Key

A. Is g differentiable at $x = 2$? Justify your answer.

$\lim_{x \to 2^-} g(x) = 3(2)^2 - 5(2) + 2 = 4$ $\quad \lim_{x \to 2^-} g'(x) = \lim_{x \to 2^-} (6x - 5) = 7$ $\quad$ No, because

$\lim_{x \to 2^+} g(x) = \sqrt{5(2)+6} = 4$ $\quad \lim_{x \to 2^+} g'(x) = \lim_{x \to 2^+} \frac{5}{2\sqrt{5x+6}} = \frac{5}{8}$ $\quad \lim_{x \to 2^-} g'(x) \neq \lim_{x \to 2^+} g'(x)$.

B. Which is greater: the average rate of change of k on $[1,4]$ or the instantaneous rate of change of k at $x = 1$? Explain.

The instantaneous rate of change at $x=1$ because the slope of the tangent line at $x=1$ is greater than the slope of the secant line between $x=1$ and $x=4$.

C. Let $m(x) = h(f(x))$. Find $m'(3)$.

$m'(x) = f'(x) \cdot h'(f(x))$ $\quad m'(3) = f'(3) \cdot h'(f(3)) = -1 \cdot h'(1) = -1 \cdot \frac{1}{2}$

$= -\frac{1}{2}$

D. Let $n(x) = h(x) \cdot f(x)$. Find $n'(3)$.

$n'(x) = h(x) \cdot f'(x) + f(x) \cdot h'(x)$

$n'(3) = h(3) \cdot f'(3) + f(3) \cdot h'(3) = 5 \cdot -1 + 1(-4) = -5 - 4 = -9$

E. Use a right Riemann sum with the three subintervals indicated by the table to estimate $\int_2^8 h(x)\,dx$.

$\int_2^8 h(x)\,dx \approx 1(5) + 2(-5) + 3(14) = 37$

F. Write the equation of the line tangent to the graph of h at $x = 8$.

$y - 14 = -2(x - 8)$

G. At which x-value(s) does the graph of j have horizontal tangent lines?

When is $j'(x) = 0$? $\quad j'(x) = 12e^{3x} - \frac{5}{2}$ $\quad \frac{5}{2} = 12e^{3x}$

$0 = 12e^{3x} - \frac{5}{2}$ $\quad \frac{5}{24} = e^{3x}$

$\frac{\ln(5/24)}{3} = x$

H. Find $\lim_{h \to 0} \frac{j(2+h) - j(2)}{h}$.

Find $j'(2)$. $\quad j'(2) = 12e^6 - \frac{5}{2}$

I. Let $p(x) = \frac{k(x)}{h(x)}$. Find $p'(3)$.

$p'(x) = \frac{h(x)k'(x) - k(x)h'(x)}{(h(x))^2}$ $\quad p'(3) = \frac{5(0) - 4(-4)}{5^2} = \frac{16}{25}$

J. Find $\lim_{x \to 5} \frac{k(x) - k(5)}{x - 5}$.

$\lim_{x \to 5} \frac{k(x) - k(5)}{x - 5} = k'(5) = -4$

K. Find $g'(6)$ and write a sentence interpreting its meaning.

For $x>2$, $g'(x) = \frac{5}{2\sqrt{5x+6}}$ $g'(6) = \frac{5}{2\sqrt{36}} = \frac{5}{12}$

At $x=6$, the instantaneous rate of change of g is $\frac{5}{12}$ (or, slope of tangent line to g at $x=6$ is $\frac{5}{12}$)

L. Are we guaranteed a value c for $4 < c < 7$ such that $f'(c) = \frac{1}{3}$? Explain.

No, f is not differentiable on the interval so the MVT does not apply.

M. Does h have a relative maximum, minimum, or neither at $x = -1$? Justify your answer.

rel max because $h'(-1)=0$ and $h''(-1)<0$ (2nd deriv test)

N. Find $\lim_{x\to 2^-} f'(x)$.

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O. Give two x-values where the graph of h is above the x-axis and h is decreasing at an increasing rate.

$x=-3$ and $x=3$ ($x=8$ also works)

P. Let q be a function such that $q'(x) = g(x)$. At which x-values does the graph of q have a point of inflection? Explain.

$q''(x) = g'(x)$ $q''=g'$ changes signs when $g'(x)=0$

$g'(x) = 6x-5$ for $x \le 2$

$g'(x) > 0$ for $x>2$

$6x-5=0 \Rightarrow x=\frac{5}{6}$

Q. Let $w(x) = \int_{-2}^{x} j(t)dt$.Find $w'(0)$.

$w'(0) = j(0) = 4e^0 - \frac{5}{2}(0) = 4$

R. Let $r(x) = \cos(\pi x) \cdot f(x)$. Find $r'(4.5)$.

$r'(x) = \cos(\pi x) \cdot f'(x) + f(x) \cdot -\pi \sin(\pi x)$

$r'(4.5) = \cos\left(\frac{9\pi}{2}\right) \cdot f'(4.5) + f(4.5) \cdot \left(-\pi \sin\left(\frac{9\pi}{2}\right)\right) = 0 + -0.5(-\pi \cdot 1)$

$= \boxed{\frac{\pi}{2}}$

S. For $x \le 2$, find $\lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$.

For $x \le 2$, $g'(x) = 6x-5$

T. Would the line tangent to the graph of h at $x = 5$ give an under- or over-approximation for $h(5.1)$? Explain.

underapproximation; the graph of h is concave up at $x=5$ so the tangent line lies below the curve at $x=5$.

U. Find $\int_1^3 f'(2x)\,dx$.

$$\frac{1}{2}f(2x)\Big]_1^3 = \frac{1}{2}\left(f(6) - f(2)\right) = \frac{1}{2}(1-2) = \frac{-1}{2}$$

V. Let $w(x) = \int_{-2}^{x} j(t)dt$. Find $w(0)$.

$$w(0) = \int_{-2}^{0}\left(4e^{3x} - \frac{5}{2}x\right)dx$$

$$= \frac{4}{3}e^{3x} - \frac{5x^2}{4}\Big]_{-2}^{0} = \frac{4}{3} - \left(\frac{4}{3}e^{-6} - 5\right)$$