-\beta) = \cos\beta$.

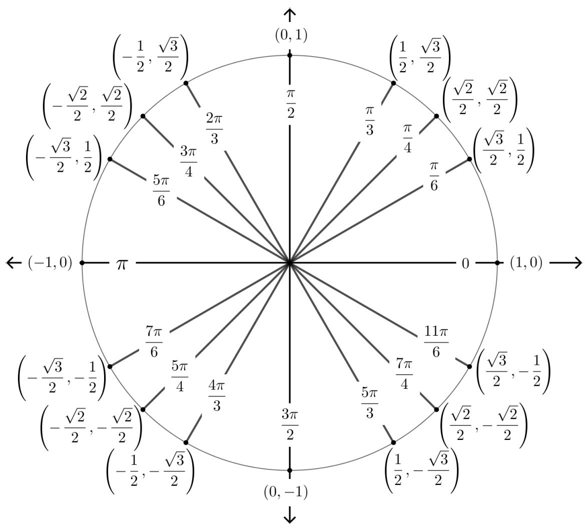
Topic 3.12 Equivalent Representations of Trigonometric Functions (Daily Video 3)

AP Precalculus

In this video, we will use trigonometric identities to solve trigonometric equations and inequalities.

Let's look an EXAMPLE!

(the interval on the video is written incorrectly. The interval is corrected below)



Solve for θ when $0 \leq \theta < 2\pi$. Show your work.

$\sin(2\theta) - \sin\theta = 0$

Solve for θ when $0 \leq \theta < 2\pi$. Show your work.

$\sin(2\theta) - \sin\theta > 0$

* The note on the video has a typo. It should read: 0 is not included in the solutions since $\sin(2 \cdot 0) - \sin(0) = 0$.

Let's PRACTICE!

Using a sum identity, find all values of θ on $[0, 2\pi)$ where $\sin\left(\theta + \frac{\pi}{3}\right) = 0$.

What should we take away?

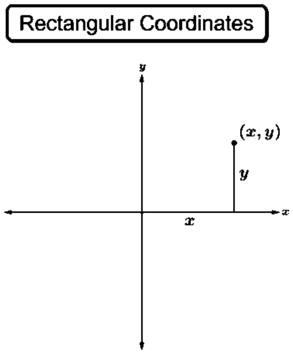
We can solve trigonometric equations and inequalities using appropriate trigonometric identities.

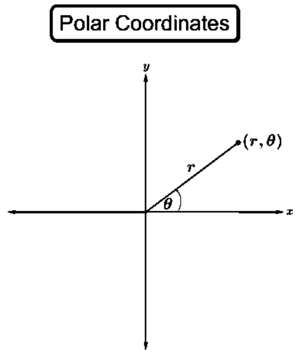
Topic 3.13 Trigonometry and Polar Coordinates (Daily Video 1)

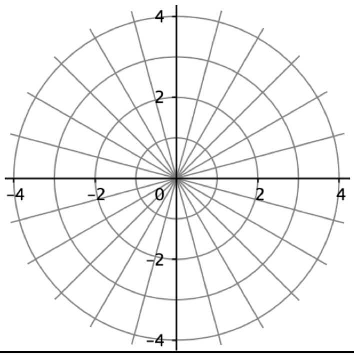
AP Precalculus

In this video, we will introduce and discuss the purpose of polar coordinates, as well as practice converting from polar to rectangular coordinates.

What Are Polar Coordinates?







Representing Points in Polar Coordinate

There are multiple ways to represent the same point in polar coodinates!

A full turn is _____ and a half turn is _____ radians.

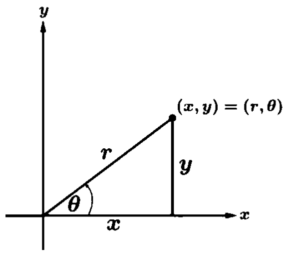
1. $\theta \pm 2\pi$ and r stays the same

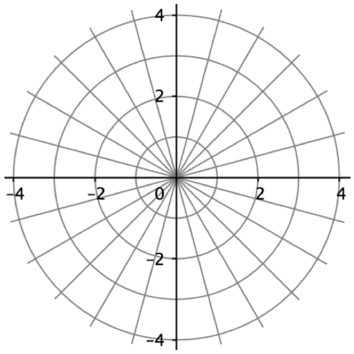
2. $\theta \pm \pi$ and $r \to -r$

Let's look at an EXAMPLE!

a) Plot the polar coordinate $\left(3, \frac{\pi}{3}\right)$ on the grid provided.

b) Write the polar coordinate two additional ways.





