

Vector, Polar, and Parametric Functions

$x(t)$  and  $y(t)$  are  
parametric functions

$$\frac{dy}{dx} =$$



Vector, Polar, and Parametric Functions

$x(t)$  and  $y(t)$  are  
parametric functions

$$\frac{d^2y}{dx^2} =$$



Vector, Polar, and Parametric Functions

Velocity vector  
if position vector is  
 $\langle x(t), y(t) \rangle$



Vector, Polar, and Parametric Functions

Acceleration vector  
if position vector is  
 $\langle x(t), y(t) \rangle$



Vector, Polar, and Parametric Functions

Speed  
if position vector is  
 $\langle x(t), y(t) \rangle$



Vector, Polar, and Parametric Functions

Given an initial position  $x(a)$   
and a horizontal velocity  $x'(t)$ ,  
 $x(b) =$



Vector, Polar, and Parametric Functions

Given an initial position  $y(c)$   
and a vertical velocity  $y'(t)$ ,  
 $y(d) =$



Vector, Polar, and Parametric Functions

Total distance traveled  
from  $t = a$  to  $t = b$   
(parametric)



Vector, Polar, and Parametric Functions

Arc Length of a curve  $f$   
on  $[a, b]$   
(rectangular)



Vector, Polar, and Parametric Functions

Arc Length of a  
parametric curve  $\begin{cases} x(t) \\ y(t) \end{cases}$   
from  $t = a$  to  $t = b$



$$\frac{\frac{d}{dx} \left( \frac{dy}{dx} \right)}{\frac{dx}{dt}}$$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$a(t) = \langle x''(t), y''(t) \rangle$$

$$v(t) = \langle x'(t), y'(t) \rangle$$

$$x(b) = x(a) + \int_a^b x'(t) dt$$

$$\sqrt{(x'(t))^2 + (y'(t))^2}$$

$$\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

$$y(d) = y(c) + \int_c^d y'(t) dt$$

$$\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

$$\int_a^b \sqrt{1 + (f'(x))^2} dx$$

Vector, Polar, and Parametric Functions

Slope of tangent line  
to a polar function  $r(\theta)$



Vector, Polar, and Parametric Functions

Polar Area  
(one function)



Vector, Polar, and Parametric Functions

Polar Area  
(two functions)



Infinite Series

Sum of a  
Geometric Series



Infinite Series

Converge or diverge?

$$\sum_{n=1}^{\infty} \frac{1}{n}$$



Infinite Series

Converge or diverge?

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$



Infinite Series

Converge or diverge?

$$\sum_{n=1}^{\infty} ar^n$$



Infinite Series

Nth Term Test for

$$\sum_{n=1}^{\infty} a_n$$



Infinite Series

Limit Comparison Test

for  $\sum_{n=1}^{\infty} a_n$   
(compare to  $\sum_{n=1}^{\infty} b_n$ )



Infinite Series

Ratio Test for

$$\sum_{n=1}^{\infty} a_n$$



$$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} (r(\theta))^2 d\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

(Use  $x = r \cos \theta$  and  $y = r \sin \theta$  to convert first)

$$S = \frac{1}{1-r}$$

$$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} (r_{outer}(\theta))^2 - (r_{inner}(\theta))^2 d\theta$$

Converges if  $p > 1$   
Diverges if  $p \leq 1$   
P-series

Diverges Harmonic series

If  $\lim_{n \rightarrow \infty} a_n \neq 0$ ,  
the series diverges

Converges if  $|r| < 1$   
Geometric series

If  $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} < 1$ ,  
the series converges

If  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n}$  is a finite, positive  
value, both series converge  
or both diverge

Infinite Series

Alternating Series Test  
for  $\sum_{n=1}^{\infty} a_n$



Infinite Series

Absolute  
Convergence



Infinite Series

Conditional  
Convergence



Infinite Series

Nth term  
of a Taylor Polynomial  
centered at  $x = a$



Infinite Series

Power Series  
for  $\sin x$



Infinite Series

Power Series  
for  $\cos x$



Infinite Series

Power Series  
for  $e^x$



Integration Techniques

Integration by Parts

$$\int u dv =$$



Integration Techniques

Improper Integrals



Integration Techniques

Which method would you use?

$$\int x \sin x dx$$



<p>Both <math>\sum_{n=1}^{\infty}  a_n </math> and <math>\sum_{n=1}^{\infty} a_n</math> converge</p>	<p>If the series is alternating and <math>\lim_{n \rightarrow \infty} a_n = 0</math>, the series converges</p>
$\frac{f^n(a)}{n!} (x - a)^n$	<p>Either <math>\sum_{n=1}^{\infty}  a_n </math> or <math>\sum_{n=1}^{\infty} a_n</math> converges</p>
$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + \frac{(-1)^n x^{2n}}{(2n)!}$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!}$
$uv - \int v du$	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$
<p>Integration by Parts</p>	$\int_a^{\infty} f(x) dx = \lim_{b \rightarrow \infty} \int_a^b f(x) dx$

Integration Techniques

Which method would you use?

$$\int_0^2 \frac{1}{x} dx$$



Integration Techniques

Which method would you use?

$$\int \frac{3x}{x^2 - x - 12} dx$$



Integration Techniques

Which method would you use?

$$\int \ln x dx$$



Integration Techniques

Which method would you use?

$$\int_2^{\infty} \frac{3}{x^2} dx$$



Integration Techniques

Which method would you use?

$$\int x^4 e^{2x} dx$$



Integration Techniques

Which method would you use?

$$\int \frac{1}{(2x - 1)^2} dx$$



Partial Fractions	Improper Integral
Improper Integral	Integration by Parts
U-Substitution	Integration by Parts