Converting From Polar To Rectangular

$\cos(\theta) = \frac{\_\_\_\_}{r}$ $\sin(\theta) = \frac{\_\_\_\_}{r}$

$x =$ ____________ $y =$ ____________

Let's look at an EXAMPLE!

Convert the following polar coordinates to rectangular coordinates.

a) $\left(3, \frac{\pi}{3}\right)$

b) $\left(-2, \frac{3\pi}{2}\right)$

What should we take away?

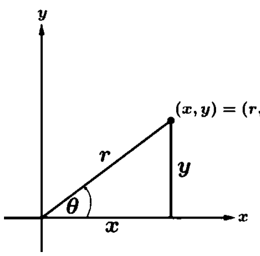
1. Polar coordinates can be written in different ways and represent the same point.
2. We can convert from polar coordinates to rectangular coordinates using the conversion:

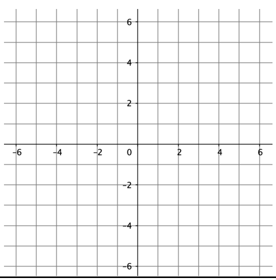
$x =$ ____________ $y =$ ____________

Topic 3.13 Trigonometry and Polar Coordinates (Daily Video 2)

AP Precalculus

In this video, we will learn to convert from rectangular coordinates to polar coordinates, as well as how to represent complex numbers in both rectangular and polar coordinates.





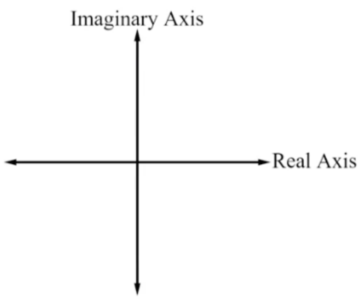
Converting From Rectangular To Polar

$x^2 + y^2 =$____________ $\tan\theta = \frac{\quad}{\quad}$

Let's look at an EXAMPLE!

Convert the following rectangular coordinates to polar coordinates.

a) $(5,5)$ b) $(1, -\sqrt{3})$



Complex Numbers

Rectangular Coordinates $(a, b) \rightarrow a + bi$

Polar Coordinates $(r, \theta) \rightarrow$____________________________

Write the complex number $-3i$ using polar coordinates.

Convert $z = 4\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$ to rectangular coordinates.

What should we take away?

1. We can convert from rectangular coordinates to polar coordinates:

   $x^2 + y^2 =$____ $\tan\theta = \frac{\quad}{\quad}$

2. We can write complex numbers in both rectangular and polar forms:

   $(a, b) \rightarrow$__________ $(r, \theta) \rightarrow$________________________

Pro Tip
Always sketch out the point first! This will help ensure your answer is reasonable.

Topic 3.14 Polar Function Graphs (Daily Video 1)

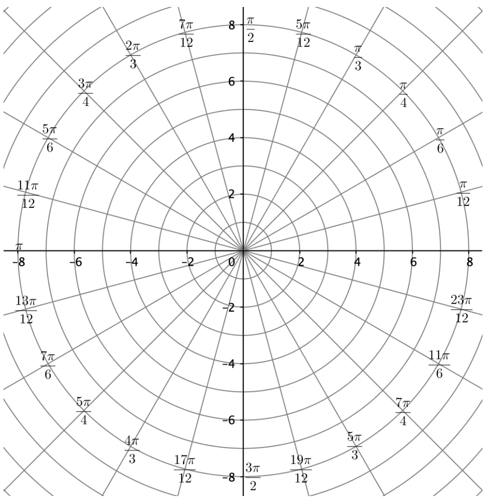
AP Precalculus

In this video, we will introduce how polar functions may be graphed by looking at several examples of basic polar graphs and practicing creating polar curves .

Functions	
Rectangular	**Polar**
$y = f(x)$	$r = f(\theta)$

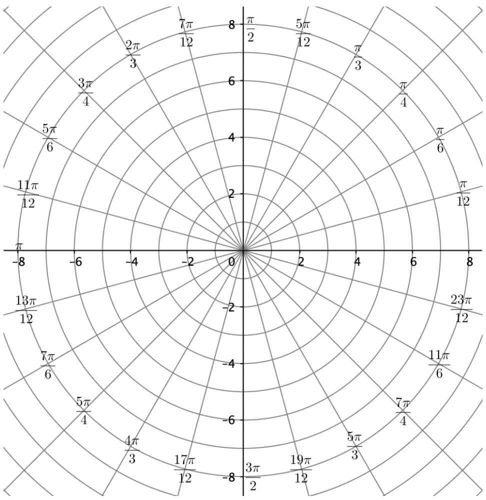
Important note: For polar functions, the dependent variable is given first! $(x, y) \rightarrow (r, \theta)$

Let's look at an Example!



θ	$r(\theta) = 3\cos(\theta)$
0	
$\pi/6$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$5\pi/6$	
π	

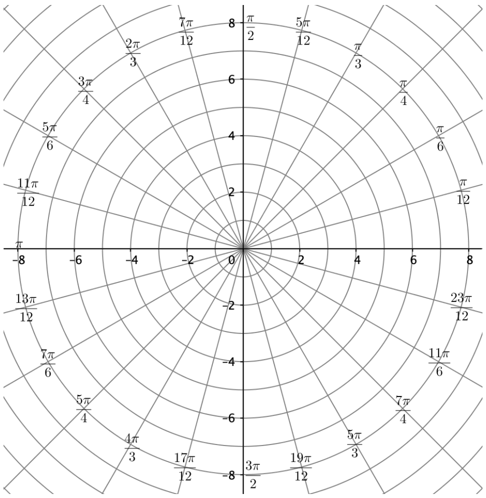
Graph $r(\theta) = 3\sin(2\theta)$



θ	$r(\theta)$
0	
$\pi/6$	
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	

θ	$r(\theta)$
π	
$7\pi/6$	
$5\pi/4$	
$4\pi/3$	
$3\pi/2$	
$5\pi/3$	
$7\pi/4$	
$11\pi/6$	
2π	

Graph $r(\theta) = 4\cos(3\theta)$



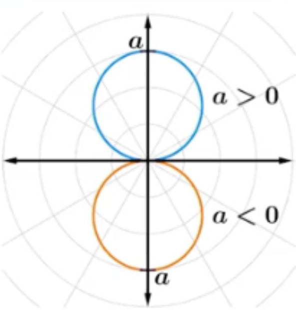
θ	$r(\theta)$
0	
$\pi/12$	
$\pi/6$	
$\pi/4$	
$\pi/3$	
$5\pi/12$	
$\pi/2$	

θ	$r(\theta)$
$7\pi/12$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
$11\pi/12$	
π	

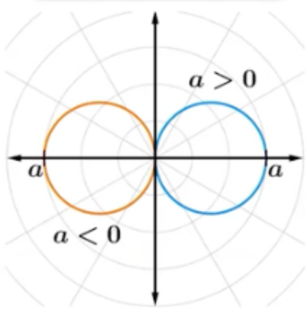
What should we take away?

1. It is helpful to recognize common polar functions.
2. Many polar curves have symmetry, which is helpful when graphing.

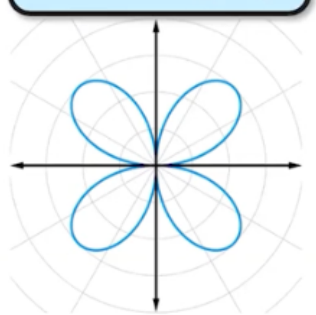
$r = a\sin(\theta)$



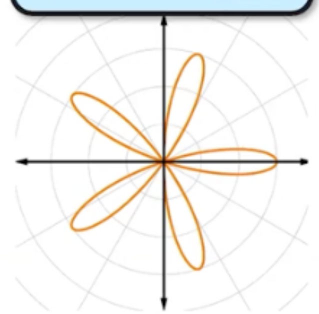
$r = a\cos(\theta)$



b is even
$r = a\sin(b\theta)$



b is odd
$r = a\cos(b\theta)$



Note: If b is even, the polar curve will have $2b$ petals. If b is odd, the polar curve will have b petals.

Topic 3.14 Polar Function Graphs (Daily Video 2)

AP Precalculus

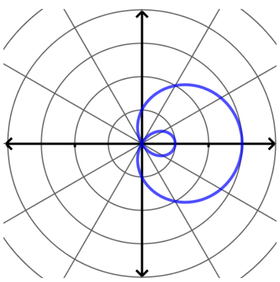
In this video, we will continue to develop our understanding of polar function graphs while identifying common characteristics found in many of these graphs .

Additional Common Polar Graphs

Graphs of the form $r(\theta) = a \pm b\cos(\theta)$ and $r(\theta) = a \pm b\sin(\theta)$

Limacon

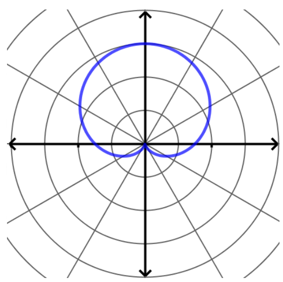
$\frac{a}{b} < 1$



$r(\theta) = 1 + 2\cos(\theta)$

Cardoid

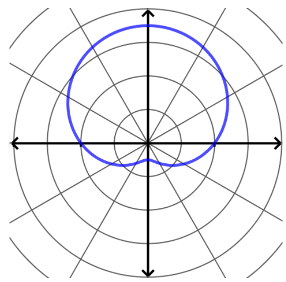
$\frac{a}{b} = 1$



$r(\theta) = 1.5 + 1.5\sin(\theta)$

Dimpled Cardoid

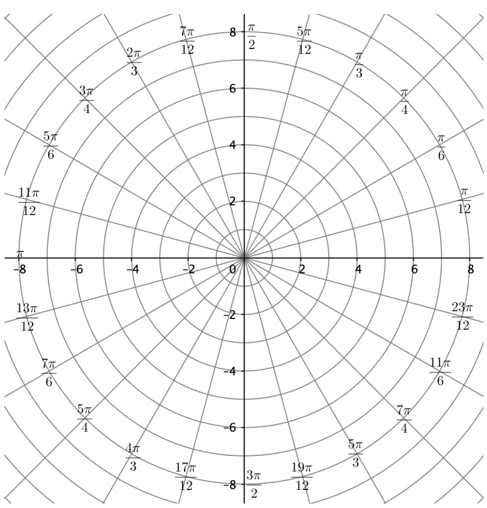
$1 < \frac{a}{b} < 2$



$r(\theta) = 2 + 1.5\sin(\theta)$

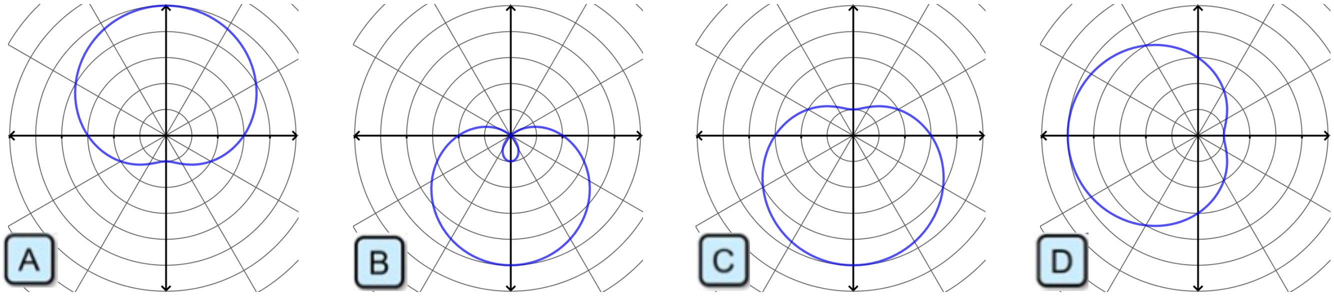
Note: The names of common polar graphs are not tested on the AP Exam.

Let's look at an Example!

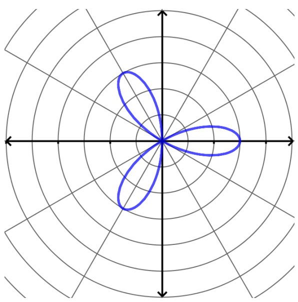
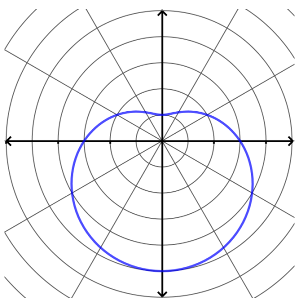
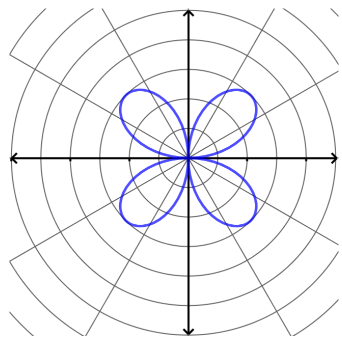


θ	$r(\theta) = 2 - 2\cos(\theta)$
0	
$\pi/6$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$5\pi/6$	
π	

Which of the following is the graph of $r(\theta) = 3 - 2\sin\theta$?



$r(0) =$ ____________ $\qquad r\left(\frac{\pi}{2}\right) =$ ____________

Symmetry about ...		
Polar Axis	$\theta = \frac{\pi}{2}$	Pole
$r(-\theta) = r(\theta)$	$-r(-\theta) = r(\theta)$	$-r(\theta) = r(\theta)$
		

Show that the polar function $r(\theta) = 2 - 2\cos(\theta)$ is symmetric about the polar axis.

We must show that ______________________.

$r(-\theta) =$ ______________________________

$r(-\theta) =$ _________________________ $= r(\theta)$

Identity: Since cosine is even, we know that

$\cos(-\theta) =$ __________

What should we take away?

1. We can use symmetry to help when graphing certain polar curves.
2. Knowing common polar curves is helpful when graphing and describing polar functions.

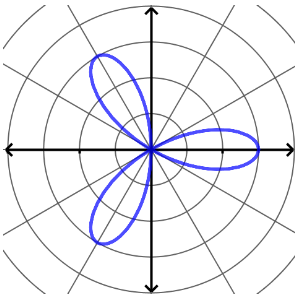
Topic 3.15 Rates of Change in Polar Functions (Daily Video 1)

AP Precalculus

In this video, we will learn how the distance to the origin of a polar graph may increase or decrease. Also, we will introduce the concept of relative extrema in polar functions.

Changes in the distance to the Origin

Consider the polar function $r(\theta) = 3\cos(3\theta)$ graphed below.



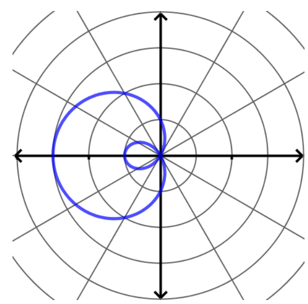
a) $\theta = \frac{\pi}{6}$ $\quad r(\theta) =$ ________________

b) $\theta = \frac{\pi}{4}$ $\quad r(\theta) =$ ________________

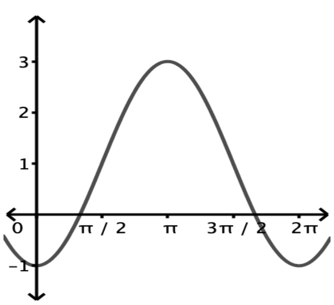
c) $\theta = \frac{\pi}{3}$ $\quad r(\theta) =$ ________________

Distance to Origin	
Increasing	**Decreasing**
r is positive and increasing	r is positive and decreasing
r is negative and decreasing	r is negative and increasing

Let's look at an Example!



The graph of $r(\theta) = 1 - 2\cos(\theta)$ is shown in the figure to the right, along with selected values of r and θ in the table below. Determine if the polar curve is getting closer to the origin, further from the origin, or neither when $\theta = \frac{\pi}{6}$.

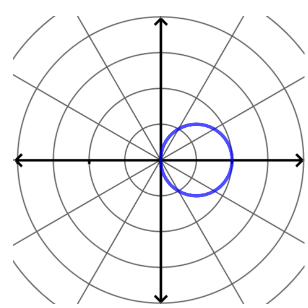


θ	$\pi/6$	$\pi/2$	π	$2\pi/3$
r	$1-\sqrt{3}$	1	3	2

The polar curve is getting ____________________ when $\theta = \frac{\pi}{6}$.

Give a reason for your answer.

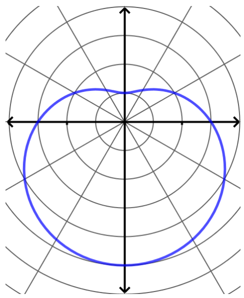
Relative Extrema



If a polar function $r = f(\theta)$ changes from increasing to decreasing or from decreasing to increasing, then the function has a ____________ ____________ corresponding to a point that is relatively furthest or closest to the origin.

Let's look at an Example!

Consider the curve $r(\theta) = -3 - 2\sin(\theta)$. Determine if the polar curve has a relative extremum on the following interval. If the curve does have a relative extremum, classify it as a relative maximum or relative minimum.

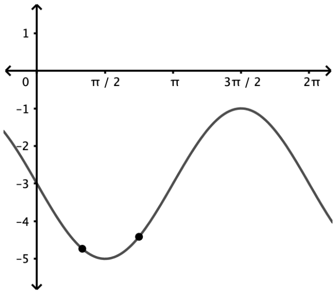


On the interval $\left[\frac{\pi}{3}, \frac{3\pi}{4}\right]$ $r\left(\frac{\pi}{3}\right) =$ ________

$r\left(\frac{3\pi}{4}\right) =$ ________

The graph of r has a relative ______________

on the interval $\left[\frac{\pi}{3}, \frac{3\pi}{4}\right]$. Justify your answer.



What should we take away?

1. We can determine whether a polar curve is getting closer or further from the origin based on the behavior of the function.
2. Using the behavior of a polar function, we can often determine when the graph reaches a relative minimum or relative maximum distance from the origin.

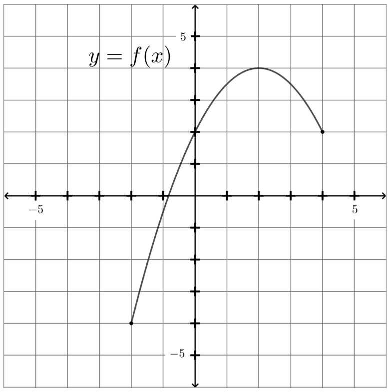
Distance to Origin	
Increasing	**Decreasing**
r is positive and ___________	r is ________ and decreasing
r is ________ and decreasing	r is negative and __________

Topic 3.15 Rates of Change in Polar Functions (Daily Video 2)

AP Precalculus

In this video, we will learn how that the average rate of change of a polar function helps us understand how r values change as θ changes.

Let's REVIEW! Average Rate of Change



AROC of $f(x)$ over the interval $[a, b]$

$\text{AROC} =$ ______________

Calculate the average rate of change of $f(x)$ over the interval $[-2,4]$. Show your work.

Sketch the "linear path" to verify that your answer above is correct.

Average Rate of Change: Polar Functions

Average rate of change of $r(\theta)$ over the interval $[\theta_1, \theta_2]$ is $\text{AROC} =$ ______________.

Let's look at an Example!

Selected values of θ and r are shown in the table to the right. Find the average rate of change of $r(\theta)$ over the interval $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$. Show all your work.

θ	$\pi/6$	$\pi/2$	π	$7\pi/6$
r	$-2-\sqrt{3}$	-3	7	$5\ -2\sqrt{3}$

Write an interpretation of the AROC calculated above.

Interpretation: The AROC tells us that r is ____________ by approximately $\frac{3(-1+\sqrt{3})}{\pi}$ units per radian over the interval ____________.

Approximating Values of Polar Functions

Selected values of θ and r are shown in the table to the right. The AROC of $r(\theta)$ over the interval $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ is ________ .

θ	$\pi/6$	$\pi/3$
r	-2	-3

We can use the AROC of $r(\theta)$ to estimate values for r inside the interval.

$$r(\theta_2) \approx r(\theta_1) + \text{AROC} \cdot (\theta_2 - \theta_1)$$

Use the AROC of $r(\theta)$ over the interval $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ to approximate $r\left(\frac{\pi}{4}\right)$ using the values in the table to the right. Show all your work.

θ	$\pi/6$	$\pi/3$
r	-2	-3

$r(\theta_2) \approx r(\theta_1) + \text{AROC} \cdot (\theta_2 - \theta_1)$

$r\left(\frac{\pi}{4}\right) \approx$ ________ + ________ $\cdot$ (   )

$r\left(\frac{\pi}{4}\right) \approx$ ________

What should we take away?

1. We can find the average rate of change of polar functions in a similar way to how we find average rate of change of rectangular functions.

$$\text{AROC} = \frac{r(\theta_2) - r(\theta_1)}{\theta_2 - \theta_1}$$

2. The average rate of change of a polar function can be used to approximate values of polar functions.

$$r(\theta_2) \approx r(\theta_1) + \text{AROC} \cdot (\theta_2 - \theta_1)$$